Roll call :- 39

F.Y. BSc - A

Subject: Financial Maths

ASSIGNMENT - 1

1) $d^{(4)} = 8\%$

(i) $\left(1+\frac{i^{(4)}}{4}\right)^4 = \left(1-\frac{d^{(4)}}{4}\right)^{-4}$

$\Rightarrow \left(1+\frac{i^{(4)}}{4}\right) = \left(1-\frac{d^{(4)}}{4}\right)^{(-1)}$

$\Rightarrow i^{(4)} = 4\left[\left(1-\frac{d^{(4)}}{4}\right)^{(-1)} - 1\right]$

$= 8.1632653\%$

$= \boxed{8.16\%}$

(ii) $(1+i) = \left(1+\frac{i^{(4)}}{4}\right)^4$

$\Rightarrow i = \left[\left(1+\frac{i^{(4)}}{4}\right)^4 - 1\right]$

$= 8.4165785\%$

$= \boxed{8.42\%}$

(iii) $\left(1-\frac{d^{(12)}}{12}\right)^{12} = \left(1-\frac{d^{(4)}}{4}\right)^4$

$\Rightarrow 1-\frac{d^{(12)}}{12} = \left(1-\frac{d^{(4)}}{4}\right)^{1/3}$

$\Rightarrow \frac{d^{(12)}}{12} = 1-\left(1-\frac{d^{(4)}}{4}\right)^{1/3}$

$\Rightarrow d^{(12)} = 12\left[1-\left(1-\frac{d^{(4)}}{4}\right)^{1/3}\right]$

$= 8.0539339\%$

$= \boxed{8.05\%}$

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2 $\delta(t) = 0.004t + 0.0002t^2$; for all t

$n = 10$ years , $C = 1000$ Rs.

i)

$\rightarrow PV_{10} = C \times V(10)$

$= C \times V^{10}$

$= C \times (e^{-\int_0^{10} \delta dt}) = [\int_0^{10} 0.004t + 0.0002t^2 dt]$

$= C \times e$

$= C \times e^{-[0.004\frac{t^2}{2} + 0.0002\frac{t^3}{3}]_0^{10}}$

$= 1000 \times e^{-[(0.2666667)]}$

$= 765.9283383$ Rs

$\approx \boxed{765.93 \text{ Rs}}$

ii) $(1+i) = e^{\int_0^{10} \delta(t) dt}$

$\Rightarrow i = e^{[0.004\frac{t^2}{2} + 0.0002\frac{t^3}{3}]_0^{10}} - 1$

$= 0.305605172$

$= \boxed{30.56\ \%}$

3. Here, the effective rate of discount $= d$, effective rate of interest $= i$.

$\rightarrow$ Now, according to the equivalent interest rate, $V(t) = 1/A(t)$

$\Rightarrow \frac{1}{1+i} = 1-d$

$\Rightarrow d = 1 - \frac{1}{1+i}$

$\Rightarrow d = \frac{1+i-1}{1+i}$

$\Rightarrow d = \frac{i}{1+i}$

$\Rightarrow \quad d = i \times \frac{1}{1+i}$

$\Rightarrow \quad \boxed{d = i.v} \qquad (\because v = \frac{1}{1+i})$

Hence, proved.

(ii) The nominal rate of discount per annum convertible half yearly for.

(a) an effective rate of discount of 2.25% per quarter is.

$\rightarrow$ Given, $\frac{d^{(4)}}{4} = 2.25\%$

$$\rightarrow \quad \left(1 - \frac{d^{(2)}}{2}\right)^2 = \left(1 - \frac{d^{(4)}}{4}\right)^4$$

$$\therefore \quad 1 - \frac{d^{(2)}}{2} = \left(1 - \frac{d^{(4)}}{4}\right)^2$$

$$\therefore \quad d^{(2)} = 2\left[1 - \left(1 - \frac{d^{(4)}}{4}\right)^2\right]$$

$$= 2\left[1 - (1 - 2.25\%)^2\right]$$

$$= 0.088987500$$

$$= \boxed{88.99\%}$$

(b) a nominal rate of discount of 5% convertible every 2 years is

$\rightarrow$ Given, $d^{(1/2)} = 5\%$

$$\Rightarrow \quad \left(1 - \frac{d^{(2)}}{2}\right)^2 = \left(1 - \frac{d^{(1/2)}}{1/2}\right)^{1/2}$$

$$\Rightarrow \quad 1 - \frac{d^{(2)}}{2} = \left(1 - 2d^{(1/2)}\right)^{1/4}$$

$$\Rightarrow \quad d^{(2)} = 2\left[1 - \{1 - 2(5\%)\}^{1/4}\right]$$

$$= 0.051992507$$

$$= \boxed{5.20\%}$$

(iii) → $n = 91$ days
→ $i = 5\%$
→ $d = ?$

$\Rightarrow \quad 1+i = (1-d)^{-1}$

$\Rightarrow \quad i = (1-d)^{-1} - 1$

& $1-d = (1+i)^{-1}$
$\Rightarrow \quad d = 1 - \frac{1}{1+i}$

$\therefore \quad d = \frac{1+i-1}{1+i} = \frac{i}{1+i}$

$\therefore \quad d = \frac{5\%}{1+5\%}$

$= \frac{0.05}{1.05}$

$= 0.047619048$

$= \boxed{4.76\%}$

4.(i) Given, Effective rate of discount $= d$,
Effective rate of interest $= i$.

→ For the equivalent rates,
$\therefore$ Discounting factor $= \frac{1}{\text{Acculumlating factor}}$

$\therefore \quad V(0,t) = \frac{1}{A(0,t)}$

$\therefore \quad 1-d = \frac{1}{1+i}$

$\therefore \quad d = 1 - \frac{1}{1+i}$

$\therefore \quad d = \frac{1+i-1}{1+i}$

$\therefore \quad d = \frac{i}{1+i} = i\left(\frac{1}{1+i}\right)$

$\therefore \quad \boxed{d = i.v} \qquad \left(\because v = \frac{1}{1+i}\right)$

Hence, proved.

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(ii) $i = 0.08 = 8\%$

(a) For $d^{(12)}$,

$\Rightarrow \left(1 - \frac{d^{(12)}}{12}\right)^{-12} = 1 + i$

$\therefore \left(1 - \frac{d^{(12)}}{12}\right)^{12} = (1+i)^{(-1)}$

$\therefore 1 - \frac{d^{(12)}}{12} = (1+i)^{\left(-\frac{1}{12}\right)}$

$\therefore d^{(12)} = 12\left[1 - (1+i)^{\left(-\frac{1}{12}\right)}\right]$

$= 12\left[1 - (1+0.08)^{(-0.083333)}\right]$

$= 0.07671477$

$= \boxed{7.67\%}$

(b) For $i^{(365)}$,

$\Rightarrow \left(1 + \frac{i^{(365)}}{365}\right)^{365} = 1 + i$

$\Rightarrow i^{(365)} = 365\left[(1+i)^{\frac{1}{365}} - 1\right]$

$= 365\left[(1+0.08)^{\frac{1}{365}} - 1\right]$

$= 0.076969155$

$= 7.696\%$

$\approx \boxed{7.70\%}$

(c) $\delta = \ln(1+i)$

$= \ln(1+0.08)$

$= 0.07696104$

$= 7.696\%$

$\simeq \boxed{7.70\%}$

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(d) For $i^{(0.5)}$,

$\Rightarrow \left(1+\frac{i^{(0.5)}}{0.5}\right)^{0.5} = (1+i)$

$\Rightarrow \left(1+\frac{i^{(0.5)}}{0.5}\right) = (1+i)^{1/0.5}$

$\Rightarrow i^{(0.5)} = 0.5\left[(1+i)^2 - 1\right]$

$= 0.5\left[(1+0.08)^2 - 1\right]$

$= 0.0832$

$= \boxed{8.32\%}$

5. (i) $d^{(4)} = 6\%$

(a) The equivalent rate of interest per annum convertible half yearly,

$\Rightarrow \left(1+\frac{i^{(2)}}{2}\right)^2 = \left(1-\frac{d^{(4)}}{4}\right)^{-4}$

$\Rightarrow 1+\frac{i^{(2)}}{2} = \left(1-\frac{d^{(4)}}{4}\right)^{-2}$

$\Rightarrow i^{(2)} = 2\left[\left(1-\frac{d^{(4)}}{4}\right)^{(-2)} - 1\right]$

$= 0.061377516$

$= \boxed{6.14\%}$

(b) For the equivalent rate of discount per annum convertible monthly,

$\Rightarrow \left(1-\frac{d^{(12)}}{12}\right)^{12} = \left(1-\frac{d^{(4)}}{4}\right)^4$

$\Rightarrow 1-\frac{d^{(12)}}{12} = \left(1-\frac{d^{(4)}}{4}\right)^{1/3}$

$\Rightarrow d^{(12)} = 12\left[1-\left(1-\frac{d^{(4)}}{4}\right)^{1/3}\right]$

$\therefore \quad d^{(12)} = 0.060302525$

$\quad = \boxed{6.03\%}$

(ii) The time line for the loan will be

~~+ 2,00,000 Rs~~ 2,20,000

15 th April, 2005 15 th April, 2006

$\Rightarrow$ The interest for the time period of one year will be,

$\Rightarrow AV = C \times (1+i)$

$\Rightarrow (1+i) = \dfrac{AV}{C}$

$\therefore \quad i = \dfrac{2,20,000}{2,00,000} - 1$

$= 0.1$

$= 10\%$

$\rightarrow$ He repaid the loan early on 17 th July, 2005.

$\therefore \quad n = 0.25$ years.

$\Rightarrow AV_{0.25} = C \times (1+i)^{0.25}$

$= 2,00,000\,(1 + 10\%)^{0.25}$

$= 2,04,809.3672$

$= \boxed{2,04,809.37 \text{ Rs}}$

6. The effective annual rate of discount offered by the manufacturers to the retailers who,

(a) Pay cash.

$\rightarrow$ Cash payment : 30% below recommended retail price (given)

→ who accept credit terms = 25% below recommended retail price

→ let's assume that the retail price is Rs. C

⇒ ~~The~~ cash price will be 30% of C
$= C - 0.3C = 0.7C$

⇒ The six month credit price will be 25% of C
$= C - 0.25C = 0.75C$

Now, the effective rate of discount per annum d will be obtained by the equation.

$$\Rightarrow 0.7C = 0.75C \times (1-d)^{1/2} \quad (\because \text{ it is 6-month credit})$$

$$\Rightarrow 1-d = \left(\frac{0.7}{0.75}\right)^2$$

$$\Rightarrow d = 1 - \left(\frac{0.7}{0.75}\right)^2$$

$$= \cancel{0.305555556} = 0.128888889$$

$$= \cancel{\boxed{30.56\%}} = \boxed{12.89\%}$$

(ii) $d = 12.89\%$

→ $d^{(12)}$ will be obtained from the equation,

$$(1-d) = \left(1 - \frac{d^{(12)}}{12}\right)^{12}$$

$$\Rightarrow 1 - \frac{d^{(12)}}{12} = (1-d)^{1/12}$$

$$\Rightarrow d^{(12)} = 12\left[1 - (1-d)^{1/12}\right]$$

$$= 12\left[1 - (1 - 12.89\%)^{1/12}\right]$$

$$= 0.137208048$$

$$= \boxed{13.72\%}$$

→ For $i^{(2)}$, the equation of equivalent discount rate will be,

$$\rightarrow \left(1+\frac{i^{(2)}}{2}\right)^{2} = (1-d)^{-1}$$

$$\Rightarrow 1+\frac{i^{(2)}}{2} = (1-d)^{-1/2}$$

$$\Rightarrow i^{(2)} = 2\left[(1-d)^{-1/2} - 1\right]$$

$$= 2\left[(1-12.89\%)^{-1/2} - 1\right]$$

$$= 0.142870809$$

$$= \boxed{14.29\%}$$

→ The force of ~~disco~~ interest δ will be,

$$\delta = \ln(1-d)^{(-1)}$$

$$= \ln(1-12.89\%)^{(-1)}$$

$$= 0.137998498$$

$$= \boxed{13.8\%}$$

7. The time line for the investment is

20,000		26000
t=0	12 M	t=17 months

(i) The nominal rate of interest per annum convertible quarterly,

$n = 17$ months

$= \frac{17}{12}$ years

⇒ The annual effective interest rate,

$$\therefore i = \left(\frac{A.V.}{C.}\right)^{1/n} - 1$$

$$\therefore i = \left(\frac{26,000}{20,000}\right)^{12/17} - 1$$

$$= 0.20345706 7$$

$$= \boxed{20.35\%}$$

$$\Rightarrow i^{(4)} = 4\left[(1+i)^{1/4} - 1\right]$$

$\therefore \quad i^{(4)} = 4\left[(1+20.35\%)^{1/4} - 1\right]$

$= 0.189552545$

$= \boxed{18.96\%}$

(ii) The nominal rate of discount per annum convertible half yearly,

$\Rightarrow \quad d^{(2)} = 2\left[1-(1+i)^{(-1/2)}\right]$

$= 2\left[1-(1+20.35\%)^{-1/2}\right]$

$= 0.176882353$

$= \boxed{17.69\%}$

(iii) For the simple rate of interest per annum, A.V = 26,000

C = 20,000

$n = \frac{17}{12}$ years

$\Rightarrow \quad A.V = C\left[1 + i_{S.I}(n)\right]$

$\Rightarrow (n) i_{S.I} = \left(\frac{A.V}{C}\right) - 1$

$\Rightarrow i_{S.I} = \dfrac{\left(\frac{A.V}{C}\right) - 1}{n} = \dfrac{\frac{26,000}{20,000} - 1}{\frac{17}{12}}$

$= 0.211764706$

$= \boxed{21.18\%}$

(iv.) From the above derived quantities, the simple interest rate is the highest.

→ The value of nominal rate of discount per annum convertible half yearly is the lowest.

→ The order of the quantities from lowest to highest is

$$d^{(2)} < i^{(4)} < i < i_{S.I}$$

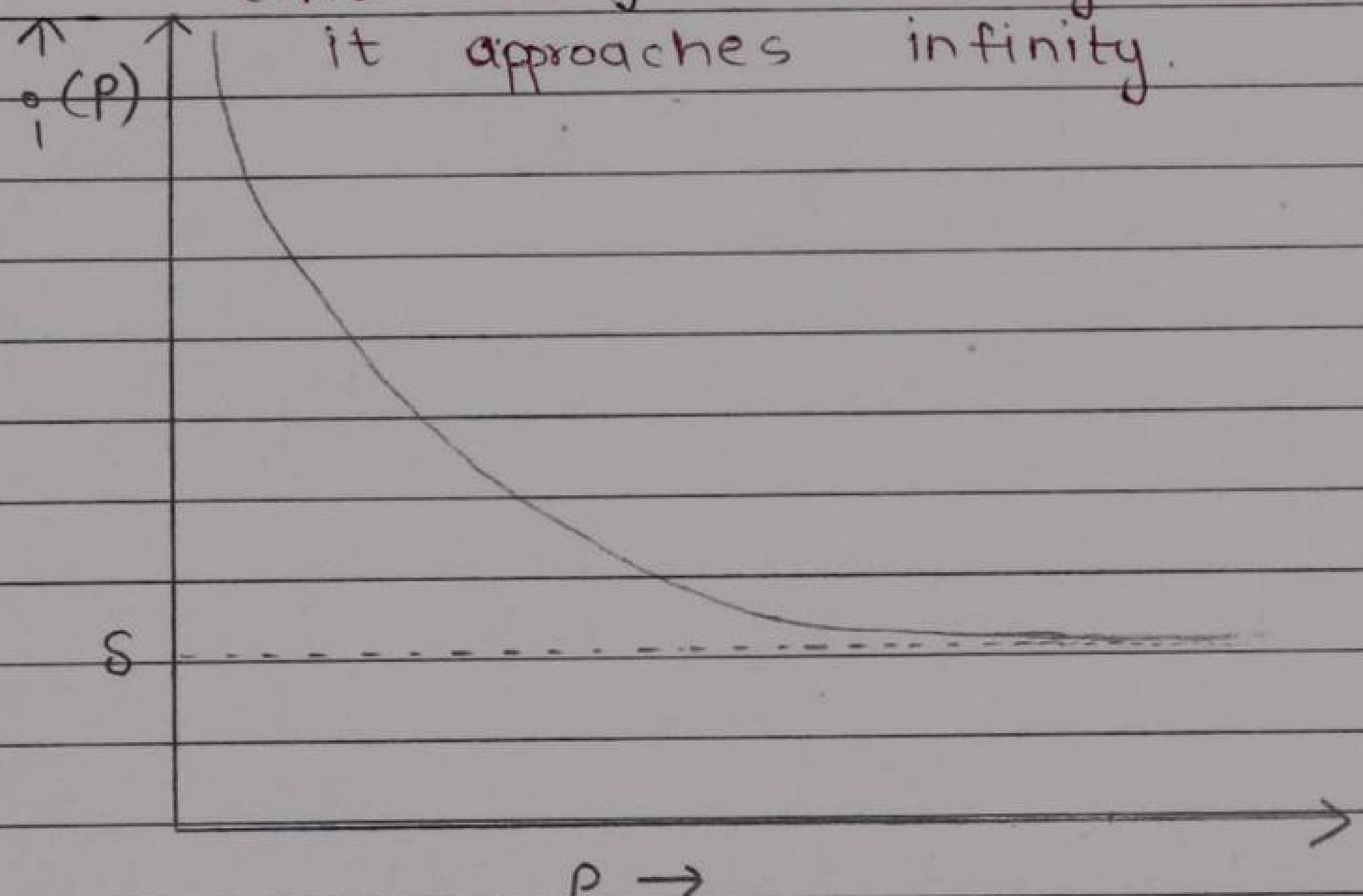
8. The effective rate of interest per annum given is, $i = 8\%$.

→ Now, for plotting the graph of $i^{(p)} \to p$,

$i^{(p)} = p\left[(1+i)^{1/p} - 1\right]$, values of $i^{(p)}$ at different p is.

→ $i^{(2)} = 0.078460969 = 7.85\%$
→ $i^{(3)} = 0.077956704 = 7.80\%$
→ $i^{(4)} = 0.077706188 = 7.77\%$
→ $i^{(5)} = 0.077556392 = 7.76\%$
→ $i^{(10)} = 0.077254952 = 7.73\%$
→ $i^{(25)} = 0.077079623 = 7.71\%$
→ $i^{(100)} = 0.076990664 = 7.70\%$
→ $i^{(365)} = 0.076969155 = 7.70\%$
→ $i^{(1000)} = 0.076964003 = 7.70\%$

∴ The graph of $i^{(p)} \to p$ is a curve continuously decreasing and constant as it approaches infinity.



⇒ Thus, as p is increasing, the value of $i^{(p)}$ is decreasing and as ~~p~~ approaches infinity, the change in value is less significant

$$\Rightarrow \lim_{p\to\infty} i^{(p)} = i^{(\infty)} = \delta \quad (\because \delta = \text{Force of interest})$$

9.(i) Force of interest is the measurement of interests at individual moments of time or infinitely small intervals of time.

⇒ It is represented by δ

⇒ It is also known as the nominal interest rate per unit time convertible continuously or momently.

⇒ The value of ~~the~~ force of interest is,

$$\delta = \lim_{p \to \infty} i^{(p)}$$

⇒ For constant force of interest, its value is,

$$\delta = \ln(1+i) = \ln\left(1 + \frac{i^{(p)}}{p}\right)^p$$

(ii) Given, $i^{(2)} = 0.0775 = 7.75\%$

→ For i, $(1+i) = \left(1 + \frac{i^{(2)}}{2}\right)^2$

$$\Rightarrow i = \left[\left(1 + \frac{i^{(2)}}{2}\right)^2 - 1\right]$$

$$= \left[\left(1 + \frac{7.75\%}{2}\right)^2 - 1\right]$$

$$= 0.079001563$$

$$= \boxed{7.9\%}$$

→ $\delta = \ln\left(1 + \frac{i^{(2)}}{2}\right)^2$ or $= \ln (1+i)$

$$= 0.076034686$$

$$= \boxed{7.6\%}$$

→ $d^{(12)} = 12\left[1 - \left(1 + \frac{i^{(2)}}{2}\right)^{-\cancel{2}/12 \; 6}\right]$

$\therefore\ d^{(12)} = 12\left[1 - \left(1 + \frac{0.0775}{2}\right)^{-1/6}\right]$

$= 0.075795747$

$\to = \boxed{7.58\ \%}$

$\to\ i^{(1/2)} = \frac{\left(1 + \frac{i^{(2)}}{2}\right)^{4} - 1}{2}$

$= \frac{\left(1 + \frac{0.0775}{2}\right)^{4} - 1}{2}$

$= 0.082122186$

$\to = \boxed{8.21\ \%}$

10. The force of interest per annum,

$$\delta(t) = \begin{cases} 0.08 & \text{for} \quad 0 \le t \le 5 \\ 0.06 & \text{for} \quad 5 \le t \le 10 \\ 0.04 & \text{for} \quad t \ge 10 \end{cases}$$

→ The amount due is 1.

→ For time $t;\ 0 \le t < 5,\quad v(t) = e^{-\delta}$

$= e^{-0.08}$

$= 0.923116346 = \boxed{0.92}$

→ For time $t;\ 5 \le t < 10,\quad v(t) = (e^{-\delta})(e^{-\delta})$

$= (e^{-0.06})(e^{-0.08})$

$= 0.869358235 = \boxed{0.87}$

→ For time $t;\ t \ge 10,\quad v(t) = \cancel{e^{-\delta}}\ v(0,5) \cdot v(5,10) \cdot v(10,t)$

$= \cancel{e^{-0.04}}$

$= (e^{-0.08})(e^{-0.06})(e^{-0.04})$

$= 0.835270211 = \boxed{0.84}$

11. (a) $C = 50,000$
$d = 18\%$ p.a
$n = 9$ months
$= 0.75$ years

$\rightarrow$ Present value at time 0.

$$PV_0 = C \times v(0, 0.75)$$
$$= C(1-d)^{0.75}$$
$$= 50,000 \times (1 - 18\%)^{0.75}$$
$$= 43,085.42623$$
$$= \boxed{43,085.43}$$

(b) i) Given, $\delta = 6\%$

$$\rightarrow d^{(12)} = 12\left[1 - e^{-\delta/12}\right]$$
$$= 12\left[1 - e^{-0.005}\right]$$
$$= 0.059850250$$
$$= \boxed{5.99\%}$$

ii) Given, $d^{(4)} = 6\%$

$$\rightarrow i^{(12)} = 12\left[\left(1 - \frac{d^{(4)}}{4}\right)^{-1/3} - 1\right]$$
$$= 12\left[\left(1 - \frac{6\%}{4}\right)^{-1/3} - 1\right]$$
$$= 0.060607089$$
$$= \boxed{6.061\%}$$

$$\rightarrow i^{(4)} = 4\left[\left(1 - \frac{d^{(4)}}{4}\right)^{-1} - 1\right]$$
$$= 4\left[\left(1 - \frac{6\%}{4}\right)^{-1} - 1\right]$$
$$= 0.060913706$$
$$= \boxed{6.091\%}$$

(c) For the ascending order of the above quantities, is :-

$$d < \delta < i^{(4)} < i$$

→ The Force of discount or interest (δ) is obtained when the value approaches to infinity.

$$\Rightarrow \lim_{p \to \infty} d^{(p)} = \delta \quad , \Rightarrow \lim_{p \to \infty} i^{(\infty)} = \delta$$

→ However, $d^{(p)}$ tends to the limit from below whereas $i^{(p)}$ tends to this limit from above.

(d) If we Substite the value of v in the equation then it gives,

$$1 - d = \frac{1}{1+i}$$

where, $1-d$ is the discounting factor and $1+i$ is the accumulating factor.

⇒ They can be substituted to find the principal value or accumulated value and the anwer will be equal because the are equivalent rates.

12.

7,049	AV_2	$AV(2, 4.5)$	$AV(4.5, 6)$
0	2	4.5	6

→ C = 7,049

→ For the first two years,

$n = 2$ years

$i^{(12)} = 6\%$ p.a.

$$\Rightarrow AV_2 = C \times A(0,2)$$

$$= C \left[\left(1 + \frac{i^{(12)}}{12}\right)^{12} \right]^2$$

$$= 7{,}049 \left[\left(1 + \frac{6\%}{12}\right)^{12} \right]^2$$

$$= 7945.349262$$

$$= \boxed{7945.35}$$

→ For the next two and a half years,

$n = 2.5$ years

$= 5.5$ years

$i_{S.I} = 1.5\%$ per quarter.

$C = 7945.349262$

→ In terms of quarters,

$AV_{5.5} = C \times A(5.5)$

$= 7945.349262\ [1 + 1.5\%(5.5)]$

$= 8600.840577$

$= \boxed{8600.84}$

⇒ For the next one and a half years,

→ $n = 1.5$ years

→ $d^{(12)} = 6\%$ p.a

→ $C = 8600.840577$

$$\Rightarrow AV_{(1.5)} = C \times A(1.5)$$

$$= 8600.840577 \left[\left(1 - \frac{d^{(12)}}{12}\right)^{-12} - 1\right]^{1.5}$$

$$= 8600.840577 \left[\left(1 - \frac{6\%}{12}\right)^{-12} - 1\right]^{1.5}$$

$$= 9412.943336$$

$$= \boxed{9412.94}$$

⇒ The accumulated amount after 6 years is 9412.94.

13.

x	i	$2x$
$t=0$	$n = ?$	$t=t$

i) For the first situation,

$i = 10\%$ p.a.

$n = ?$

⇒ $AV = C \cdot A(0,t)$

$\therefore 2x = x(1+i)^n$

$\therefore 2 = (1 + 6\%)^n$

Taking log on both sides,

$\therefore \log 2 = n \log(1.06)$

$\therefore n = 11.89566105$

$\therefore \boxed{n \approx 11.9 \text{ years}}$

(ii) For $d^{(12)} = 10\%$ p.a.

$$i = \left[\left(1 - \frac{d^{(12)}}{12}\right)^{(-12)} - 1\right]$$

$$= \left[\left(1 - \frac{10\%}{12}\right)^{(-12)} - 1\right]$$

$$= 0.105634077$$

$$= \boxed{10.56\%}$$

$\Rightarrow$ $A.V = C \times A(0, n)$

$\Rightarrow$ $2x = x\,(1+i)^n$

$\Rightarrow$ $2 = (1 + 10.56\%)^n$

$\because$ Taking log on both sides.

$\Rightarrow$ $\log 2 = n \log 1.1056$

$\Rightarrow$ $n = \dfrac{\log 2}{\log 1.1056}$

$$= 6.902550414 \text{ years}$$

$$\approx \boxed{6.9 \text{ years}}$$

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