Financial Mathematics

1.

(i) APR = $\frac{\text{average annual interest}}{\text{average loan amount}}$

$= \frac{2 \text{ average annual interest}}{\text{original loan}}$

The Original loan amount = ₹ 20000

$20000 = 12 \times 427.9\, a_{\overline{5}|}^{(12)}$

$= a_{\overline{5}|}^{(12)} = 3.85998$

Flat rate = $\frac{\text{Total Interest}}{\text{original loan} \times \text{Total years}}$

$= \left[\frac{5 \times 12 \times 427.90 - 20000}{20000 \times 5}\right] = 5.68\%$

So the APR $\approx 2 \times 6.67 = 11\%$

at

$i = 11\%,\ a_{\overline{5}|}^{(12)} = 3.8788$

$i = 10\%,\ a_{\overline{5}|}^{(12)} = 3.96154$

by interpolation,

$I = \frac{P - 10\%}{3.8499 - 3.96155} = \frac{11\% - 10\%}{\quad}$

$i = 10.8\%$

At 10.8%, $12 \times 427.9\, a_{\overline{5}|}^{(12)} = 20000.2$

Now
at 10.9%, $12 \times 427.96 \, a_{\overline{5}|}^{(12)} = 19958.2$
So $i = 10.89$ is closer

The APR = 10.8%

(ii) Now we know
APR = 10.8%

The capital after one year $= 12 \times a_{\overline{4}|}^{(12)}$ @ 10.8 x 427.9

$= 16775.98$

Now if t denotes the term of the new loan

$16775.98 = 12 \times 274.49 \, a_{\overline{t}|}^{(12)}$

$16775.98 = \dfrac{3294.88\,(1 - 1.108^{-t})}{12 \times (1.108^{1/12} - 1)}$

$1.108^{-t} = 0.4754$, $t = 7.25$ years

(iii) 4 years of payment remaining on the original loan,
the total interest $= 4 \times 12 \times 427.9 - 16775.68$
$= 3763.22$

The term of the new loan is 7.25 years
Total interest $= 7.25 \times 12 \times 274 - 16775.6$
$= 7105.63$

2.
(i) (a) Discounted payback period : It is the amount of time that it takes for the initial cost of a project to equal to discounted value of expected cash flows. It is the period in which the cumulative net present value of a project equals zero.

(ii) (b) Payback period : It can be termed as the length of time it takes to recover the cost of an investment or the length of time an investor needs to reach a breakeven point.

(iii) (a) Internal rate of return
The IRR of project A is the rate of interest which satisfies the equation of value

$$17000 = 20000v - 10000v^2 + 20000v + 20000v^2 = 0$$

$$10v^2 + 20v^3 - 170$$

$$10v^2 + 20v^3 - 170 = 0$$

Now,

$0.9 \rightarrow -16.1$

$0.93 \rightarrow -0.474$

$0.94 \rightarrow 4.9538$

$\therefore v = 0.9309$ $\quad \left(\frac{1}{1+i} = 0.9309\right.$ $\quad i = 0.074224$

So $i = 7.4\%$

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(b) NPV

The net present value would be discounted at 8%.

Project A : NPV $= -170000 + 10000v^2 + 200000v^3 = 682$

B $= -200000 + 14000 a_{\overline{6}|} + 200000v^6 = 9834.7$

3. Value as on 1/11/1985

Annuity 1 $= 200 \times v^{1/4} \times \ddot{a}_{\overline{22}|}$ @ 8%

Here $v^{1/4} = 0.98094S$

$a_{\overline{22}|} = 10.2009$ & $d = 0.07407$

$$= \frac{200 \times 0.980945 \times 10.2009 \times 0.08}{0.07407}$$

$= 2161.36631$

Annuity 2 $= 320 \times (1+i)^{1/12} \times a^{(4)}_{\overline{25}|}$ @ 8%

$= 320 \times 1.006434 \left(\frac{1 - 0.92592^{25}}{0.08770}\right)$

where $(1+i)^{1/12} = 1.006434$

$v = 0.925926$

$i^{4} = 0.7790$

$= 320 \times 1.006434 \times 9.18475$

$= 2957.8708$

Annuity 3 $= 180 \times a^{(12)}_{\overline{18.75}|}$

when $i^{(12)} = 0.077208$

$v = 0.925926$

$= 180 \times \left[\dfrac{1-(0.925926)^{18.75}}{0.077208} \right]$

$= 1780.664167$

Now let required annuity be A

Value of it is

$x(1+i)^{1/4} \times a^{(2)}_{\overline{21.5}|}(1+i)^{1/4}$

Now

$A = \dfrac{6899.9}{1.019427 \times 10.3088}$

$= 656.56$

4. To derive $(I\ddot{a})_{n}$ for $i>0$

timeline

1 2 3 4 ... n

0 ... n-1 n

Now,

$$(I\ddot{a})_{\overline{n}|} = 1+2v+3v^2+\dots+nv^{n-1} \rightarrow (1)$$

$$(I\ddot{a})_{\overline{n}|} = (1+i)(Ia)_{\overline{n}|}$$

$$= (1+i)\left(\frac{\ddot{a}_{\overline{n}|}-nv^n}{i}\right)$$

$$= \frac{\ddot{a}_{\overline{n}|}-nv^n}{\frac{i}{1+i}}$$

So $\boxed{(I\ddot{a})_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|}-nv^n}{d}}$

5.

(i) For the year 2014

Annuity fund $\frac{16.4}{12.4}-1 = 0.3226$ i.e. 32.26

Equity fund $\frac{15.5}{12.1}-1 = 0.2810$ i.e. 28.10.

(ii)
(a) Now lets say that investor buys 100 units per quarter. Then the equation of value will be given by

$1240(1+i) + 1310(1+i)^{3/4} + 1480(1+i)^{1/2} +$
$+ 1580(1+i)^{1/4} = 4 \times 1640$

Now, let $1+i = x$
we get

$1240x + 1310x^{3/4} + 1480x^{1/2} + 158x^{1/4}$
$-6580 = 0$

For the following rates,
$1 \to -95$
$1.03 \to 2.145$
$1.01 \to 29.51$

$1.029 \to -0.99$
$1.293 \to -0.049$
$1.2932 \to \underline{0.0126}$

$x = 1.2932$
Hence $i = \underline{\underline{29.32\%}}$

(b) $400 \times \ddot{s}_{\overline{1}|}^{(4)} = 100 \times 15.5 \left(\frac{1}{12.1} + \frac{1}{9.2} + \frac{1}{10.8} + \frac{1}{13.1} \right)$

$\ddot{s}_{\overline{1}|}^{(4)} = \frac{15.5}{4} \left(0.087264 + 0.1086956 52 + 0.09708737 8 \right.$
$\left. + 0.076335877 \right)$

$= \frac{15.5}{4} (0.3647635)$

$\ddot{s}_{\overline{1}|}^{4} = 1.4134587$

We know that

$\frac{(1+i)-1}{d^{4}} = 1.4134587$

$\frac{(1+i)-1}{4[1-(1/1+i)^{1/4}]} = 1.4134587$

Now if $1+i = x$

$\frac{x-1}{4[1-(1/x)^{1/4}]} = 1.4134587$

For the following interest rates:

$1.7 \to -0.0046$
$1.8 \to 0.0493$

$1.71 \to 0.00055$
$1.709 \to 0.0000035$

$1+i = 1.709$
$i = 0.709 = i = 70.9$

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6.

(i) Let x be the initial anual repayment

$160000 = x\, a_{\overline{10}|}$ @ 8%

$x = \frac{160000}{a_{\overline{10}|}} = \frac{160000}{6.7101} = 23,844.65$

(ii) outstanding loan immediately after 4th payment

$23844.6\, a_{\overline{6}|}$ @ 8% $= 23844.65 \times 4.6229 = 110,231.41$

Let y be the next annual installment.

Hence the equation

$y\, a_{\overline{6}|} = 110231.44$ @ 10%

$y = \frac{110231.442}{a_{\overline{6}|}} = \frac{110231.442}{4.35} = 25308.72$

(iii) outstanding after 7th payment

$25309.72\, a_{\overline{3}|}$ @ 10%

$= 25309.72 \times 2.4869 = 62942.74$

Now z be the next annual installment

~~$Z a_{\overline{3}|} = 629942.74$~~ @ 9%

$Z = \frac{62942.74}{a_{\overline{3}|}} = \frac{62942.74}{2.5313} = 24865.77$

8.

(i) PV of the outflow is

$2.7 + ^{0} a_{\overline{3}|}$ @ i

The expected value of the income

$1.06v^3 + \bar{a}_{\overline{10}|}(0.25v + 0.2v^2 + 0.1v^3)$ @ i

equating both we get,

~~2.70~~ $2.7 + 0.2 a_{\overline{3}|} = 0.1v^3 + \bar{a}_{\overline{10}|}(0.25v + 0.2v^2 + 0.1v^3)$

(ii) Based on the PV of company's income

$0.1v^3 + 0.85\bar{a}_{\overline{10}|}v^3$

$0.1v^3 + 0.85\bar{a}_{\overline{10}|} - 2.7 - 0.2a_{\overline{3}|}$ @ 10%

$= 0.1 \times 0.75134 + 0.85 \times 0.7513 \times 6.4469 - 2.7 - 0.2 \times 204868$

$= 4.19 - 3.19$

$= 1.0089$

Since the NPV is positive the project can be invested in

9.

(ii) TWRR

$$(1+i)^3 = \frac{33}{30.5} \times \frac{41.05-4.5}{39} \times \frac{45.6}{41.05} \times \frac{47}{45.6-2.5}$$

$$(1+i)^3 = 1.0819671 \times 0.9371794 \times 1.110840 \times 1.090$$

$$(1+i)^3 = 1.228133$$

Hence $i = 0.07095128$ = $\underline{\underline{7.095128\ \%}}$

(iii) TWRR gives as fair basis for assessing the investment performance as it eliminates the cash grow amount an dtiming for the fund. whereas MWRR is volatile to the amount of the net cash that grows.

10. Now if the loan is L

$L = 180000\, a_{\overline{15}|} + 20000\, (Ia)_{\overline{15}|}$ @12%

$L = 180000 \times 6.8109 + 20000 \times 40.77310$

$= 180000 \times 6.8109 + 20000 \times \left(\frac{4.8877604}{0.12} \right)$

$= 1225962 + 20000 \times 40.731337$

$= 1225962 + 814626.7478$

$= 2040588$

Total cost $= \frac{2040588}{0.8} = 2550735$

(ii) loan outstanding at the beginning of 9th

$L = (180000v + 180000v^2 + \ldots 180000v^7) + (10 \times 20000v^2 \ldots\ldots (15 \times 20000v^{37})]$

$L(9) = 340000 \times 4.5638 + 20000 \left(\frac{5.11\ 14 - 7 \times 0.4523}{0.12} \right)$

$L(9) = 1875850.33$

Loan schedule

Year	O/S	instalment	interest	Capital
9	18,75,850	360000	225102.04	134897.9
10	17,40,952	380000	209814.2	171086
11	1569866			

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(i) TWRR

$$(1+i)^2 = \frac{500}{460} \times \left(\frac{550+50}{540}\right) \times \frac{x}{556}$$

$$(1+i)^2 = \frac{600 \times x}{46 \times 540 \times 11}$$

$$(1+i)^2 = 1.44$$

$$\frac{1.44 \times 46 \times 540 \times 11}{600} = x$$

$x = 655.77$