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# SRM-4 - ASSIGNMENT

① (i) Let $T_i$ be the time ~~the~~ until default of the bond $i$ where $i = 1,2,3$.

Joint probability : $P(T_1 \leq 1, T_2 \leq 1, T_3 \leq 1) = C(u_1, u_2, u_3)$
where $u_i = P(T_i \leq 1) = 0.1$ for $i = 1,2,3$

Using the Gumbel copula w/ parameter $\alpha = 2$, we have:
$$P(T_1 \leq 1, T_2 \leq 1, T_3 \leq 1) = C(u_1, u_2, u_3)$$
$$= \exp\{-((-\ln u_1)^2 + (-\ln u_2)^2 + (-\ln u_3)^2)^{1/2}\}$$
$$= 0.0185$$

(ii) The Gumbel copula exhibits (non-zero) upper-tail dependence, the degree of which can be varied by adjusting the single parameter. However, it exhibits no lower tail dependence

⇒ Hence, the Gumbel copula is appropriate if we believe the three investments are likely to behave similarly as the term approaches 5 years but not at early durations.

⇒ This is unlikely to be the case though. If one bond defaults early on then it may be indicative of problems in the industry sector or the economy & so the other investments may also be likely to default early on.

② (i) The 3 parameters of GEV are:

a) The location parameter $\alpha$

b) A scale parameter $\beta$, this is always greater than zero

c) A shape parameter $\gamma$

ii) The 1st 2 parameters rescale the distribution & determine shift & stretch of the distⁿ. The 3rd parameter determines overall shape of the distribution and its sign

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iii) These are analogous to following in the standard distⁿ?

1) mean
2) standard deviation
3) skewness

(3) (i) 4 ways of measuring the tail weight of a distⁿ:

- The existence of moments
- Limiting density ratios
- hazard rate
- mean residual life.

(ii) $$\lim_{x \to \infty} \frac{f_{\alpha=0.2}(x)}{f_{\alpha=0.3}(x)} = \lim_{x\to\infty} \left\{ \frac{0.2\lambda^{0.2}}{(\lambda+x)^{1.2}} \Big/ \frac{0.3\lambda^{0.3}}{(\lambda+x)^{1.3}} \right\}$$

$$= \frac{2}{3}\left(\frac{1}{\lambda^{0.1}}\right) \lim_{x\to\infty} (\lambda+x)^{0.1}$$

$$= \infty$$

Then the distⁿ w/ $\alpha = 0.2$ has a much thicker tail

(4) Given that ${}_{30}p_{50} = 0.6$ for Male & ${}_{30}p_{50} = 0.65$ for female

$\Rightarrow$ The probability of death for the same period is ${}_{30}q_{50} = 0.4$ for male & ${}_{30}q_{50} = 0.35$ for female

Let X be the random variable representing future lifetime of male & Y be the random variable representing the future lifetime of female. Then:

$P(X \leq 30) = 0.4$ & $P(Y \leq 30) = 0.35$

$\Rightarrow P(X \leq 30, Y \leq 30) = ?$

(i) Gumbel Copula w/ $\alpha = 3.5$

$u = 0.4$ and $v = 0.35$

$$C(u,v) = \exp\left\{ - \left\{ (-\ln u)^{\alpha} + (-\ln v)^{\alpha} \right\}^{(1/\alpha)} \right\}$$

Applying the values of $\alpha$, $u$ & $v$, $C(u,v) = 0.2996$

ii) ~~The~~ Clayton Copula w/ $\alpha = 3.5$

$C(u,v) = (u^{-\alpha} + v^{-\alpha} - 1)^{(-1/\alpha)}$

applying the values of $\alpha$, $u$ & $v$, $C(u,v) = 0.30595$

iii) The Frank copula w/ $\alpha = 3.5$

$$C(u,v) = (-1/\alpha)\ln\left\{1 + \frac{(e^{-\alpha u}-1)(e^{-\alpha v}-1)}{(e^{-\alpha}-1)}\right\}$$

Applying the values of $\alpha$, $u$ & $v$, $C(u,v) = 0.2273$

iv) Clayton copulas gives the highest probability of claims because it exhibits lower tail dependence. This means that if one life does not survive for long then there is high probability that the other life will also not survive for long.

If the deaths are independent then the probability of paying benefit is 0.4.

However, 2 lives covered are related, so we would expect the prob. of paying benefit to be higher than under the assumption of independence. Hence clayton copula is more appropriate.

5) i) $\lambda = 0.125$

mean residual life of 2 yr. old phone

$$= \frac{\int_x^\infty \{1 - F(y)\}\,dy}{1 - F(x)}$$

$$= \int_x^\infty e^{-\lambda y}\,dy \,/\, e^{-\lambda x}$$

$$= 1/\lambda = 8 \text{ years}$$

(ii) The expected life time of a new phone $= 1/\lambda = 8$ years
& the expected life time of a 2 yr. old phone is also 8 years.
This is due to the memory loss property of exponential dist$^n$.
Hence the exponential dist$^n$ is not appropriate for measuring Mean Residual life of a product.

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