Probability & Statistics Assignment

Q1.

(1) 4, 7, ~~8~~9, 9, 11, 15, 18, 18, 18, 25

mean $\Rightarrow \bar{x} = \frac{\Sigma x}{n} = \frac{134}{10} = 13.4$

median $\Rightarrow \frac{\left(\frac{n}{2}\right)^{th} \text{term} + \left(\frac{n+1}{2}\right)^{th} \text{term}}{2} = \frac{11+15}{2} = 13$

mode $\Rightarrow$ 18

standard deviation $\Rightarrow$

x	$(x-\bar{x})$	$(x-\bar{x})^2$
4	-9.4	88.36
7	-6.4	40.96
9	-4.4	19.36
9	-4.4	19.36
11	-2.4	5.76
15	1.6	2.56
18	4.6	21.16
18	4.6	21.16
18	4.6	21.16
25	11.6	134.56
		374.4

$S.D = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{374.4}{10}}$

$S.D. = \sqrt{37.44}$

$SD = 6.12$

Inter quartile range $\Rightarrow$

$$Q_1 = \frac{2^{nd}\text{ term} + 3^{rd}\text{ term}}{2} = \frac{7+9}{2} = 8$$

$$Q_3 = \frac{7^{th}\text{ term} + 8\text{th term}}{2} = \frac{18+18}{2} = 18$$

$$I.Q.R \Rightarrow Q_3 - Q_1 = 18 - 8$$
$$\Rightarrow 10$$

(ii)

no. of household	0	1	2	3	4	(x)
no. of people	5	11	26	15	3	$= 60$ (f)
$f \cdot x$	0	11	52	45	12	$= 120$
$x - \bar{x}$	-2	-1	0	1	2	
$(x-\bar{x})^2$	4	1	0	1	4	$= 10$
$f(x-\bar{x})^2$	20	11	0	15	12	$= 58$

$$\text{mean} \Rightarrow \bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{120}{60} = 2$$

$$\text{median} \Rightarrow \frac{\left(\frac{60}{2}\right)\text{th term} + \left(\frac{60}{2}+1\right)^{th}\text{ term}}{2}$$

$$= 30th + 31st = \frac{2+2}{2}$$

$$= 2$$

mode $\Rightarrow$ 2.

$$S.D = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\frac{58}{60}} = \sqrt{0.9667}$$

SD = 0.98320903

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③

I.Q.R. $\Rightarrow$

$Q_1 = 15^{th}$ term $= 1$
$Q_3 = 45^{th}$ term $= 3$

$IQR = Q_3 - Q_1 = 3-1$
$I.Q.R = 2$

Q2. $\lambda = 0.5$

(i) $\because$ Poisson distribution is used,

$P(\text{no claim}) = P(X=0)$

$$P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$\therefore P(X=0) = \frac{(0.5)^0 e^{-0.5}}{0!} = e^{-0.5}$$

$P(X=0) = 0.60653066$

(ii) Given conditions make it a binomial distribution,

$\therefore$ success $\Rightarrow$ one or more claims
$\therefore$ failure $\Rightarrow$ no claim

$P(\text{success}) = P(X>0) \Rightarrow 1 - P(X=0)$
$= 1 - 0.60653066$
$= 0.39346934$

$P(\text{failure}) = P(X=0) = 0.60653066$ [from (i)]

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$$P(X=x) = {}^{n}C_{x}\, p^{x} q^{n-x}$$

$$\therefore P(x=1) = {}^{3}C_{1}\,(0.39346934)^{1}\,(0.60653066)^{2}$$

$$= 3 \times (0.39346934)\,(0.367879441\,5)$$

$$P(X=1) = 0.434247843\,3$$

(III) given condition makes this exponential distribution,

let time $= t$

$$P(x>2) = 1 - P(x \leq 2) = 1 - F(2)$$

$$= 1 - (1 - e^{-\lambda x})$$

$$= e^{-0.5 \times 2} = e^{-1}$$

$$P(x>2) = 0.36787944$$

Q3.] $\alpha = 2$, $\lambda = 1/2 = 0.5$, $0 < x < \infty$

(I) $\mu = \dfrac{\alpha}{\lambda}$

$\mu = \dfrac{2}{0.5} = 4$ (mean)

$\sigma^2 = \dfrac{\alpha}{\lambda^2}$

$\therefore \sigma = \sqrt{\dfrac{\alpha}{\lambda^2}} = \dfrac{\sqrt{\alpha}}{\lambda}$

$\sigma = \dfrac{\sqrt{2}}{0.5}$

$\sigma = 2\sqrt{2} = 2.818$

(II) skewness $= \dfrac{2}{\sqrt{\alpha}} = \dfrac{2}{\sqrt{2}} = \sqrt{2}$

$= 1.414$

$\therefore$ It is positive skewed data

$\therefore$ Graph will ~~show~~ be right skewed.

(III) $f_X(x) = \dfrac{\lambda^{\alpha}}{\Gamma(\alpha)} \cdot x^{\alpha-1} \cdot e^{-\lambda x}$

$= \dfrac{(1/2)^2}{(2-1)!} x^{2-1} \cdot e^{-x/2}$

$= \dfrac{1}{4} x e^{-x/2}$

$F_X(x) = \dfrac{1}{4} \displaystyle\int_0^x x e^{-x/2} \, dx = \dfrac{1}{4} \left[x e^{-x/2}(-2) + 2e^{-x/2} \right.$

$F_X(x) = \dfrac{1}{2} \left(e^{-x/2} - x e^{-x/2} - 1 \right)$

Q4.] male = 10 , female = 8
$\therefore$ total = 18

actuary males = 6 , actuary females = 7

$n(S) = 2$ out of 18
$= {}^{18}C_2 = \frac{18 \times 17}{2}$

~~n(A) = 2 females qualified females - qualified~~
~~= 2 out of 18 out of which 2 females~~
$\cancel{= {}^{7}C_2 = \frac{7 \times 6}{2}}$

$\cancel{P(A) = \frac{n(A)}{n(S)} = \frac{7 \times 6}{18 \times 17} = 7}$
$= {}^{18}C_2$

$n(A) = 2$ females out of 7 qualified
$= {}^{7}C_2 = \frac{7 \times 6}{2}$

$P(A) = \frac{n(A)}{P(A)} = \frac{7 \times 6}{18 \times 17}$

$P(A) = \frac{7}{51}$

Q5.] ~~P(Lvl 1)~~

let $P(\text{level } 1) = P(l_1) \Rightarrow 0.4$

similarly $P(l_2) = 0.5$

$P(l_3) = 0.1$

let P(late by l_1) = ~~P(A) = 0~~ $P(A/l_1) = 0.2$

similarly $P(A/l_2) = 0.4$

$P(A/l_3) = 0.1$

~~P(A/B)~~ =

To find - $P(l_2/A) = \dfrac{P(A/l_2)\cdot P(l_2)}{P(l_1)\cdot P(A/l_1) + P(l_2)\cdot P(A/l_1) + P(l_3)\cdot P(A/l_3)}$

$= \dfrac{(0.5)(0.4)}{(0.4)(0.2) + (0.5)(0.4) + (0.1)(0.1)}$

$= \dfrac{0.2}{0.08 + 0.2 + 0.01} = \dfrac{0.2}{0.29}$

$P(l_2/A) = 0.689655172$

Q6.] $X \sim U(1,2)$ & $y = \dfrac{3}{6-x}$

$\therefore \alpha = 1, \beta = 2$ $\therefore x = 6 - \dfrac{3}{y}$

$f_X(x) = \dfrac{1}{\beta - \alpha} = \dfrac{1}{2-1} = 1$

$\dfrac{dx}{dy} = \dfrac{d}{dy}\left(6 - \dfrac{3}{y}\right)$

$\dfrac{dx}{dy} = \dfrac{3}{y^2}$

$$f_Y(y) = f_X^{\,x}\left[\frac{dx}{dy}\right]$$

$$f_Y(y) = \frac{3}{y^2}$$

Q7.] (i) $F(3)$

$$F(3) = P(X \leq 3)$$
$$= P(X=1) + P(X=2) + P(X=3)$$
$$= 0.05 + 0.15 + 0.35$$
$$F(3) = 0.55$$

(ii) $E(X)$

$$E(X) = \Sigma p_i x_i$$
$$= 1(0.05) + 2(0.15) + 3(0.35) + 4(0.4) + 5(0.05)$$
$$E(X) = 3.25.$$

(iii) for $V(X)$, we need to find $E(X^2)$

$$\therefore E(X^2) = \Sigma p_i x_i^2$$
$$= 1(0.05) + 4(0.15) + 9(0.35) + 16(0.4) + 25(0.05)$$
$$E(X^2) = 11.45$$

$$V(X) = E(X^2) - [E(X)]^2$$
$$= 11.45 - (3.25)^2$$
$$= 11.45 - 10.5625$$
$$V(X) = 0.8875$$

Q8.] let success = claim of more than 1000

$P(\text{sucess}) = 0.2 = p$

$P(\text{failure}) = 0.8 = q$

$n = 15$

$P(X<3) = P(X=0) + P(X=1) + P(X=2)$

$P(X=x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$

$\therefore P(X<3) = {}^{15}C_0 (0.2)^0 (0.8)^{15} + {}^{15}C_1 (0.2)(0.8)^{14} + {}^{15}C_2 (0.2)^2 (0.8)^{13}$

$P(X<3) = 0.035184372 + 0.131941395 + 0.230897442$

$P(X<3) = 0.398023209$

Q9.] let success = winning

$\therefore p = 0.01,\ q = 0.99$

$n = 7$

$P(X=x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$

$P(X=3) = {}^{7}C_{3} (0.01)^3 (0.99)^4$

$= \frac{7\times6\times5}{3\times2} (0.01)^3 (0.99)^4$

$P(X=3) = 0.000033620 8$

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Q10] $\lambda = \frac{2}{5} = 0.4$

~~P(X>2)~~

$P(X>2) = 1 - P(X \leq 2)$

$= 1 - [P(X=0) + P(X=1) + P(X=2)]$

$\rightarrow \therefore P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}$

$\rightarrow \therefore P(X=0) = \frac{\lambda^0 e^{-0.4}}{0!} = e^{-0.4}$

$\rightarrow P(X=0) = 0.670320046$

$\rightarrow P(X=1) = \frac{(0.4)^1 \cdot e^{-0.4}}{1} = 0.4e^{-0.4}$

$P(X=1) = 0.268128018$

$\rightarrow P(X=2) = \frac{(0.4)^2 \cdot e^{-0.4}}{2} = 0.08 \cdot e^{-0.4}$

$= 0.053625604$

$\therefore P(X \leq 2) = 0.670320046$
$+ 0.268128018$
$+ 0.053625604$

$P(X \leq 2) = 0.992073668$

$\therefore P(X>2) = 1 - 0.992073668$

$P(X>2) = 0.007926332$

Q11.] $P(B) = P(G) = 1/2$

let A be ~~the~~ ~~outcome~~ event of 3 boys.

$\therefore P(A) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

Q12.] $\lambda = 10$, $\alpha = 1$

$\therefore$ it is an exponential distribution ($\because \alpha = 1$)

let time $(t) = 0.1$ day

we need less than 0.1 days

$\therefore P(T < 0.1)$

$$P(T<0.1) = F(0.1)$$
$$= 1 - e^{-\lambda t} = 1 - e^{-10 \times 0.1}$$
$$= 1 - e^{-1}$$
$$= 1 - (0.367879441)$$
$$P(T<0.1) = 0.632120559$$

Q13.] $X \sim U(75, 120)$ given $\alpha = 75$, $\beta = 120$

$$f_X(x) = \frac{1}{120-75} = \frac{1}{45}$$

$$P(80 < X < 100) = \frac{100-80}{120-75}$$

$$P(80 < X < 100) = \frac{20}{45} = \frac{4}{9}$$

Q15] $f_X(x) = \dfrac{k}{(x+5)^4}$, $x>0$

(i) k

$$\int_0^\infty f_X(x) = 1$$

$$\int_0^\infty \frac{k}{(x+5)^4}\, dx = 1$$

$$k\int_0^\infty \frac{1}{(x+5)^4}\, dx = 1 \Rightarrow k\int_0^\infty (x+5)^{-4}\, dx = 1$$

$$k\left[\frac{(x+5)^{-3}}{-3}\right]_0^\infty = 1$$

$$-\frac{3}{k} = \left[\frac{1}{(x+5)^3}\right]_0^\infty$$

$$-\frac{3}{k} = \left[\frac{1}{\infty} - \frac{1}{(5)^3}\right] \Rightarrow \cancel{-}\frac{3}{k} = \cancel{-}\frac{1}{125}$$

$\therefore k = 3 \times 125$

$k = 375$

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(ii) $F(10) = P(X \le 10)$

$$= \int_0^{10} \frac{375}{(x+5)^4}\, dx$$

$$= -\frac{375}{3}\left[\frac{1}{(x+5)^3}\right]_0^{10}$$

$$= -125 \cdot \left[\frac{1}{15^3} - \frac{1}{5^3}\right]$$

$$= -125\left[\frac{1}{3375} - \frac{1}{125}\right]$$

$$= -\frac{125}{125}\left[\frac{1}{27} - 1\right]$$

$$= -1\left[-0.96296296\right]$$

$$F(10) = 0.96296296$$

(iii) $E(X) = \int_0^{\infty} x \cdot f_X(x)\, dx$

$$= 375 \int_0^{\infty} \frac{x}{(x+5)^4}\, dx$$

$$= 375\left[\int_0^{\infty} \frac{x+5}{(x+5)^4} - \frac{5}{(x+5)^4}\, dx\right]$$

$$= 375\left[\int_0^{\infty} \frac{1}{(x+5)^3}\, dx - 5\int_0^{\infty} \frac{1}{(x+5)^4}\, dx\right]$$

$$= 375\left[\left\{\frac{1}{-2(x+5)^2}\right\}_0^{\infty} - 5\left\{\frac{1}{-3(x+5)^3}\right\}_0^{\infty}\right]$$

$$= 375\left[\frac{1}{50} - 5\left(\frac{1}{375}\right)\right]$$

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$$E(x) = 375 \left[\frac{1}{50} - \frac{5}{375} \right]$$

$$= \frac{375}{50} - 5 = \frac{75}{10} - 5$$

$$= 7.5 - 5$$

$$E(x) = 2.5$$