

Numerical methods and algebra

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Roll:- 16

1 $\sqrt{7 + \sqrt{7 + \sqrt{7} \dots}}$

$$y = \sqrt{7 + \sqrt{7 + \sqrt{7} \dots}}$$

$$y = \sqrt{7 + y}$$

$$y^2 = 7 + y$$

$$y^2 - y - 7 = 0$$

$$y = \frac{1 \pm \sqrt{29}}{2}$$

$$\Rightarrow \boxed{\frac{1 + \sqrt{29}}{2}} \text{ as } y \text{ is always } > 0$$

2 $7x^2 - 6y^2 = -11x^2 - 4y^2$

$$18x^2 = 2y^2$$

$$\frac{x^2}{y^2} = \frac{2}{18}$$

$$\boxed{\frac{x}{y} = \sqrt{\frac{2}{18}}}$$

$$3) \quad x + 3y = 4$$

$$x^2 + y^2 - 4y - 12 = 0 \rightarrow (1)$$

$$x = 4 - 3y \rightarrow (2)$$

(2) in (1)

$$(4 - 3y)^2 + y^2 - 4y - 12 = 0$$

$$16 + 9y^2 - 24y + y^2 - 4y - 12 = 0$$

$$10y^2 - 28y + 4 = 0$$

$$5y^2 - 14y + 2 = 0$$

$$y = \frac{14 \pm \sqrt{156}}{10}$$

$$= \frac{14 \pm 12.49}{10}$$

$$= 2.649$$

$$= 2.649, -0.151$$

$$x_1 = 4 - 3(2.649)$$

$$= -3.9470$$

$$x_2 = 4 - 3(-0.151)$$

$$= 8.53$$

The pairs are

$$(-3.9470, 2.649) \quad (8.53, -0.151)$$

4) $d = 2x^2 - 16x + 34 \Rightarrow$ density

i) $x = 3.5$

$d = 2.5$ mill particles/ ft^3

ii) $d = 2x^2 - 16x + 34 = 0$

differentiation

$4x - 16 = 0 \rightarrow \textcircled{1}$

$x = 4$ (400 RPM)

differentiation of $\textcircled{1}$

4

minima

$d = 2(4)^2 - 16(4) + 34$

$d = 2$

$\therefore$ the no. of revolution for minimum

Smoke = 4

iii) $d = 100$

$100 = 2x^2 - 16x + 34$

$30 = x^2 - 8x + 17$

$0 = x^2 - 8x + (-33)$

$x = (x-11)(x+3)$

$\boxed{x = 11}$

the speed must be 11 revolution per minute

5) $C_1 = 21500 + 25(\text{km})$
 $C_2 = 2100 + 22(\text{km})$

i) Cost to rent car from Co. 1
 $C_1 = 1500 + 25n$

ii) Cost to rent car from Co. 2
 $C_2 = 2100 + 22n$

iii) $C_2 < C_1$
 $1500 + 25n > 2100 + 22n$

iv) $3n > 600$
 $n > 200$

it is less expensive to take C_2 when the number of km is more than 200.

6) $x^3 - 7x^2 + 14x - 6 = 0$

a) $a = 0$, $b = 1$

a	b	$a+b$	$f(a)$	$f(b)$	$f(\frac{a+b}{2})$
0	1	0.5	-6	2	-0.625
0.5	1	0.75	-0.625	2	0.9844
0.5	0.75	0.625	-0.628	0.9844	0.2598

$x = 0.625$

b) $a=1$ $b=3.2$ $f(x) = x^3 - 7x^2 + 16x - 6$

a	b	$a+b/2$	$f(a)$	$f(b)$	$f(a+b/2)$
1	3.2	2.1	2	1.791	-0.421
2.1	3.2	2.65	1.791	-0.42	0.5521
2.65	3.2	2.925	0.5521	-0.112	0.0852

$x = 2.925$

c) $[3.2, 4]$

a	b	$a+b/2$	$f(a)$	$f(b)$	$f(a+b/2)$
3.2	4	3.6	-0.112	2	0.336
3.2	3.6	3.4	-0.112	0.336	-0.016
3.4	3.6	3.5	-0.016	0.336	0.125
3.4	3.5	3.45	-0.016	0.125	0.046125
3.4	3.45	3.425	-0.016	0.0461	0.0130

$x = 3.425$

7 $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$
 $f'(x) = 920x^3 + 54x^2 + 18x - 221$

i	x_i	$f(x_i)$	$f'(x_i)$	$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$
1	-1	156.1398	-1105	-0.60811
2	-0.6081	43.994	-418.898	-0.23541
3	-0.23541	1.537	-234.246	-0.04759
4	-0.04759	0.0004	-221.704	-0.04066
5	-0.04066	~0	-221.70443	-0.04066
	-0.04066	~0	-221.70443	-0.04066

$x = -0.04066$

i	x_i	$f(x_i)$	$f'(x_i)$	$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$
1	0.9	-36.585	509.62	0.97179
2	0.97178	6.37668	691.80	0.96259
3	0.9625	0.11523	666.87	0.9624
	0.9624	0.3267	666.411	0.9624
	0.9624			

$x = 0.9624$

8 a Average = $\frac{70+86+81+83+x}{5}$

$= \frac{320+x}{5}$

b Average = B

$80 < \frac{320+x}{5} < 90$

C $80 < \frac{320+x}{5} < 90$

$$400 < 320+x < 450$$

$$80 < x < 130$$

$$\boxed{80 < x < 130}$$

9 $x_1 = 0.04$ $x_2 = 0.2$ $x = ?$
 $f(x_1) = 8.54$ $f(x_2) = -11.81$ $f(x) = ?$

$$\frac{x - x_1}{x_2 - x_1} = \frac{f(x) - f(x_1)}{f(x_2) - f(x_1)}$$

$$\frac{x - 0.04}{0.16} = \frac{f(x) - 8.54}{-20.35}$$

$$f(x) = 13.6275$$

10 $R(x) = 32x - 0.21x^2$
 $C(x) = 195 + 12x$

for profit $R(x) > C(x)$

$$\therefore 32x - 0.21x^2 > 195 + 12x$$

$$20x - 195 - 0.21x^2 > 0$$

$$0.21x^2 - 20x + 195 < 0$$

$$0.21x^2 - 20x + 195 \neq 0$$

$$x = \frac{20 \pm \sqrt{212.8}}{0.42}$$

$$0.42$$

$$x = 82.35, 12.88$$

$$(x - 82.38)(x - 12.88) < 0$$

$$x \in (12.88, 82.38)$$