CALCULUS ASSIGNMENT.

1. Let $I = \int_{1/50}^{2} \frac{e^{2/w}}{w^2} \, dw.$

$\therefore$ put $\frac{2}{w} = t$.

$\therefore \frac{dt}{dw} = \frac{-2}{w^2}$

$\therefore \frac{dw}{w^2} = \frac{dt}{-2}$

when, $w = 2$, $t = 1$

and when $w = 1/50$, $t = 100$

$\therefore \quad I = \int_{1}^{100} \frac{e^t}{-2} \, dt.$

$= \frac{-1}{2} \int_{1}^{100} e^t \, dt.$

$= \frac{-1}{2} [e^{100} - e^1]$

$\therefore I = \frac{-e}{2} [e^{99} - 1]$

2. $$I = \int \frac{2t^3+1}{(t^4+2t)^3}\,dt$$

put $m = t^4 + 2t$

$$\frac{dm}{dt} = 4t^3 + 2$$

$$\frac{dm}{2} = (2t^3+1)\,dt$$

$$\therefore\ I = \int \frac{1}{2m^3}\,dm$$

$$= \frac{1}{2}\int m^{-3}\,dm$$

$$= \frac{1}{2}\cdot\frac{m^{-2}}{-2} + k$$

$$= \frac{-1}{4m^2} + k$$

$$\therefore\ I = \frac{-1}{4(t^4+2t)^2} + k.$$

3. $f'(x) = x^4 + 3x - 9$

We know that

$$\int f'(x) = f(x)$$

$$\therefore f(x) = \int x^4 + 3x - 9 \, dx.$$

$$= \int x^4 \, dx + \int 3x \, dx - \int 9 \, dx.$$

$$\therefore f(x) = \frac{x^5}{5} + \frac{3x^2}{2} - 9x + K.$$

4. $f(x) = \begin{cases} 6 & x > 1 \\ 3x^2 & x \leq 1 \end{cases}$

Let $I = \int_{-2}^{3} f(x) \, dx.$

$$= \int_{-2}^{1} 3x^2 \, dx \cdot \int_{2}^{3} 6 \, dx.$$

$$= \left[\frac{\cancel{3}x^3}{\cancel{3}} \right]_{-2}^{1} \times \left[6x \right]_{2}^{3}$$

$$= [1 - (-8)] \times [18 - 12]$$

$$\therefore I = 54.$$

5. Let $I = \int 3(8y-1)\, e^{4y^2-y}\, dy$.

put $t = 4y^2 - y$.

$$\frac{dt}{dy} = 8y - 1$$

$\therefore \quad dt = (8y-1)\,dy$.

$$\therefore I = \int 3 . e^t \, dt$$

$$= 3 \int e^t \, dt$$

$$= 3 . e^t + k .$$

$$\therefore I = 3 . e^{4y^2-y} + k .$$

6. $f''(x) = 15\sqrt{x} + 5x^3 + 6$ $\qquad f(1) = -5/4$, $f(4) = 404$

integrating on both sides,

$$\int f''(x) = \int 15\sqrt{x}\, dx + \int 5x^3\, dx + \int 6\, dx .$$

$$f'(x) = \frac{15 x^{3/2}}{3/2} + \frac{5x^4}{4} + 6x + c$$

$$f'(x) = 10 x^{3/2} + \frac{5x^4}{4} + 6x + c$$

integrating on both sides again,

$$\int f'(x) = \int 10 x^{3/2}\, dx + \int \frac{5x^4}{4}\, dx + \int 6x\, dx + \int c\, dx$$

$$f(x) = \frac{\cancel{10}^{2} x^{5/2}}{\cancel{5}/2} + \frac{\cancel{5} x^5}{4 \times \cancel{5}} + \frac{\cancel{6}^{3} x^2}{\cancel{2}} + cx + k$$

$$\therefore f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 + cx + k$$

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taking $x = 1$.

$f(1) = 4(1)^{5/2} + \frac{(1)^5}{4} + 3(1)^2 + c(1) + k$

$\frac{-5}{4} = 4 + \frac{1}{4} + 3 + c + c$

$\frac{-5}{4} - \frac{1}{4} = 7 + c + k$

$\frac{-6}{4} - 7 = c + k$

$\cancel{\frac{-34}{8} = c}$

$\therefore \frac{-34}{8} = c + k$ — (1)

now, taking $x = 4$

$f(4) = 4(4)^{5/2} + \frac{(4)^5}{4} + 3(4)^2 + c(4) + k$

$404 = 128 + 256 + 48 + 4c + k.$

$-28 = 4c + k$

$-4(7 + c) = k$ — (2)

Substituting value of k in eq. (1)

$\frac{-34}{8} = c - 4(7 + c) = c - 28 - 4c$

$23.75 = -3c \quad \therefore c \approx -7.91667$

now, $k = -4(7 + c) = -4(7 - 7.91667)$

$\therefore k \approx 3.667$

8. $\dfrac{dr}{d\theta} = \dfrac{r^2}{\theta}$ $\qquad r(1) = 2.$

$$\frac{dr}{r^2} = \frac{d\theta}{\theta}$$

integrating on both sides,

$$\int \frac{1}{r^2}\,dr = \int \frac{1}{\theta}\,d\theta$$

$$\frac{-1}{r} + c = \log|\theta| + c$$

$$\therefore \log|\theta| = \frac{-1}{r}$$

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7. $f(x+iy) + g(x-iy)$

differentiating w.r.t. x.

$A = f'(x+iy)(1) + g'(x-iy)(1)$

$\therefore A = f'(x+iy) + g'(x-iy)$

differentiating w.r.t. y.

$B = f'(x+iy)(i) + g'(x-iy)(-i)$

Differentiating A w.r.t. x again

$A' = f''(x+iy)(1) + g''(x-iy)(1)$

$\therefore A' = f''(x+iy) + g''(x-iy)$

Differentiating B, w.r.t. y again

$B' = f''(x+iy)(i)^2 + g''(x-iy)(-i)(-i)$

$= f''(x+iy)(i^2) + g''(x-iy)(i^2)$

$\therefore B' = (i^2)[f''(x+iy) + g''(x-iy)]$

$\therefore B' = (i^2)A$.

9. $\frac{dv}{dt} = 9.8 - 0.196v.$

$$\frac{1}{dt} = \frac{9.8 - 0.196v}{dv}.$$

$$\frac{1}{dt} = \frac{9.8}{dv} - \frac{0.196v}{dv}$$

reciprocating,

$$dt = \frac{1}{9.8}dv - \frac{1}{0.196v}dv.$$

integrating on both sides,

$$\int 1.dt = \frac{1}{9.8}\int 1 dv - \frac{1}{0.196}\int \frac{1}{v} dv.$$

$$\therefore t = \frac{v}{9.8} - \frac{1}{0.196}\log|v| + c$$

12. $f(x) = x^3 - 10x^2 + 6$ for $x = 3$

$f'(x) = 3x^2 - 20x = -33$

$f''(x) = 6x - 20 = -2$

$f'''(x) = 6$

$x = a = 3$

$$x = f(a) + (x-a)\frac{f'(x)}{1!} + (x-a)^2\frac{f''(x)}{2!} + (x-a)^3\frac{f'''(a)}{3!} + \dots n.$$

$$= -57 + (x-3)(-33) + (x-3)^2\frac{-2}{2} + (x-3)^3$$

$$\therefore x = -57 + (-33)(x-3) - (x-3)^2 + (x-3)^3 \dots$$

13. $f(x) = (1+x)^{\mu}$

$f'(x) = \mu(1+x)^{\mu-1}$

$f''(x) = \mu(\mu-1)(1+x)^{\mu-2}$

$f'''(x) = \mu(\mu-1)(\mu-2)(1+x)^{\mu-3}$

let $x = a = 0$

$\therefore f(x) = 1^{\mu} = 1$

$f'(x) = \mu$

$f''(x) = \mu(\mu-1)$

$f'''(x) = \mu(\mu-1)(\mu-2)$

$$\therefore f(x) = (1+x)^{\mu} = 1 + \mu x + \frac{\mu(\mu-1)}{2!}\cdot x^2 + \frac{\mu(\mu-1)(\mu-2)}{3!}\cdot x^3 + \ldots$$

14. $f(x) = \sqrt{1+x} = (1+x)^{1/2} = 1.$

$f'(x) = \frac{1}{2}(1+x)^{-1/2} = \frac{1}{2}$

$f''(x) = \frac{1}{2} x - \frac{1}{2}(1+x)^{-3/2} = \frac{-1}{4}$

$f'''(x) = \frac{-1}{4}\cdot\frac{-3}{2}(1+x)^{-5/2} = \frac{3}{8}$

$$\therefore f(x) = 1 + \frac{1}{2}(x-0)^1 + \frac{(x-0)^2}{2!}\left(\frac{-1}{4}\right) + \frac{3}{8}\frac{(x-0)^3}{3!} \ldots$$

$$\therefore f(x) = 1 + \frac{1}{2}x - \frac{1x^2}{8} + \frac{\cancel{3}x^3}{8\times\cancel{6}2}$$

$$\therefore f(x) = 1 + \frac{1}{2}x - \frac{1x^2}{8} + \frac{x^3}{16} \ldots$$

15 $f(x) = e^{-6x}$ about $x = -4$

$f'(x) = e^{-6x}(-6) = -6e^{-6x}$

$f''(x) = 36e^{-6x}$

$f'''(x) = -216e^{-6x}$

put $x = a = -4$.

$f(a) = e^{24}$

$f'(a) = -6e^{24}$

$f''(a) = 36e^{24}$

$f'''(a) = -216e^{24}$

by taylor series

$$e^{-6x} = e^{24} + (x+4)\left(\frac{6e^{24}}{1}\right) + (x+4)^2\left(\frac{6e^{(24)}}{2!}\right) + (x+4)^3\left(\frac{6e^{24}}{3!}\right) \dots$$

$$\therefore e^{-6x} = e^{24} + (x+4)(6e^{24}) + (x+4)^2(3e^{24}) + (x+4)^3(e^{24}) + \dots$$

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16. $f(x) = \ln(3+4x)$ about $x=0$

$$f'(x) = \frac{1}{3+4x}(4)$$

$$f''(x) = -4(3+4x)^{-2}(4) = -16(3+4x)^{-2}$$

$$f'''(x) = 8(-4)(3+4x)^{-3}(4) = 128(3+4x)^{-3}$$

put $x = a = 0$

$$f(x) = f(a) + (x-a)\frac{f'(a)}{1!} + (x-a)^2\frac{f''(a)}{2!} + (x-a)^3\frac{f'''(a)}{3!}$$

$$\therefore f(x) = \ln 3 + \frac{4x}{3} - \frac{8x^2}{9} + \frac{64x^3}{81} + \cdots$$

11. $y' = \dfrac{xy^3}{\sqrt{1+x^2}}$ $\qquad y(0) = -1$ — (1)

$$\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$$

$$\frac{1}{y^3} \cdot dy = \frac{x}{(1+x^2)^{1/2}}\, dx.$$

integrating on both sides,

$$\int \frac{1}{y^3}\, dy = \int \frac{x}{(1+x^2)^{1/2}}\, dx$$

$$\int y^{-3}\, dy = \frac{1}{2}\int \frac{2x}{\sqrt{(1+x^2)}}\, dx$$

$$\frac{y^{-2}}{2} = \frac{1}{2} \times 2\sqrt{1+x^2} + C$$

$$\frac{y^{-2}}{2y^2} = \sqrt{1+x^2} + C$$

put $x = 0$, $y = -1$ from (1)

$$\frac{(-1)^2}{2} = \sqrt{1+0^2} + C$$

$$\frac{1}{2} = 1 + C$$

$$\therefore \boxed{C = -1/2}$$

$$\frac{1}{2y^2} = \frac{\cancel{2}\sqrt{1+x^2} - 1}{\cancel{2}}$$

$$\therefore y = \pm \frac{1}{\sqrt{2\sqrt{1+x^2} - 1}}$$

Above DE is valid when,

$1 + x^2 \geq 0$ which is true for all values of x.

and,

$$2\sqrt{1+x^2} - 1 > 0$$

$$2\sqrt{1+x^2} > 1$$

$x^2 > -3/4$ which is not possible

$\therefore y = \pm \dfrac{1}{\sqrt{2\sqrt{1+x^2} - 1}}$ is defined for all real values of x

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10. $ty' + 2y = t^2 - t + 1 \qquad y(1) = 1/2.$

$\cancel{ty}\ t\dfrac{dy}{dt} + 2y = t^2 - t + 1$

dividing by t.

$\dfrac{dy}{dt} + \dfrac{2}{t}y = t - 1 + \dfrac{1}{t}$

$\therefore\ P = 2/t \quad , \quad Q = t - 1 + 1/t.$

ODE is linear — [given]

$\therefore$ Integrating factor $= e^{\int P\,dt}$

$= e^{\int 2/t\,dt} = t^2.$

General solution

$y.\ IF = \int Q\ IF\ dt$

$y \cdot t^2 = \int (t - 1 + 1/t) \times t^2\ dt$

$= \int t^3 - t^2 + t\ dt.$

$y.\ t^2 = \dfrac{t^4}{4} - \dfrac{t^3}{3} + \dfrac{t^2}{2} + C \quad — (1)$

Put $t = 1$ & $y = 1/2.$

$\dfrac{1}{2} \times 1^2 = \dfrac{1}{4} - \dfrac{1}{3} + \dfrac{1}{2} + C$

$\therefore\ C = \dfrac{1}{12} \quad — (2)$

From (1) and (2)

$yt^2 = \dfrac{t^4}{4} - \dfrac{t^3}{3} + \dfrac{t^2}{2} + \dfrac{1}{12}.$