Financial Engineering

1) i) Arbitrage opportunity is the situation when we can make Riskless profit.

An arbitrage opportunity means that,
1] Immediate profit with zero probability of future loss.

2] ~~Immediate 0 loss with future probability of profit.~~
2) Zero cost with zero probability of future loss and Non-zero probability of profit.

ii) law ~~on~~ of one price states that any two portfolio that behaves in same manner should have the same price. If this were not, we could buy cheap & sell expensive to create Risk free profit

iii] a] $T = 3$ months.
$S = 125$, $K = 120$.
$C = 30$, $r = 5\%$ p.a.
Dividend, $q = 15\%$ p.a
$P_t = ?$

using put - call parity.

$$S + P = C + Xe^{-rt} + (D)e^{-r(T-t)}$$

$$125 + P = 30 + 120 \times e^{-0.05 \times \frac{3}{12}} + \frac{18.75}{4} \times e^{-0.05 \times \frac{3}{12}}$$

$$125 + P = 30 + 118.509 + \frac{18.51708}{4}$$

$$P = 30 + 118.509 + 4.6875 - 125$$

$$P = 28.11$$

b) As, price of put option is only 23, then we buy cheap.

So,

We will buy cheap & sell expensive.

i.e. we buy put option and shares and sell call option and cash.

Q) 2] $dx_t = A_t dt + B_t dz_t$.

$A_t = \alpha \mu (T-t),\ B_t = \sigma\sqrt{T-t}$ — (1)

$$dF = \frac{\partial f}{\partial x} B_t dz_t + \left(\frac{\partial f}{\partial t} + \frac{\partial f}{\partial x} A_t + \frac{1}{2}\frac{\partial^2 f}{\partial x^2} B_t\right)$$

$$= -f B_t dz_t + f \frac{\partial m}{\partial t}(T-t)dt - f A_t dt + \frac{1}{2} f B_t^2 dt.$$

$$dF = f\left(\frac{\partial m}{\partial t}(T-t) - A_t + \frac{1}{2} B_t^2\right)dt - f B_t dz_t$$

For f to be martingle,

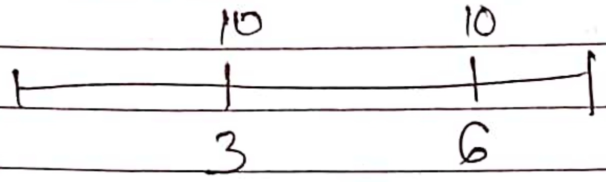
$$\frac{\partial m}{\partial t}(T-t) - A_t + \frac{1}{2} B_t^2 = 0$$

Thus.

$$\frac{\partial m}{\partial t}(T-t) = A_t - \frac{1}{2} B_t^2$$

Q.4 $T = 10$ months, $r = 9.5\%$ p.a.
$K = 350$, $D = 10$ (3 monthly quarterly)

Dividend.



1] Call option and dividend are inversely related to each other.
2) As on ex-dividend date price of option goes down, which lead ~~to~~ the ~~dividend~~ share price to decreased.
3) as share price decrease call option price will also decrease.
4) Also, holder of the call option doesn't get benefit of dividend which is going issue, so it is NOT optimal to exercise the option on either of two dividend dates.

5) $\sigma = 35\%$, $S = 254$,
$K = 258$, $T = 6$ months.

Geometric B.M = exp(General Brownian motion).

a) Assuming price will greater than 258

$$S_t = e^{Bt}$$

$$S_t = \log N\left(B_0 + \mu t, \sigma^2 t\right)$$

$B_0 = 254$,
μ = increasing trend
$\mu = 0.16$
So,

$$S_t = \log N\left[\left\{254 + \left(0.16 - \frac{(0.35)^2}{2}\right)\right\} \times 0.5, \; 0.35 \times \sqrt{0.5}\right]$$

$$S_t = \log N\left[254.049, 0.247487\right]$$

$\ln(258)$

$$1 - N\left(\frac{5.55 - 254.049}{0.247}\right) =$$

$1 - N(-1006)$

Q-7 $dC = \mu . C dt + \sigma C dW_t$

i) Binomial process

for up moves =

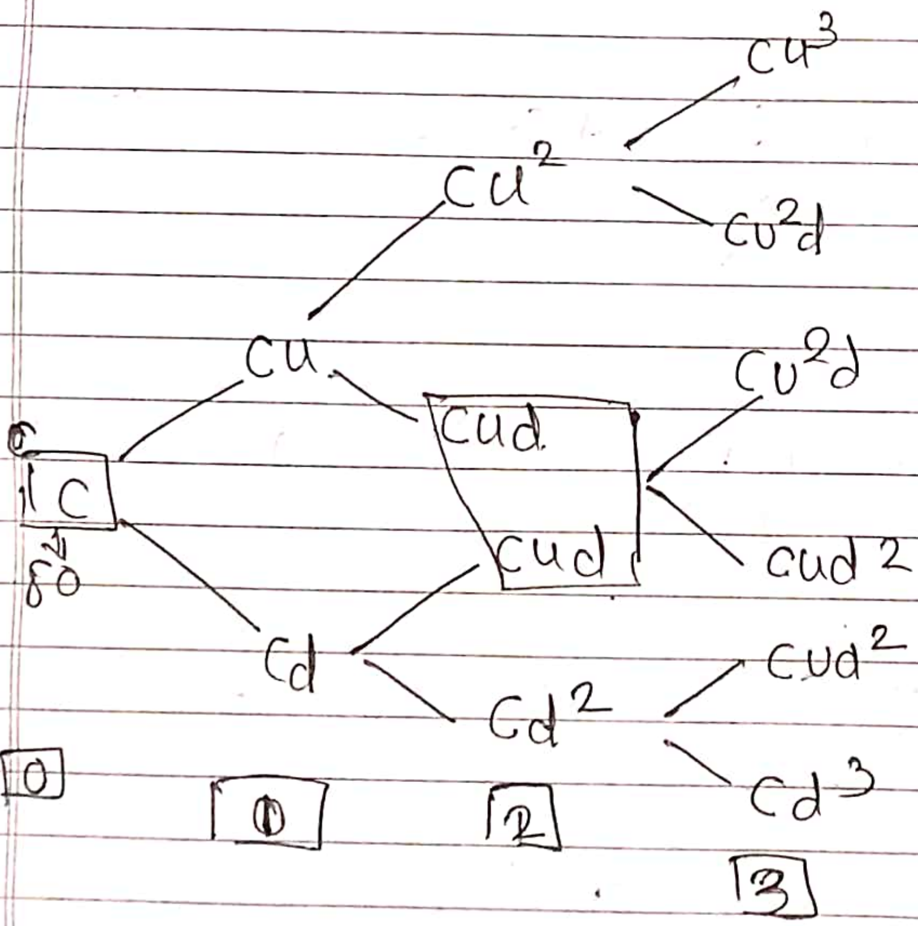
$S_{T(1)} = S_0 \times u$

~~$S_{T(1)}$~~ for down move

$S_{T(1)} = S_0 \times d$.

ii) $C = 80$,

$\sigma = 15\%$, $r = 0$.



b) $T = 9$ month.

Call option = ?

$S = K = 80$.

~~$U = \sigma\sqrt{t} = 0.18\sqrt{\frac{9}{12}} = 0.1290038$~~

~~$d = \frac{1}{U} = 7.698$~~

using put call parity.

as, put will expire worthless

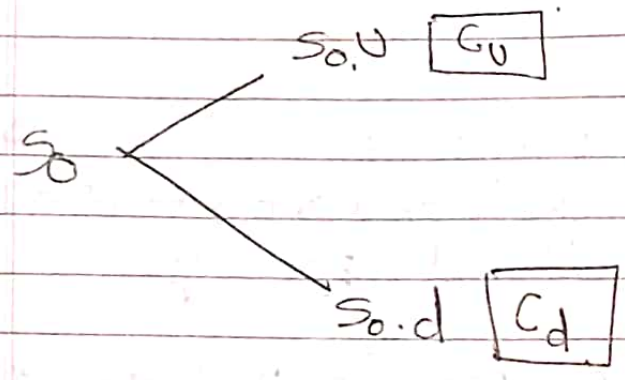
$S_0 + P = C + Xe^{-rT}$

$C = S_0 - Xe^{-rT}$

$= 80 - 80 \times e^{-0}$

Q8

Derivation of the Risk neutral prob
[1 time step]



$$C_u = \phi S_0 . u + \psi e^r$$

$$C_d = \phi S_0 d + \psi e^r$$

$$C_u - C_d = \phi [S_0 u - S_0 d]$$

$$\boxed{\phi = \frac{C_u - C_d}{S_0[u-d]}} \quad \text{—— } (1)$$

Substitute in eq^n (1)

$$C_u = \frac{C_u - C_d}{S_0[u-d]} \times S_0 . u + \psi e^r$$

$$C_u = \frac{u C_u - u . C_d}{u-d} + \psi e^r$$

$$\psi e^r = C_u - \frac{u . C_u - u . C_d}{u-d}$$

$$= \frac{U.C_u - d.C_u - UC_u + UC_d}{U-d}$$

100

$$\boxed{\psi = e^{-r}\left[\frac{U.C_d - d.C_u}{U-d}\right]} \longrightarrow (2)$$

$$C_0 = \phi S_0 + \psi$$

$$= \cancel{C_u - C_d}$$

$$= \frac{C_u - C_d}{S_0[U-d]} . S_0 + e^{-r}\left[\frac{U.C_d - d.C_u}{U-d}\right]$$

$$= \frac{C_u - C_d + e^{-r}U.C_d - d.C_u e^{-r}}{U-d}$$

$$= \frac{C_u - d.C_u e^{-r} + e^{-r}U.C_d - C_d}{U-d}$$

$$= C_u\left[\frac{1 - d.e^{-r}}{U-d}\right] + C_d\left[\frac{e^{-r}U - 1}{U-d}\right]$$

Multiply & divide by e^{r}

$$C_0 = \frac{1}{e^{r}}\left[C_u\left[\frac{e^{r} - d}{U-d}\right] + C_d\left[\frac{U - e^{r}}{U-d}\right]\right]$$

$$C_0 = e^{-r}\left[C_u\left[\frac{e^{r} - d}{U-d}\right] + C_d\left[\frac{U - e^{r}}{U-d}\right]\right]$$

iii) $E[S_1] = q S_0 \cdot u + (1-q) S_0 d$

$$= S_0 \left[\frac{e^r - d}{u-d} \times u + \frac{u - e^r}{u-d} \times d \right]$$

$$= S_0 \left[\frac{e^r u - ud + ud - e^r d}{u-d} \right]$$

$$= S_0 e^r \left[\frac{u-d}{u-d} \right]$$

$$E(S_1) = S_0 e^r$$

~~So,~~ Hence, proved that underlying Stock price grown on average at Risk free rate.

iv) As investor are risk averse, as risk increase, probability of upmove also increase.

$$0 < q < 1$$

$$0 < \frac{e^r - d}{u - d} < 1$$

$$0 < e^r - d < u - d$$

$$d < e^r < u$$

this create ~~the~~ the No ~~arbi~~ Arbitrage situation.

Q.9) 3-month Forward

$F = S_0 \times e^r$

S = stock price.

r = risk free rate of interest.

So, put call parity states that;

$$C + K \cdot e^{-rt} = p + s$$

c = call option price.

p = price of put option.

$t = 0.25$

$$13.334 + 70 e^{-0.25r} = 0.120 + S$$

$$8.869 + 75 \cdot e^{-0.25r} = 0.568 + S$$

Solving simultaneous eqn

$S = 82$, $r = 7\%$

$$F = 82 \times e^{0.07 \times 0.25}$$

$$= 83.45$$

ii) To find forward rate (r_f)

$2.569 + 90 \times e^{-0.5 \times r_6}$

$= 7.909 + 82$

$r_6 = 6\%$, $r_f = 5\%$

iii) using put-call parity

$6.890 + ae^{-0.07 \times 0.25} = 1.055 + 82$

$b + 80 \times e^{-0.07 \times 0.25} = 1.789 + 82$

$2.594 + 85 e^{-0.07 \times 0.25} = c + 82$

$$\boxed{a = 77.5 , b = 5.177 \quad c = 4.119}$$

10. Assumed $i = 0$.

$$q = \frac{(1-d)}{u-d},$$

u = up move, d = down move

for recombining tree, $d = \frac{1}{u}$.

So,

$$q = (1 - 1/u) / (u - 1/u) = 1/(u+1)$$

For No-arbitrage to hold, we must have

$$u > 1 > d$$

then, $u > 1 \Rightarrow u + 1 > 2$

$q = 1/u+1 < 1/2$, Hence proved.

Q.11 Recombing Binomial tree is one in which the size of up-steps and down steps are assumed to be same under all states.

- q is Constent
- It has only (n+1) possible state of time.

Disadvantages:-
- It assume that volatility & drift parameters of the underlying asset price are constant over time.

Q.14 i) $S = 500$, $T = 6$.
$U = 6\%$, $d = 5\%$, $r = 5\%$.

$K = 510$

i] using put-call parity

$S + P = C + Xe^{-rt}$
$P = 0$

$500 = C + 510 \times e^{-0.05 \times \frac{6}{12}}$

$C = 500 - 497.4$
$C = 2.6$

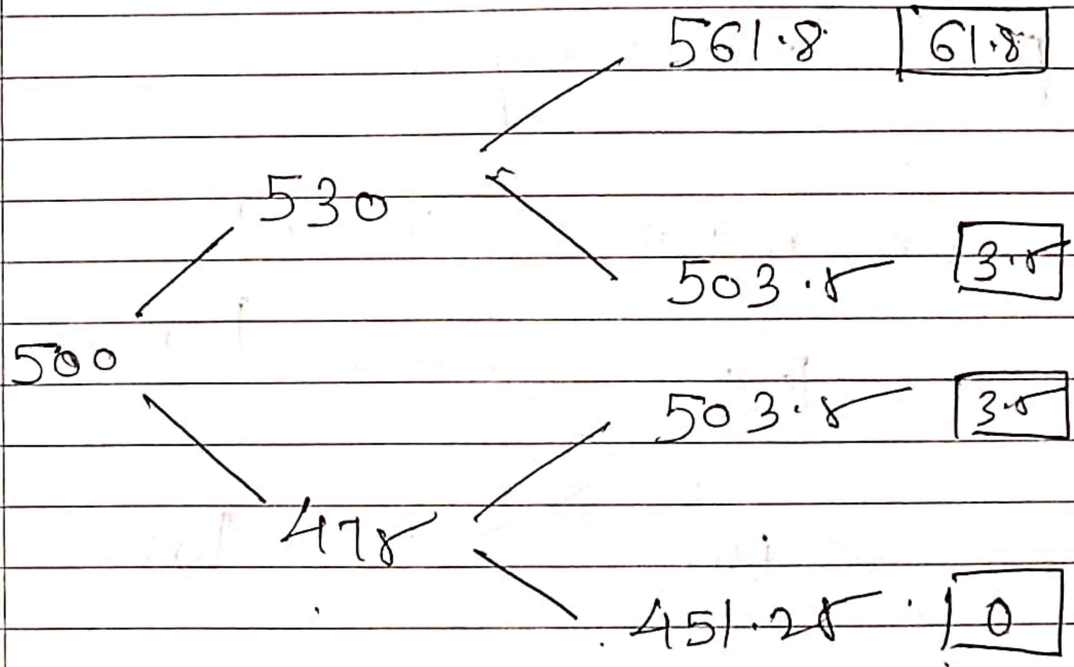
~~iii)~~

Q.14. $S = 500$, $u = 1.06$, $d = 0.95$.
$r = 5\%$.

$$q = \frac{e^{r} - d}{u - d} = \frac{e^{0.05} - 0.95}{1.06 - 0.95}$$

~~500~~

$$\boxed{q = 0.920646}$$



$$C = \left[61.8 \times q^{2} + 3.5(1-q)q + 3.5 (1-q)q + 0\right] \times e^{-2r}$$

$$= 52.8924 \times e^{-2r}$$

$$C_t = \underline{\underline{47.859}}$$

ii) european put option, $X = 510$.

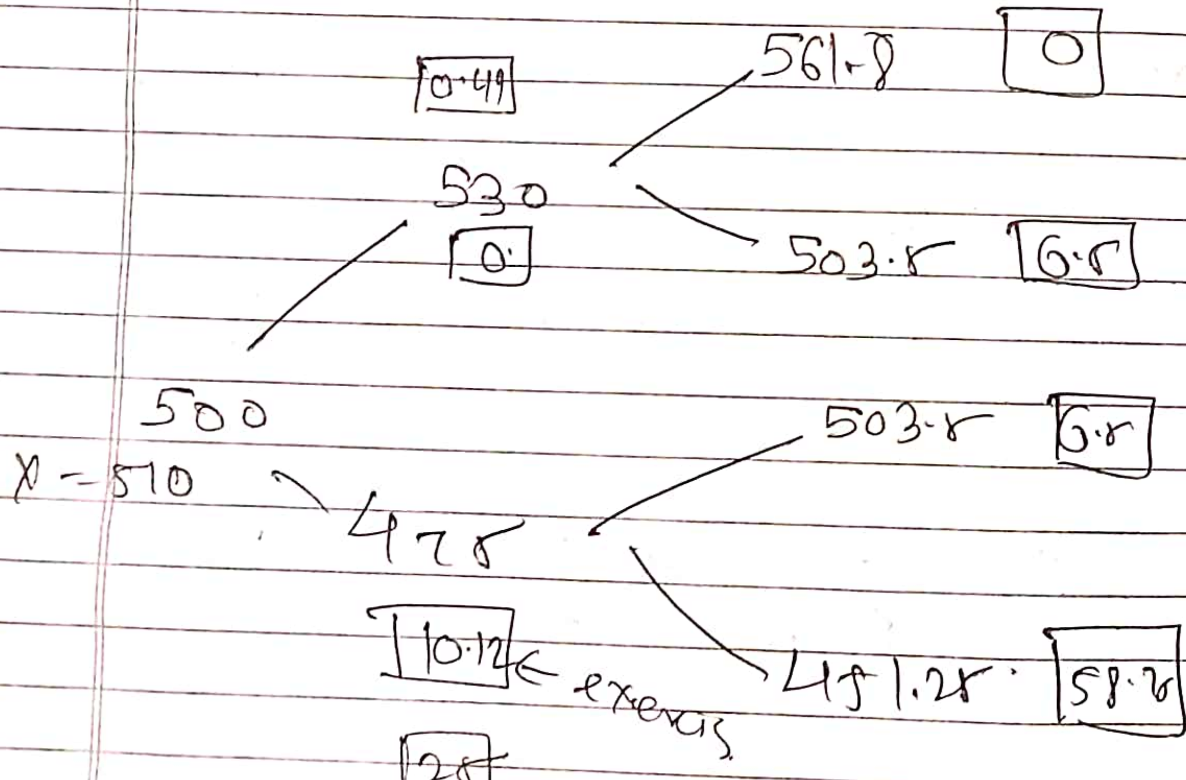
	Payoff $(X - S_T)$
1	0
2	6.5
3	6.5
4	58.75

$q = 0.920646$

$$P_t = 6.5(1-q)q \times 2 + 58.75(1-q)^2$$

$$\boxed{P_t = 1.401554}$$

iii) American put $(X - S_T)$



Value at 1 Node up.

$= [0 \times q + 6.5 \times (1-q)] \times e^{-r}$

$= 0.4906$

Value at 1 down down

$= [6.5 \times q + 58.75 (1-q)] \times e^{-r}$

$= 10.1270$

So, we will exercise the contract.

$P_t = [0.49 \times q + 25 \times (1-q)] \times e^{-r}$

$\boxed{P = 2.316211}$