Assignment 2:

1. $X \sim N(800, 100^2)$ home

$Y \sim N(1200, 300^2)$ car

$$P((Y_1 + Y_2 + Y_3 > (X_1 + X_2 + X_3\} + X_4) + 800)$$

$$\Rightarrow P((Y_1 + Y_2 + Y_3) - (X_1 + X_2 + X_3 + X_4) > 800)$$

$$(Y_1 + Y_2 + Y_3) - (X_1 + X_2 + X_3 + X_4)$$

$$\sim (400, 310000)$$

$$\therefore P\left(Z > \frac{84000}{\sqrt{310000}}\right)$$

$$= P(Z > 0.718) = 1 - \phi(0.718)$$

$$= \boxed{0.236}$$

$$Y = X_1 + X_2 + X_3 + \cdots X_N$$

2. $M_Y(t) = M_N(\ln M_X(t))$

$$= M_N(\ln(1 - t/\lambda)^{-a})$$

$$= M_N(-6\ln(1 - t/2))$$

$$\therefore \left(\frac{0.9e^{t}}{1-0.1e^{t}}\right)^{k}$$

$$= \left(\frac{0.9e^{\ln(1-t/2)^{-6}}}{1-0.1e^{\ln(1-t/2)^{-6}}}\right)^{k}$$

2) $$M_Y(t) = \left(\frac{0.9(1-0.5t)^{-6}}{1-0.1(1-0.5(t))^{-6}}\right)^{3}$$

$E(Y) = M'_Y(0)$ $\quad$ $V(Y) = M''_Y(0) - (M'_Y(0))^2$

3. $M_X(t) = E(e^{tx})$

$$= \int_{-a}^{\infty} e^{tx}\cdot \lambda e^{-\lambda(x-a)}\,dx$$

$$= \lambda e^{-\lambda a}\left(\frac{e^{x(t-\lambda)}}{(t-\lambda)}\right)_{0}^{\infty}$$

$$M_X(t) = \boxed{\frac{-\lambda e^{ta}}{t-\lambda}}$$

4. $G_v(t) = p^k(1-qt)^{-k}$

$G_v(t) = t^0 P(x=0) + t^1 P(x=1) +$

$\frac{t^2}{2!} P(x=2) + \frac{t^3}{3!} P(x=3)$

$+ \frac{t^4}{4!} P(x=4)$

$G_v(t) = p^k(1-(1-p)t)^{-k}$

$= p^k(1-t+pt)^{-k}$

$G'_v(t) = p^k(-k(1+qt)^{-k-1} \cdot (-q))$

$= p^k qk((1-qt)^{(-k-1)})$

$G''_v(t) = p^k q^4 k(k+1)(k+2)(k+3)$
$(1-qt)^{-(k+4)}$

$P(x=4) = \frac{t^4}{4!} \times p^k q^4 k(k+1)(k+2)$
$(k+3)$

$q = 1-p$

5. $E(Y/X) = \int_{10}^{20} y \cdot \frac{ax(x+y}{10ax(x+15)} dy$

$= \frac{1}{10(x+15)} \left(\frac{300x}{2} + \frac{7000}{3} \right)$

~~P(Y/X)~~

$P(Y > X/3 + 10)$

$= \int_0^5 \int_{y=10}^{10+x/3} ax^2 + axy \; dy \; dx$

$= \int_{10}^{10+x/3} \frac{125a}{3} + \frac{25a}{2} y \; dy$

$= \frac{125a}{3} \left(\frac{10 + x}{3} - 10 \right) +$

$\frac{25a}{2} \left(\frac{10+x^2}{3} - 50 \right)$

$= \boxed{\frac{125ax}{9} + \frac{25a^2x}{6} + \frac{25ax^2}{36}}$

6. $X \sim Poi(\lambda) \quad Y \sim Poi(\mu)$

$X - Y = 2$

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$$M_2(t) = E(e^{tz}) = E(e^{t(x-y)})$$

$$= E(e^{tx} \cdot e^{-ty})$$

$$= M_x(t) \cdot M_y(-t)$$

$$= e^{\lambda(e^t-1)} \cdot e^{\mu(e^{-t}-1)}$$

hence author statement is wrong as the MGF is not of Poisson

$$z = x+y$$

$$M_z(t) = E(e^{t(x+y)})$$

$$= M_x(t) \cdot M_y(t)$$

$$= e^{\lambda(e^t-1)} \cdot e^{\mu(e^t-1)}$$

$$\boxed{= e^{(\lambda+\mu)(e^t-1)}}$$

on comparing $\quad x+y \sim Poi(\lambda-\mu)$

7. $$V(X/Y=2) = E(X^2/Y=2) - (E(X/Y=2))^2$$

$$= ((1^2\times 0) + (2^2\times 0.3)) + (0.1\times 3^2) + (4^2\times 0.2)) - ((1\times 0) + (2\times 0.3) + (3\times 0.1) + (4\times 0.2))$$

$= \boxed{3.6}$

$E(U/V = v) = \int_0^1 \mu f(u/v = v)$

$= \boxed{\dfrac{8v-3}{12v-4}}$

8. $X \sim \text{Gamma}(3, 2)$

$E(Y/X = x) = 3x + 1$

$V(Y/X = x) = 2x^2 + 5$

$E(Y) = E(E(Y/X = x))$

$= E(3x + 1) = \boxed{5.5}$

$V(Y) = E(V(Y/X = x)) + V(E(Y/X = x))$

$= E(2x^2 + 5) + V(3x + 1)$

$= \boxed{\dfrac{71}{4}}$

$Z = X + Y$

9. $E(X) = 3$ $\quad V(X) = 2^2$

$E(Y) = 4$ $\quad V(Y) = 1$

$E(Z) = E(X+Y)$

$= \boxed{7}$

$\text{covv}(X, Y) = -0.3$

$\text{cov}(X, Z) = E(XZ) - E(X)E(Z)$

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$= 24.4 - (3 \times 7) = \boxed{3.4}$

$V(Z) = cov(X+Y, X+Y)$

$= V(X) + V(Y) + 2cov(X, Y)$

$= \boxed{3.8}$

$cov(X, Y) = E(X, Y) - E(X)E(Y)$

$E(X, Y) = \boxed{11.4}$

10. $f(x, y) = kx^{-a} e^{-y/\beta}$

$$1 = \int_1^{\infty} \int_1^{\infty} k x^{-a} e^{-y/\beta} \, dy \, dx$$

$$1 = k \int_1^{\infty} x^{-a} \left(\frac{e^{-y/\beta}}{-y/\beta} \right)_1^{\infty} dx$$

$$1 = k \frac{1}{\beta} e^{-1/\beta} \left[\frac{x^{-a+1}}{a+1} \right]_1^{\infty}$$

$$\frac{k e^{-1/\beta}}{\beta} \cdot \frac{1}{(a+1)} = 1$$

$$k = \frac{(a+1)\beta}{e^{-1/\beta}} = \boxed{(a+1)\beta e^{1/\beta}}$$