

Q1) $\lim_{h \rightarrow 0} \frac{2(-3+h)^2 - 18}{h}$

$$\lim_{h \rightarrow 0} \frac{2(h^2 + 9 - 6h) - 18}{h}$$

$$\lim_{h \rightarrow 0} \frac{18 + 2h^2 - 12h - 18}{h}$$

$$\lim_{h \rightarrow 0} \frac{h(2h - 12)}{h}$$

$$\boxed{-12}$$

Q2) $g(x) = \begin{cases} 2x & x < 6 \\ x-1 & x \geq 6 \end{cases}$

a) $x=4$

$$\lim_{x \rightarrow 4^+} 2(4+h) = 8$$

$$\lim_{x \rightarrow 4^-} 2(4-h) = 8$$

$$\lim_{x \rightarrow 4} 2(4) = 8$$

Hence continuous

b) $x=6$

$$\lim_{x \rightarrow 6^+} 6 + h - 1$$

$$= 5$$

$$\lim_{x \rightarrow 6^-} 2(6-h)$$

$$= 12$$

~~lim~~ $x \rightarrow 6$ Hence discontinuous

Q3)

$$\lim_{x \rightarrow 9} \frac{x-9}{3-\sqrt{x}}$$

$$\lim_{x \rightarrow 9} \frac{(\sqrt{x})^2 - 3^2}{3 - \sqrt{x}}$$

$$\lim_{x \rightarrow 9} \frac{(\sqrt{x} + 3)(\sqrt{x} - 3)}{-(\sqrt{x} - 3)} = \lim_{x \rightarrow 9} -(\sqrt{x} + 3)$$

$$= -(3+3) = \textcircled{+6}$$

$$-(-3+3) = \textcircled{0}$$

$$\begin{aligned} \text{Q2)} \quad f(x) &= \frac{4x+5}{9-3x} \\ \lim_{x \rightarrow -1^+} & \quad \frac{4(x+h)+5}{9-3(x+1)} \\ & \quad \frac{4(-1)+5}{9-3(-1)} \end{aligned}$$

$$\text{Q4)} \quad f(x) = \frac{4x+5}{9-3x}$$

$$\text{a)} \quad \lim_{x \rightarrow -1} \frac{4(x)+5}{9-3(x)}$$

$$= \frac{-4+5}{9+3}$$

$$= \frac{1}{12}$$

$$\text{b)} \quad \lim_{x \rightarrow 0} \frac{4(x)+5}{9-3x}$$

$$= \frac{0+5}{9-0}$$

$$= \frac{5}{9}$$

$$\text{c)} \quad \lim_{x \rightarrow 3} \frac{4x+5}{9-3x}$$

$$\frac{4(3)+5}{9-9}$$

Undefined

Q.5) $\lim_{x \rightarrow \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$

$$\lim_{x \rightarrow \infty} \frac{e^{4x}}{e^{4x}} \left(\frac{6 - e^{-6x}}{8 - e^{-2x} + 3e^{-5x}} \right)$$

$\frac{6}{8} = \frac{3}{4}$

Q.6) $g(y) = \begin{cases} y^2 + 5 & \text{if } y < -2 \\ 1 - 3y & \text{if } y \geq -2 \end{cases}$

a) $\lim_{y \rightarrow 6} g(y)$

$$\lim_{y \rightarrow 6} 1 - 3(y) = 1 - 3(6) = -17$$

b) $\lim_{x \rightarrow -2} g(x)$

$$\lim_{x \rightarrow -2^+} 1 - 3(-x + h) = 1 - 3(-2 + 0) = 7$$

$$\lim_{x \rightarrow -2^-} 1 - 3(x - h)^2 + 5 = 1 - 3(0)^2 + 5 = 5$$

$5 \neq 7$

Not continuous

$$Q7) \quad F(t) = (4t^2 - t) \overset{\text{diff w.r.t } t}{(t^3 - 8t^2 + 12)}$$

$$F'(t) = (4t^2 - t)(3t^2 - 16t) + (t^3 - 8t^2 + 12)(8t - 1)$$

$$= 12t^4 - 64t^3 - 3t^3 + 16t^2 + 8t^4 - 64t^3 + 96t - t^3 + 8t^2 - 12$$

$$= 20t^4 - 132t^3 + 24t^2 + 96t - 12$$

$$Q8) \quad f(w) = \frac{3w + w^4}{2w^2 + 1}$$

diff w.r.t w

$$= \frac{(2w^2 + 1)(3 + 4w^3) - (3w + w^4)(4w)}{(2w^2 + 1)^2}$$

$$= \frac{6w^2 + 6w^5 + 3 + 4w^3 - 12w^2 - 4w^5}{(2w^2 + 1)^2}$$

$$= \frac{2w^5 - 6w^2 + 4w^3 + 3}{(2w^2 + 1)^2}$$

$$= \frac{2w^5 - 6w^2 + 4w^3 + 3}{4w^4 + 1 + 4w^2}$$

09) $f(x) = 2e^x - 8^x$
diff. wert x

$$f'(x) = 2e^x - 8^x \log 8$$

010) $y = z^5 - e^z \ln(z)$

diff wert z

$$y' = 5z^4 - e^z \ln(z) - \frac{e^z}{z}$$

011) a) $f(x) = (6x^2 + 7x)^4$
diff. wert x

$$f'(x) = 4(6x^2 + 7x)^3 (12x + 7)$$

b) $f(t) = 5 + e^{4t+t^2}$
diff wert t

$$f'(t) = 0 + e^{4t+t^2} (4 + 2t)$$

012) a) $z = \ln(7 - x^3)$
diff wert x

$$z' = \frac{-3x^2}{7 - x^3}$$

$$= \frac{(7-x^3)(-6x) - (-3x^2)(7-x^3)}{(7-x^3)^2} - \frac{-6x + 3x^2}{7-x^3}$$

$$z'' = \frac{-42x + 6x^4}{(7-x^3)^2} - \frac{-3x^2}{7-x^3} = \frac{-42x - 3x^4}{(7-x^3)^2}$$

b) $q(v) = \frac{2}{(6+2v-v^2)^4}$

diff. w.r.t v

$$q'(v) = \frac{(6+2v-v^2)(0) - (2)(4)(6+2v-v^2)^3(2-v)}{(6+2v-v^2)^8}$$

$$= -16(1-v)(6+2v-v^2)^{-5}$$

diff. w.r.t v

$$q''(v) = -16[(1-v)(5)(6+2v-v^2)^{-6}(2-v) + (-1)(6+2v-v^2)^{-5}]$$

$$= (716)(6+2v-v^2)^{-5} - (1-v)(2-v)(5)(6+2v-v^2)^{-6}$$

$$q''(v) = (16)(6+2v-v^2)^{-5} - (1-v)(2-v)(5)(6+2v-v^2)^{-6}$$

8)

$$P(t) = \ln(1+t^2)$$

diff. wert t

$$P'(t) = \frac{2t}{1+t^2}$$

$$= \frac{(1+t^2)(2) - (2t)(2t)}{(1+t^2)^2}$$

diff. wert t

$$P''(t) = \frac{2+2t^2-4t^2}{(1+t^2)^2} = \boxed{\frac{2-2t^2}{(1+t^2)^2}}$$

13)

$$f(x) = x^3 - 3x^2 - 9x + 12$$

$$f'(x) = 3x^2 - 6x - 9$$

$$3(x^2 - 2x - 3) = 0$$

$$x^2 - 3x + x - 3 = 0$$

$$x(x-3) + 1(x-3) = 0$$

$$(x-3)(x+1) = 0$$

$$\boxed{x = -1}$$

$$\boxed{x = 3}$$

$$f''(x) = 6x - 6$$

$$\textcircled{x = -1} \rightarrow \text{max}$$

$$f''(-1) = 6(-1) - 6 = -12 \rightarrow \text{max}$$

$$f''(3) = 6(3) - 6 = 12 \rightarrow \text{min}$$

$$f(-1) = 17$$

$$f(3)$$

Q14)

$$f(x) = 4x^3 - 18x^2 + 24x - 7$$

$$f'(x) = 12x^2 - 36x + 24$$

$$12(x^2 - 3x + 2) = 0$$

$$x^2 - 2x - x + 2 = 0$$

$$x(x-2) - 1(x-2)$$

$$(x-1)(x-2)$$

$$x=1$$

$$x=2$$

$$f''(x) = 12(2x-3)$$

$$= 12(2(1)-3) = -12$$

$$f''(2) = 12(2(2)-3) = 12$$

$$f(1) = 3$$

$$f(2) = \text{EBO} 1$$

Q15) $f(x) = x^2(x+3)$
 $= x^3 + 3x^2$

diff w.r.t x

$$f'(x) = 3x^2 + 6x$$

$$\text{let } f'(x) = 0$$

$$3x(x+2) = 0$$

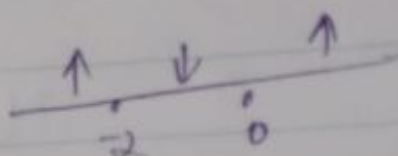
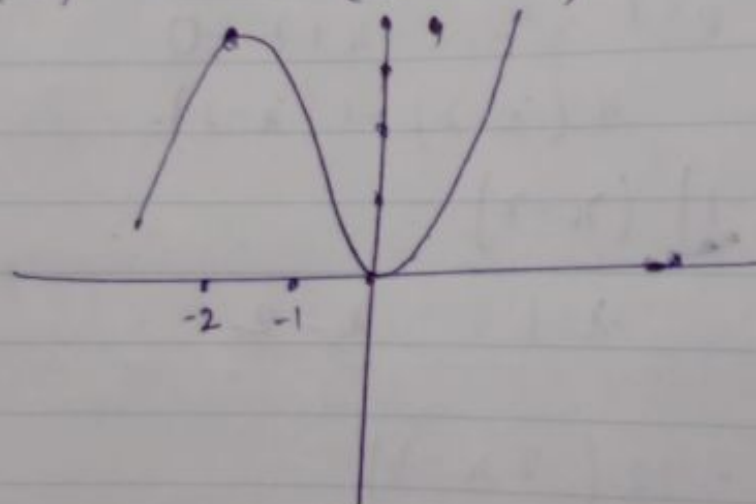
$$x = -2$$

$$x = 0$$

$$f''(x) = 6x + 6$$

$$f''(0) = 6 \text{ (minimum)}$$

$$f''(-2) = -6 \text{ (maximum)}$$



Q16)

$$f(x) = 3x^2 - 6x$$

diff w.r.t x

$$f'(x) = 6x - 6$$

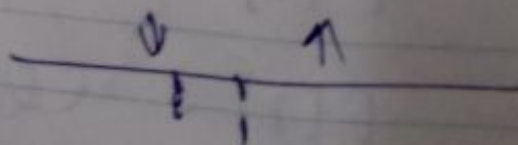
$$f'(x) = 0$$

$$6(x-1)$$

$$x = 1$$

$$f(1) = 3 - 6 = -3$$

$$f(0) = 0$$



$$f''(x) = 6 \text{ (minimum)}$$

