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Financial Mathematics
Assignment - 2

Q1.i) Loan Amount = 20000
Monthly Payments = 427.9
Time = 5 years

$$20000 = 427.9(12)\, a_{\overline{5}|}^{12} \quad @\ i\%$$

$$\Rightarrow \quad 20000 = 427.9(12)\left(\frac{1-(1+i)^{-5}}{(1+i)^{\frac{1}{12}}-1)\,12}\right)$$

At $i = 10\%$ RHS = 20341
$i = 11\%$ RHS = 19916

$\therefore$ By interpolating, $i = 10.8\%$
$\therefore$ APR = 10.8%

ii) $PV_1 = 427.9(12)\, a_{\overline{47}|}^{(12)} \quad @\ i = 10.8\%$

$$= 427.9(12)\left(\frac{1-(1.108)^{-4}}{((1.108)^{1/12}-1)12}\right)$$

$$= 427.9(39.205369) = 16775.97778$$

$$16775.97778 = 274.49(12)\, a_{\overline{t}|}^{(12)} \quad @\ 10.8\%$$

$$\frac{16775.97728}{274.49} = \frac{(1-(1.108)^{-t})(12)}{((1.108)^{1/12}-1)\,12}$$

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$$1-(1.108)^{-t} = 0.52456676$$
$$-t = -7.2499$$
$$t = 7.2499$$

iii) Total interest of initial loan $\Rightarrow 427.9 \times 48 - 16775.97728$
$\Rightarrow 3763.22$

Total interest of restructed loan $\Rightarrow 7.2499 \times 274.49 \times 12$
$- 16775.97728$
$= 7104.3233$

∴ They will pay 3341.43 more interest

Q2i) a) Discounted Payback Period
- Discounded Payback Period is the number of years after which the cumulative discounded cash inflows cover the initial investment.

b. Payback Period
- Payback Period is the number of years necessary to recover funds invested in the project without considering the interest, both on the borrowing and investing.

ii) Unlike the NPV, neither the DPP nor the PP give any indication of how profitable a project is, as they ignore cash flows after the accumulated value of zero is reached.

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The PP can give misleading results, as it does not take into account the time value of money.

Hence, NPV is a superior measure as compared to DPP or PP.

iii) a) Project A

PV of outlay = $-170 - 20v - 10v^2$

PV of inlay = $20v + 20v^2 + 200v^3$

$NPV = -170 - 20v - 10v^2 + 20v + 20v^2 + 200v^3$

$0 = -170 + 10v^2 + 200v^3$

IRR $\Rightarrow$ $0 = -170 + 10v^2 + 200v^3$

$170 = 10(1+i)^{-2} + 200(1+i)^{-3}$

At $i = 0.07$, LHS $= 171.99$

$i = 0.08$, LHS $= 167.33$

$\therefore$ By interpolating, $i = 7.4\%$.

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Project B

PV of outlay = -200

PV of inlay = $14 a_{\overline{6}|} + 200 v^6$

$$NPV = -200 + 14\left(\frac{1-(1+i)^{-6}}{i}\right) + 200(1+i)^{-6}$$

IRR $\Rightarrow$ $$0 = -200 + 14\left(\frac{1-(1+i)^{-6}}{i}\right) + 200(1+i)^{-6}$$

$$200 = 14\left(\frac{1-(1+i)^{-6}}{i}\right) + 200(1+i)^{-6}$$

At $i = 0.07$, LHS $= 200$

By interpolating, $i = 7\%$.

b. $NPV_A = -170 + 10v^2 + 200v^3$

At $d = 6\%$.

$NPV_A = -170 + 10(1-0.06)^2 + 200(1-0.06)^3$

$NPV_A = 4.9528$

$NPV_B = -200 + 14 a_{\overline{6}|} + 200 v^6$

$$= -200 + 14\left(\frac{1-(0.94)^6}{0.06}\right)(1(0.94)) + 200(0.94)^6$$

$NPV_B = 5.9958509$

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c. For Project A, the interest rate should be below 7.4%, if above this then the project is unprofitable

For Project B, the interest rate should be below 7% if above this then the project is unprofitable.

Other factors:

- Project A (No net income during FY) - comparision time period
- Lower IRR for Project B for lower period of both projects.

Q3. PV of annuity 1 = $200\,i\,\ddot{a}_{\overline{22}|}\,(v^{3/12})$

$= (1.08)^{-1/4}(200)(10.2007)\dfrac{(0.08)}{(0.074074)}$

$= 2161.365534$

PV of annuity 2 = $320\,i\,\ddot{a}^{(4)}_{\overline{16.25}|}\,v^{2/12}$

$= 320\left(\dfrac{1-(1.08)^{-16.25}}{0.08}\right)(1.049519)(1.08)^{-1/15}$

$= 2996.048870$

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Total PV = 6953.065823

Revised annuity = $6953.065823 = X\ddot{a}^{(2)}_{\overline{21.5}|}\ @\ d^{(2)}$

$$6953.065823 = X\left(\frac{1-(1.08)^{-21.5}}{0.08}\right)(1.059615)$$

$$X = 649.0135657$$

Q4. To derive : $(I\ddot{a})_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{d}$

Proof,

$$PV = 1 + 2v + 3v^2 + \ldots\ldots + nv^{n-1} \quad \text{---} \quad (1)$$

$$(1-d)(PV) = v + 2v^2 + 3v^3 + \ldots\ldots + (n-1)v^{n-1} + nv^n \quad \text{---} \quad (2)$$

(1) − (2)

$$PV - (1-d)PV = 1 + v + v^2 + v^3 + \ldots + v^{n-1} - nv^n$$

$$d(PV) = \ddot{a}_{\overline{n}|} - nv^n$$

$$(I\ddot{a})_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{d}$$

Q5. For property fund

$$1+i = \frac{130.1}{120.4} \times \frac{140.8}{130.1} \times \frac{158}{140.8} \times \frac{164}{158}$$

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TWRR = i = 32.258645 %.

For Equity fund

$$1+i = \frac{92}{12.1} \times \frac{10.3}{9.2} \times \frac{13.1}{10.3} \times \frac{15.5}{13.1}$$

TWRR = i = 28.0.991736 %.

ii)a. $12.4(1+i) + 13.1(1+i)^{3/4} + 14.8(1+i)^{1/2} + 15.8(1+i)^{1/4}$

$= 4(16.4)$

i = 29.32 %.

iii)a) $(12.1)(1+i) + 9.2(1+i)^{3/4} + 10.3(1+i)^{1/2} + 13.1(1+i)^{1/4}$

$= 4 \times 15.5$

i = 67.31 %.

Q6 i) $160000 = X\, a_{\overline{10}|}$ @ 8 %

$$X = \frac{160000}{6.7101} = 23844.65201$$

ii) $PV_4 = 23844.65201\, a_{\overline{6}|}$ @ 8 %

$= 23844.65201\,(4.6229)$

$= 110231.4422$

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$110231.4422 = X^{1} a_{\overline{6}|} @ 10\%$

$X^{1} = \frac{110231.4422}{4.3553} = 25309.72428$

iii) $PV_7 = 25309.72428\, a_{\overline{3}|} @ 10\%.$

$= 25309.72488\ (2.4869)$

$= 62942.75331$

$62943.75331 = X^{11} a_{\overline{3}|} @ 9\%.$

$X^{11} = \frac{62942.75331}{2.5313} = 24865.78174$

Q8 i) PV of outlay $= 2.7 + 0.2\, a_{\overline{3}|}$

PV of inlay $= \bar{a}_{\overline{10}|}(0.25v + 0.2v^2 + 0.1v^3) + 0.1v^3$

$0 = NPV = \bar{a}_{\overline{10}|}(0.25v + 0.2v^2 + 0.1v^3) + 0.1v^3 - 2.7 - 0.2\, a_{\overline{3}|}$

$\therefore 0 = \frac{1-(v^{10})}{-\ln(v)} (0.25v + 0.2v^2 + 0.1v^3) + 0.1v^3 - 2.7 - 0.2 \left(\frac{1-v^3}{v^{-1}-1}\right)$

$i = 9.3\%.$

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ii) NPV $= 0.1v^2 + 0.85\bar{a}_{\overline{10}|}v^3 - 2.7 - 0.2a_{\overline{3}|}$ @ 10%

$= 0.1(1.1)^{-2} + 0.85(1.1)^{-3}\left[\frac{1-(1.1)^{-10}}{\ln(1.1)}\right] - 2.7 - 0.2\left(\frac{1-(1.1)^{-3}}{0.1}\right)$

$= 1.00089$

i) $(1+i)^3 = \frac{33}{30.5} \times \frac{38.5}{33+6} \times \frac{41.05-4.5}{38.5} \times \frac{45}{41.05} \times \frac{45.6}{45} \times \frac{47}{45.6-2.5}$

$(1+i)^3 = 1.228313$

$i = 7.0951\% = TWRR$

$30.5(1+i)^3 + 6(1+i)^{2.25} + 4.5(1+i)^{1.5} - 2.5(1+i)^{0.5} = 47$

At $i = 7\%$, RHS $= 46.74$

$i = 7.5\%$, RHS $= 47.37$

By interpolation, $i = 7.206\% = MWRR$

ii) Linked rate of return

2011 ⇒

$30.5(1+i) + 6(1+i)^{0.25} = 38.5$

$i = 6.266\%$

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2012 ⇒

$$38.5(1+i) + 4.5(1+i)^{0.5} = 45$$

$$i = 4.92\%.$$

2013 ⇒

$$45(1+i) + 2.5(1+i)^{0.5} = 47$$

$$i = 10.276\%.$$

$$(1+i)^3 = 1.06266 \times 1.0492 \times 1.10276$$

$$i = 7.13\%.$$

iii) TWRR eliminates the effect of the cashflow amount and timing and therefore gives a fairer basis for assessing the investment performance for the fund.

MWRR is sensitive to the amount and timing of the net cashflows.

Q10 i) $PV = 180000\, a_{\overline{15}|} + 20000\,(Ia)_{\overline{15}|}$ @ 12%.

$$= 180000(6.8109) + 20000(40.7313)$$

$$= 2040588$$

∴ Total cost of house $= \frac{2040588}{0.8} = 2550735$

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ii) $PV_9 = 340000\, a_{\overline{7}|} + 20000\,(Ia)_{\overline{7}|}$ @ 12%.

$$= 340000(4.5638) + 20000\left(\frac{5.1114 - 7 \times 0.4523}{0.12}\right)$$

$$= 1875850.33$$

Year	Loan	Payment	Interest	Capital Repayment
9	1875850.33	360000	225102.04	134897.96
10	1740952.37	380000	208914.28	171085.72
11	1569866.65			

iii) $1569866.65 = X\, a_{\overline{5}|}$ @ 10%.

$$X = \frac{1569866.65}{3.790786} = 414126.87$$

Total installments less loan outstandings at the end of 10th year = 630133.35

Extra Payment Made = $5(414126.87) + 0.01(1569866.05) - 1569866.65$

$$= 516466.35$$

Q11 i) $TWRR = (1+i)^2 = \frac{500}{460} \times \frac{550-50}{500+40} \times \frac{600}{550} \times \frac{x}{600}$

$$1.44 = \frac{500}{460} \times \frac{500}{540} \times \frac{x}{550}$$

$$x = 786.9312$$

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ii) MWRR $\Rightarrow$ $460(1+i)^2 + 40(1+i)^{21/12} - 50(1+i)$

$= 786.9312$

$i = 30.909\ \%$.

Q12 i) Discounted Payback Period (DPP) is the number of years after which the cumulative discounted cash inflows cover the initial investment.

ii) a. $-800(1+i)^t - 50(1+i)^{t-1} + 100\,\bar{s}_{\overline{t-2}|} = 0$ @ $i = 7\%$.

$$-800(1.07)^t - 50(1.07)^{t-1} + 100\left(\frac{(1.07)^{t-2} - 1}{\ln(1.07)}\right) = 0$$

$\therefore t = 17.767$

b. $100\,\bar{s}_{\overline{4.233}|}$ @ $6\ \%$.

$$= 100\left(\frac{(1.06)^{4.233} - 1}{\ln(1.06)}\right) = 480$$

iii) $-800(1+i)^t - 50(1+i)^{t-1} + 102.971\,s_{\overline{t-2}|}$ @ $7\% = 0$

$\Rightarrow t = 18$

$-800(1+i)^{18} - 50(1+i)^{17} + 102.971\,s_{\overline{16}|}$ @ $7\ \% = 9.7714$

$\Rightarrow 9.7714(1+i)^4 + 100\,\bar{s}_{\overline{4}|}$ @ $6\ \%$.

$= 462.79$ = Accumulated amount after 22 years

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Q13 i) $988000 = X\, a^{(12)}_{\overline{25}|} \;@\; 7\%$

$$988000 = 12 \times \left(\frac{1-(1.07)^{-25}}{(0.07)^{1/12}-1)12} \right)$$

$X = 6848$, Monthly repayment = 6848

ii) $X\, a^{(12)}_{\overline{24/3}|} = 6848 \left(a^{(12)}_{\overline{34/3}|} \;@\; 7\% \right) = 648600$

iii) PV after 10th september's payment $= X\, a^{(12)}_{\overline{83/6}|} \;@\; 7\%$.

PV after 10th October's payment $= X\, a^{(12)}_{\overline{13.75}|} \;@\; 7\%$

Capital repaid $\Rightarrow X\, a^{(12)}_{\overline{13.75}|} - X\, a^{(12)}_{\overline{83/6}|}$

$$= X \left(\frac{v^{13.75} - v^{83/6}}{i^{(12)}} \right) @\; 7\%$$

$= 26.86$

iv) Capital repayment in these 12 installments $\Rightarrow$

$$\Rightarrow X \left(a^{(12)}_{\overline{22/3}|} - a^{(12)}_{\overline{19/3}|} \right) @\; 7\%$$

$= 516.2$

∴ Total interest in these 12 installments

$12X - 51620 = 30556$

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