ARYAN TRIPATHI

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SRM 3: Assignment1 – [redacted]

① $(1.021) \times (1.012) \times (0.992) = 1.02498$

$1.012 \times 0.992 = 1.003904$

$0.992$

$1$

Incremental mid 2004 prices

| AY \ DY | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 2001 | 1276.11 | 10029 | 6057.15 | 356 |
| 2002 | 1460.68 | 1079.38 | 723 | |
| 2003 | 1554.46 | 1079 | | |
| 2004 | 1491 | | | |

Cumulative claims mid 2000 prices

| AY \ DY | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 2001 | 1276.11 | 2279.01 | 2884.13 | 3240.13 |
| 2002 | 1460.68 | 2539.98 | 3262.98 | |
| 2003 | 1559.46 | 2633.46 | | |
| 2004 | 1491 | | | |

| | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Total claims | 5782.25 | 7452.45 | 6147.11 | 3240.13 |
| (Past claims) | (1491) | (2633.46) | (3262.98) | (0) |
| | 4291.25 | 4818.99 | 2884.13 | 3240.13 |

Df 1.73666 1.2756 1.12349

| A4 \ D4 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 2001 | 1276.11 | 2279 | 2884.13 | 3240 |
| 2002 | 1460.68 | 2539 | 3262.98 | 3665.74 |
| 2003 | 1554.46 | 2633.46 | 3339.245 | 3773.89 |
| 2004 | 1491 | 2589.36 | 3302.99 | 3710 |

Incremental claims for mid 2004 policies

| A4 \ D4 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 2001 | | | | |
| 2002 | | | | |
| 2003 | | | | |
| 2004 | | 1098.36 | 718.64 | 407.4 |
| | | × | × | × |
| Inflation | | 1.025 | 1.050625 | 1.076891 |
| inflation adjust | | 1125.89 | 748.72 | 439.05 |

Total claims. $= 2313.5875$ (figures in '000s)

ii) Ultimate amount for 2004 policies = 5250000×0.75
= 3937500

| | | | | |
|---|---|---|---|---|
| Cumulative DFS | – | 2.48871 | 1.43804 | 1.12349 |
| Reverse | 1 | 1.2343 | 1.43304 | 2.48871 |
| 1/f | 1 | 0.81017 | 0.6978 | 0.4018 |
| 1-1/f | 0 | 0.1983 | 0.3022 | 0.5982 |

for 2004 AY

DY  Emerging Liability  Inflation

1  $3937\times\left(\frac{1}{1.43304}-\frac{1}{2.4887}\right)=1165.51\times1.025=1194.65$

2  $3937\left(\frac{1}{1.12349}-\frac{1}{1.433}\right)=757.24\times(1.050625)=795.57$

3  $3937.5\left(1-\frac{1}{1.2343}\right)=432.61\,(1.076891)=465.87$

Total emerging liability = 2456.09 (figure in '000s)

| 2 | 1 | 2 | 3 | Ultimate |
|---|---|---|---|---|
| 2005 | 3512 | 7210 | 9563 | 9563 |
| 2006 | 2889 | 6723 | | |
| 2007 | 3876 | | | |

| | | | |
|---|---|---|---|
| r | 2.1767 | 1.3264 | 1 |
| f | 2.8872 | 1.3264 | 1 |
| $(1-1/f)$ | 0.6536 | 0.24608 | 0 |

| AY | 2007 | 2006 | 2005 |
|---|---|---|---|
| EP | 12549 | 11814 | 11041 |
| IUL | 10917.63 | 9843.18 | 9605.67 |
| EL | 7135.76 | 2422.21 | 0 |
| RL | 3870 | 6723 | 9563 |
| UL | 11005.76 | 9145.21 | 9563 |

(in '000s)

Total UL = 29713 x 1000

$\Rightarrow$ Reserves = 29713970 - 20103000

= 9610970

③ Incremental data - Mid 2013 prices

| AY | DY 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 2010 | 137.64 | 36.288 | 31.5688 | 13 |
| 2011 | 101.39 | 28.514 | 23 | |
| 2012 | 129.33 | 35 | | |
| 2013 | 149 | | | |

Cumulative claims

| AY | DY 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 10 | 137.64 | 173.28 | 205.4968 | 218.4968 |
| 11 | 101.39 | 129.904 | 152.904 | |
| 12 | 129.33 | 164.33 | | |
| 13 | 149 | | | |

| | 1 | 2 | 3 |
|---|---|---|---|
| Development factors | 1.27094 Ⓐ | 1.17960 Ⓑ | 1.06326 Ⓒ |

Predicting future values

| AY/DY | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 10 | | | | |
| 11 | | | | 162.577 |
| 12 | | | 193.844 | 206.4 |
| 13 | | 189.37 | 223.38 | 237.512 |

Incremental claims (w/o inflation)

| AY/DY | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 10 | | | | |
| 11 | | | | 9.673 |
| 12 | | | 29.54 | 12.266 |
| 13 | | 40.37 | 34.01 | 14.112 |

1 yr

2 yr

3 yrs

__/__/____

Incremental (w/ inflation)

| AY/DY | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 10 | | | | |
| 11 | | | | 9.9632 |
| 12 | | | 30.399 | 13.013 |
| 13 | | 41.5811 | 36.085 | 15.442 |

∴ o/s claims w/ inflation = 9.9632 + 30.399 + 15.44[illegible]

= 146.4793

(4) ~~[illegible]~~

i) Outstanding claims ~~[illegible]~~ using the ACPC method:

| | DY | | | |
|---|---|---|---|---|
| AY | 0 | 1 | 2 | ultimate |
| 2005 | 530.634 | 720.744 | 830.532 | → 830.532 |
| 2006 | 491.320 | 716.092 | | → 825.1714 |
| 2007 | 753 | | | → 1221.4595 |

(5) $X \sim Exp(\lambda)$

(i) need to show that $Y = KX$ is also exponentially distributed.

Now, $P(Y<y) = P(KX<y)$

$= P(X < y/K)$

$= \int_0^{y/K} \lambda e^{-\lambda z} dz$

$= \int_0^y \lambda e^{-\lambda/K x} \frac{dx}{K}$ , noting the substitution $x = Kz$

$= \int_0^y \frac{\lambda}{K} e^{-\lambda/K x} dx$

Which is the distⁿ function of a exponential distⁿ w/ parameter $\lambda/K$. So $Y$ is exponentially distributed w/ parameter $\lambda/K$

(ii) Loss in 2006 is greater than $M$ given by $e^{-\lambda M}$

prob. of loss in 2007 is greater than $M$: $e^{-\lambda M/K}$

Likelihood of data:

$L = C \times e^{-4\lambda M} \times \prod_i (\lambda e^{-\lambda x_i}) \times e^{-6\lambda M/K} \times \prod_j (\frac{\lambda}{K} e^{-\lambda y_j/K})$

log likelihood:

$$l = \log L = c' - 4\lambda M + 10\log\lambda - \lambda\Sigma x_i - 6\lambda M/K + 12\log\lambda - \lambda/K \; \Sigma y_i$$

$$= c' - 4\lambda M + 22\log\lambda - 13{,}500 - 6\lambda M/K - 17{,}000\lambda/K$$

differentiating: $l' = -4M + \frac{22}{\lambda} - 13{,}500 - 6M/K - 17{,}000/K$

equating to zero

$$-4M + 22/\hat{\lambda} - 13{,}500 - 6M/K - 17{,}000/K = 0$$

$$22/\hat{\lambda} = 13{,}500 + 17000/K + 4M + 6M/K$$

$$\hat{\lambda} = \frac{22}{13{,}500 + 17{,}000/K + 4M + 6M/K}, \quad l'' = \frac{-22}{\lambda^2} < 0$$

(iii) a) $\hat{\lambda} = \frac{22}{13500 + 17000/1.1 + 4(1600) + 6(1600)/1.1} = 0.000499$

b) $\int_0^M x\lambda e^{-\lambda x}\,dx + M\int_M^\infty \lambda e^{-\lambda x}\,dx$

$$= [-xe^{-\lambda x}]_0^M + \int_0^M e^{-\lambda x}\,dx + M[-e^{-\lambda x}]_M^\infty$$

$$= -Me^{-\lambda M} + \left[-\frac{1}{\lambda}e^{-\lambda x}\right]_0^M + Me^{-\lambda M}$$

$$= \frac{1}{\lambda}(1 - e^{-\lambda M})$$

$$= 1102.107$$

$$1102.107 = \frac{1}{\mu}(1 - e^{-\mu R})$$

$$= \frac{1.05 \times 1.1}{0.0004999}\left(1 - e^{\frac{-0.0004999}{1.05(1.1)}}\right) \Rightarrow R = 1496.52$$

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(7) $X \sim Poi(50)$

Claim amount $\sim 100X$

$$f(x) = \frac{3}{32}(6x - x^2 - 5) \quad 1 < x < 5, \text{ otherwise}$$

(i) $E(100X) = 100\,E(X)$

$$= 100 \times \frac{3}{32}\int_1^5 x(6x - x^2 - 5)\,dx$$

$$= \frac{300}{32} \times \left[2x^3 - \frac{x^4}{4} - \frac{5x^2}{2}\right]_1^5 = 300$$

$$Var(100X) = E(100X)^2 - (E(100X))^2$$

$$\Rightarrow E(100)^2 = 100^2\,E(X^2)$$

$$= 100^2 \times \frac{3}{32} \cdot \int_1^5 x^2(6x - x^2 - 5)\,dx$$

$$= \frac{30000}{32}\left[\frac{6x^4}{4} - \frac{x^5}{5} - \frac{5x^3}{3}\right]_1^5$$

$$= 9800$$

$$Var(100X) = 98\,000 - (300)^2 = 8000$$

$Y$ = repair

$$E(Y) = 0.3 \times 200 = 60$$

$$Var(Y) = 0.3 \times 200^2 - (60)^2$$

$$= 8400$$

$Z$ = amount + repair

$$E(Z) = E(X+Y) = 300 + 60 = 360$$

$$Var(Z) = 8000 + 8400 = 16400$$

$$E(S) = 50 \times 360 = 18000$$

$$Var(S) = 50 \times 16400 + 50\,(360)^2$$

$$= 7{,}300{,}000$$