Mucharla Harsha Vardhan
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Assignment-2

1) Given,

$$\int_{\frac{1}{50}}^{2} \frac{1}{w^2} e^{\frac{2}{w}} dw = \int_{1/50}^{2} w^{-2} e^{2w^{-1}} dw$$

$$= -\frac{1}{2} \int_{1/50}^{2} -2w^{-2} e^{2w^{-1}} dw$$

It is in the form $\int f'(x) e^{f(x)} dx = e^{f(x)}$.

$$\therefore \int_{1/50}^{2} \frac{1}{w^2} e^{2/w} dw = -\frac{1}{2} \left[e^{2/w} \right]_{1/50}^{2}$$

$$= -\frac{1}{2} \left[e^{1} - e^{100} \right]$$

2) Given,

$$\int \frac{2t^3+1}{(t^4+2t)^3}\,dt$$

$$= \frac{1}{2}\int \frac{4t^3+2}{(t^4+2t)^3}\,dt$$

Let $u = (t^4+2t)^3$. Then,

$$\frac{du}{dt} = 3(t^4+2t)^2(4t^3+2)$$

$$\Rightarrow dt = \frac{du}{3(t^4+2t)^2(4t^3+2)}$$

Therefore,

$$\int \frac{2t^3+1}{(t^4+2t)^3}\,dt = \frac{1}{2}\int \frac{(4t^3+2)\,du}{3(t^4+2t)^2(4t^3+2)\,u}$$

$$= \frac{1}{2}\int \frac{du}{3\,u^{\frac{2}{3}}\cdot u}$$

$$= \frac{1}{6}\int \frac{du}{u^{\frac{5}{3}}}$$

$$= \frac{1}{6}\left[\frac{u^{-\frac{5}{3}+1}}{-5/3+1}\right] + C$$

$$= \frac{-1}{4}\, u^{2/3} + C$$

$$= \frac{-1}{4}\left[(t^4+2t)^3\right]^{-\frac{2}{3}} + C$$

$$= \frac{-1}{4(t^4+2t)^2} + C$$

3) Given,

$$f'(x) = x^4 + 3x - 9$$

Integrating on both sides we have,

$$f(x) = \int (x^4 + 3x - 9)\,dx$$

$$= \left[\frac{x^5}{5}\right] + 3\left[\frac{x^2}{2}\right] - 9\,[x] + C$$

$$= \frac{x^5}{5} + \frac{3}{2}x^2 - 9x + C$$

4) Given,

$$f(x) = \begin{cases} 6 & ; \quad x > 1 \\ 3x^2 & ; \quad x \le 1 \end{cases}$$

To find,

$$\int_{-2}^{3} f(x)\,dx = \int_{-2}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx$$

$$= \int_{-2}^{1} 3x^2\,dx + \int_{1}^{3} 6\,dx$$

$$= 3\left[\frac{x^3}{3}\right]_{-2}^{1} + 6\,[x]_{1}^{3}$$

$$= (1-(-8)) + 6(3-1)$$

$$= 9 + 12$$

$$= 21$$

5) To solve,

$$\int 3\,(8y-1)\, e^{4y^2-y}\, dy$$

We know that, $\int f'(x) e^{f(x)}\, dx = e^{f(x)} + C.$

Similarly,

$$\int 3\,(8y-1)\, e^{4y^2-y}\, dy = 3\int (8y-1)\, e^{(4y^2-y)}\, dy$$

$$= 3\int \frac{d}{dx}(4y^2-y)\, e^{(4y^2-y)}\, dy$$

$$= 3\, e^{4y^2-y} + C$$

6) Given,

$$f''(x) = 15\, x^{\frac{1}{2}} + 5x^3 + 6$$

By integrating we have,

$$f'(x) = \int (15x^{\frac{1}{2}} + 5x^3 + 6)\, dx$$

$$= 15\left[\frac{x^{3/2}}{3/2}\right] + 5\left[\frac{x^4}{4}\right] + 6\,[x] + C$$

$$= 10\, x^{3/2} + \frac{5}{4}x^4 + 6x + C$$

Again integrating we have,

$$f(x) = \int \left(10x^{3/2} + \frac{5}{4}x^4 + 6x + C\right)dx$$

$$= 10\left[\frac{x^{5/2}}{5/2}\right] + \frac{5}{4}\left[\frac{x^5}{5}\right] + 6\left[\frac{x^2}{2}\right] + C[x] + C_1$$

$$f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 + Cx + C_1$$

Its given that $f(1) = \frac{-5}{4}$ i.e.

$$\Rightarrow \quad 4(1)^{5/2} + \frac{(1)^5}{4} + 3(1)^2 + C(1) + C_1 = \frac{-5}{4}$$

$$\Rightarrow \quad 4 + \frac{1}{4} + 3 + C + C_1 = \frac{-5}{4}$$

$$\Rightarrow \quad C_1 + C = -\frac{17}{2} = -8.5 \quad \text{—(1)}$$

Also given that $f(4) = 404$. i.e,

$$\Rightarrow \quad 4(4)^{5/2} + \frac{(4)^5}{4} + 3(4)^2 + C(4) + C_1 = 404$$

$$\Rightarrow \quad 4C + C_1 = \cancel{1\,0\,9}\,-28 \quad \text{—(2)}$$

By solving ① and ② we have,

$$C = -6.5,$$

$$C_1 = -2.$$

$$\therefore \; f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 - 6.5x - 2$$

8) Given,

$$\frac{dr}{d\theta} = \frac{r^2}{\theta}$$

$$\Rightarrow \frac{dr}{r^2} = \frac{d\theta}{\theta}$$

By integration we have,

$$\frac{r^{-3}}{-3} = \log\theta + C$$

$$\Rightarrow r^{-3} = -3\log\theta - 3C$$

$$\Rightarrow r = (-3\log\theta - 3C)^{-\frac{1}{3}}$$

Given, $r(1) = 2$ i.e, $(-3\log(1) - 3C)^{-\frac{1}{3}} = 2$

$$\Rightarrow -3C = \frac{1}{8} \Rightarrow C = -\frac{1}{24}$$

$$\therefore \; r^{-3} + 3\log\theta - \frac{1}{8} = 0$$

9) Given,

$$\frac{dv}{dt} = 9.8 - 0.196\,V$$

$$\Rightarrow \frac{dv}{9.8 - 0.196v} = 1\,dt$$

Integrating both sides, we have,

$$\Rightarrow \frac{\log(9.8 - 0.196v)}{-0.196} = t + C$$

$$\Rightarrow -0.196t + \log(9.8 - 0.196v) - C = 0$$

12) Given,

$f^{(n)}(x)$	$f^{(n)}(3)$
$f^{(0)}(x) = x^3 - 10x^2 + 6$	$f(3) = -57$
$f'(x) = 3x^2 - 20x$	$f'(3) = -33$
$f''(x) = 6x - 20$	$f''(3) = -2$
$f'''(x) = 6$	$f'''(3) = 6$
$f''''(x) = 0$	$f''''(3) = 0$

We know that,

$$f(x) = f(a) + \frac{(x-a)f'(a)}{1!} + \frac{(x-a)^2 f''(a)}{2!} + \dots$$

$$\Rightarrow f(x) = f(3) + \frac{(x-3)f'(3)}{1} + \frac{(x-3)^2 f''(3)}{2} + \frac{(x-3)^3 f'''(3)}{3!}$$

$$= -57 + (x-3)(-33) + \frac{(x-3)^2(-2)}{2} + \frac{(x-3)^3(6)}{6}$$

$$\Rightarrow x^3 - 10x^2 + 6 = -57 - 33(x-3) - (x-3)^2 + (x-3)^3$$

13) To find maclaurin series for $(1+x)^{\mu}$.

Let $f(x) = (1+x)^{\mu}$. Then,

$$f'(x) = \mu(1+x)^{\mu-1};$$

$$f''(x) = \mu(\mu-1)(1+x)^{\mu-2}$$

$$f'''(x) = \mu(\mu-1)(\mu-2)(1+x)^{\mu-3}$$

$$\vdots \qquad \vdots$$

$$f^{(n)}(x) = {}^{\mu}C_{n} \times (1+x)^{\mu-n}$$

Maclaurin series for $f(x)$ is given by,

$$f(x) = \sum \frac{f^{(n)}(0) \cdot x^{n}}{n!}$$

$$\Rightarrow (1+x)^{\mu} = \sum \frac{{}^{\mu}C_{n} (1+0)^{\mu-n} \cdot x^{n}}{n!}$$

$$\Rightarrow (1+x)^{\mu} = \sum \frac{{}^{\mu}C_{n} \, x^{n}}{n!}$$

iv) To find maclaurin series for $\sqrt{1+x}$

Let $f(x) = \sqrt{1+x}$ then,

$f^{(n)}(x)$	$f^{n}(0)$
$f(x) = \sqrt{1+x}$	$f(0) = 1$
$f'(x) = \frac{1}{2}(1+x)^{-\frac{1}{2}}$	$f'(0) = \frac{1}{2}$
$f''(x) = \frac{-1}{4}(1+x)^{-3/2}$	$f''(0) = \frac{-1}{4}$
$f'''(x) = \frac{-3}{8}(1+x)^{-5/2}$	$f'''(0) = -3/8$

Then, $f(x) = \frac{f(0)}{0!} + \frac{f'(0)\,x^1}{1!} + \frac{f''(0)\,x^2}{2!} + \frac{f'''(0)\,x^3}{3!} + \cdots$

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$$\Rightarrow \sqrt{1+x} = 1 + \frac{\frac{1}{2}x}{1!} + \frac{-\frac{1}{4}x^2}{2!} + \frac{-3/8\,x^3}{3!} + \cdots$$

$$= 1 + \frac{x}{2} - \frac{1}{8}x^2 - \frac{1}{16}x^3 + \cdots$$

15) To find taylor series for $f(x) = e^{-6x}$ at $x = -4$.

Then,

$f^{n}(x)$	$f^{(n)}(-4)$
$f(x) = e^{-6x}$	$f(-4) = e^{24}$
$f'(x) = -6e^{-6x}$	$f'(-4) = -6e^{24}$
$f''(x) = (-6)^2 e^{-6x}$	$f''(-4) = (-6)^2 e^{24}$
$\vdots$	$\vdots$
$f^{(n)}(x) = (-6)^n e^{-6x}$	$f^{(n)}(-4) = (-6)^n e^{24}$

We know that,

$$f(x) = \sum \frac{f^{(n)}(a)(x-a)^n}{n!}$$

$$\Rightarrow e^{-6x} = \sum_{n=0}^{\infty} \frac{(-6)^n e^{-6x} (x+4)^n}{n!}$$

(16) To find taylor series for $f(x) = \ln(3+4x)$ at $x=0$.

Then,

$f^{(n)}(x)$	$f^{(n)}(0)$
$f(x) = \ln(3+4x)$	$f(0) = \ln(3)$
$f'(x) = 4(3+4x)^{-1}$	$f'(0) = 4/3$
$f''(x) = (4)(-4)(3+4x)^{-2}$	$f''(0) = -16 \times 3^{-2}$
$f'''(x) = (8)(4)(4)(3+4x)^{-3}$	$f'''(0) = 8(4)^2(3)^{-3}$

$$f(x) = f(0) + \frac{f'(0)(x-0)}{1!} + \frac{f''(0)(x-0)^2}{2!} + \frac{f'''(0)(x-0)^3}{3!} + \cdots$$

$$\ln(3+4x) = \ln 3 + \frac{4/3\, x}{1} + \frac{-16 \times 3^{-2} x^2}{2} + \frac{8(4)^2(3)^{-3} x^3}{6} + \cdots$$