

ASSIGNMENT - 1

1. $\lim_{h \rightarrow 0} \frac{2(-3+h)^2 - 18}{h}$

$$\lim_{h \rightarrow 0} \frac{2(9 - 6h + h^2) - 18}{h}$$

$$\lim_{h \rightarrow 0} \frac{18 - 12h + 2h^2 - 18}{h}$$

$$\lim_{h \rightarrow 0} \frac{h(2h - 12)}{h}$$

$$\lim_{h \rightarrow 0} 2h - 12$$

$$\lim_{h \rightarrow 0} 2 \times 0 - 12$$

$$= \underline{\underline{-12}}$$

2. $g(x) = \begin{cases} 2x & x < 6 \\ x-1 & x \geq 6 \end{cases}$

a) $x = 4$

$$g(4) = 2 \times 4 = 8$$

LHL

$$g(4-h) = \lim_{h \rightarrow 0^-} = 2(4-h) = 8$$

RHL

$$g(4+h) = \lim_{h \rightarrow 0^+} 2(4+h) = 2(4+0) = 8$$

Continuous

b) $x=6$

$$g(6) = 6-1 = 5$$

LHL

$$g(6-h) = \lim_{h \rightarrow 0^-} ~~2(6-h)~~ \rightarrow 2(6-h) = 12$$

RHL

$$g(6+h) = \lim_{h \rightarrow 0^+} (6+h)-1 = 5$$

Discontinuous

$$3. \quad \lim_{x \rightarrow 9} \frac{x-9}{3-\sqrt{x}}$$

$$\lim_{x \rightarrow 9} \frac{x-9}{3-\sqrt{x}} \times \frac{3+\sqrt{x}}{3+\sqrt{x}}$$

$$\lim_{x \rightarrow 9} \frac{(x-9)(3+\sqrt{x})}{9-x}$$

$$\lim_{x \rightarrow 9} \frac{-(9-x)(3+\sqrt{x})}{(9-x)} \Rightarrow \lim_{x \rightarrow 9} -(3+\sqrt{x})$$

$$\Rightarrow -3-\sqrt{9} = \underline{\underline{-6}}$$

$$4. \quad f(x) = \frac{4x + 5}{9 - 3x}$$

$$a) \quad x = -1$$

$$f(-1) = \frac{4(-1) + 5}{9 - 3(-1)} = \frac{-4 + 5}{9 + 3} = \frac{1}{12} \quad \text{Continuous}$$

$$b) \quad x = 0$$

$$f(0) = \frac{4 \times 0 + 5}{9 - 3 \times 0} = \frac{5}{9} \quad \text{Continuous}$$

$$c) \quad x = 3$$

$$f(3) = \frac{12 + 5}{9 - 3 \times 3} = \frac{17}{0} \quad \text{Discontinuous}$$

$$5. \quad \lim_{x \rightarrow \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{-2x} + 3e^{-x}}$$

$$\lim_{x \rightarrow \infty} \frac{6e^{4x} (6 - e^{-6x})}{e^{4x} (8 - e^{-2x} + 3e^{-6x})}$$

$$\lim_{x \rightarrow \infty} \frac{6 - e^{-6x}}{8 - e^{-2x} + 3e^{-6x}}$$

$$\frac{6 - 1/e^{6\infty}}{8 - 1/e^{2\infty} + 3(1/e^{6\infty})}$$

$$6. \quad g(y) = \begin{cases} y^2 + 5 & \text{if } y < -2 \\ 1 - 3y & \text{if } y \geq -2 \end{cases}$$

$$a) \quad \lim_{y \rightarrow 6} g(y) = \lim_{y \rightarrow 6} 1 - 3(6) = -17$$

$$b) \quad \lim_{y \rightarrow -2} g(y) = \lim_{y \rightarrow -2} 1 - 3(-2) = 7$$

$$7. \quad f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$$

$$f'(t) = (4t^2 - t) \frac{d}{dt}(t^3 - 8t^2 + 12) + (t^3 - 8t^2 + 12) \frac{d}{dt}(4t^2 - t)$$

$$f'(t) = (4t^2 - t)(3t^2 - 16t) + (t^3 - 8t^2 + 12)(8t - 1)$$

$$f'(t) = 12t^4 - 64t^3 - 3t^3 - 16t^2 + 8t^4 - t^3 - 64t^3 + 8t + 96t - 12$$

$$= 20t^4 - 132t^3 + 24t^2 + 96t - 12$$

$$= 4(5t^4 - 33t^3 + 6t^2 + 24t - 3)$$

$$8. \quad R(w) = \frac{3w + w^4}{2w^2 + 1}$$

$$R'(w) = \frac{(2w^2 + 1)(3 + 4w^3) - (3w + w^4)(4w)}{(2w^2 + 1)^2}$$

$$R'(w) = \frac{(6w^2 + 8w^5 + 3 + 4w^3) - (12w^2 + 4w^5)}{(2w^2 + 1)^2}$$

$$R'(w) = \frac{6w^2 + 8w^5 + 4w^3 + 3 - 12w^2 - 4w^5}{(2w^2 + 1)^2}$$

$$= \frac{4w^5 + 4w^3 - 6w^2 + 3}{(2w^2 + 1)^2}$$

9. $f(x) = 2e^x - 8^x$

$$f'(x) = 2e^x - 8^x \log 8$$

10. $y = z^5 - e^z \ln(z)$

$$\frac{dy}{dz} = 5z^4 - \left[\frac{e^z}{z} + \ln(z)e^z \right]$$

11. a) $f(x) = (6x^2 + 7x)^4$

$$f'(x) = 4(6x^2 + 7x)^3 \frac{d}{dx} (6x^2 + 7x)$$

$$= 4(6x^2 + 7x)^3 (12x + 7)$$

b) $f(t) = 5 + e^{4t+t^7}$

$$f'(t) = e^{4t+t^7} \frac{d}{dt} (4t + t^7)$$

$$= (e^{4t+t^7}) \cdot (4 + 7t^6)$$

12. a) $z = \ln(7 - x^3)$

$$\frac{dz}{dx} = \frac{1}{7-x^3} \cdot \frac{d}{dx}(7-x^3)$$

$$= \frac{1}{7-x^3} (-3x^2)$$

$$= \frac{-3x^2}{7-x^3}$$

$$\frac{d^2z}{dx^2} = \frac{(7-x^3) \cdot \frac{d}{dx}(-3x^2) - (-3x^2) \frac{d}{dx}(7-x^3)}{(7-x^3)^2}$$

$$= \frac{(7-x^3)(-6x) - (-3x^2)(-3x^2)}{(7-x^3)^2}$$

$$= \frac{-42x + 6x^4 - 9x^4}{(7-x^3)^2}$$

$$= \frac{-42x - 3x^4}{(7-x^3)^2}$$

b) $Q(v) = \frac{2}{(6+2v-v^2)^4}$

$$Q'(v) = 2(6+2v-v^2)^{-4}$$

$$= -8(6+2v-v^2)^{-5}(2-2v)$$

$$\begin{aligned}
 g''(v) &= -8 \left[(6+2v-v^2)^{-5} \frac{d}{dv} (2-2v) + (2-2v) \frac{d}{dv} (6+2v-v^2) \right] \\
 &= -8 \left[(6+2v-v^2)^{-5} (-2) + (2-2v)(2-2v) \right] \\
 &= 16(6+2v-v^2)^{-5} - 8(4+4v^2-8v) \\
 &= 16(6+2v-v^2)^{-5} - 32(1+v^2-2v) \\
 &= 16 \left[(6+2v-v^2)^{-5} - 2(1+v^2-2v) \right]
 \end{aligned}$$

$$c) f(t) = \ln(1+t^2)$$

$$\begin{aligned}
 f'(t) &= \frac{1}{(1+t^2)} \frac{d}{dt} (1+t^2) \\
 &= \frac{2t}{1+t^2}
 \end{aligned}$$

$$f''(t) = \frac{(1+t^2)(2) - (2t)^2}{(1+t^2)^2}$$

$$= \frac{2 + 2t^2 - 4t^2}{(1+t^2)^2}$$

$$= \frac{2 - 2t^2}{(1+t^2)^2}$$

$$13. \quad f(x) = x^3 - 3x^2 - 9x + 12$$

$$f'(x) = 3x^2 - 6x - 9$$

$$\text{Put } f'(x) = 0$$

$$3x^2 - 6x - 9 = 0$$

$$x^2 - 2x - 3 = 0$$

$$x^2 - 3x + x - 3 = 0$$

$$x(x-3) + 1(x-3) = 0$$

$$(x+1)(x-3) = 0$$

$$x = -1$$

$$\underline{\underline{or}} \quad x = 3$$

$$f''(x) = 6x - 6$$

$$\bullet \quad x = 3$$

$\therefore$ Minimum at $x = 3$

$$f''(3) = 18 - 6 = \underline{\underline{12}}$$

$$\bullet \quad x = -1$$

$$f''(-1) = 6(-1) - 6 = -12 \quad \therefore \text{Maximum at } x = -1$$

14. $f(x) = 4x^3 - 18x^2 + 24x - 7$

$$f'(x) = 12x^2 - 36x + 24$$

Put $f'(x) = 0$

$$12x^2 - 36x + 24 = 0$$

$$x^2 - 3x + 2 = 0$$

$$x^2 - 2x - x + 2 = 0$$

$$x(x-2) - 1(x-2) = 0$$

$$(x-1)(x-2) = 0$$

$$x = 1$$

$$\underline{\underline{09}} \quad x = 2$$

$$f''(x) = 24x - 36$$

• $x = 1$

∴ Maximum at $x = 1$

$$f''(1) = 24 - 36$$

$$= -12$$

• $x = 2$

∴ Minimum at $x = 2$

$$f''(2) = 48 - 36$$

$$= 12$$

15. $f(x) = x^2(x+3)$

$$f(x) = x^3 + 3x^2$$

$$f'(x) = 3x^2 + 6x$$

Put $f'(x) = 0$

$$3x^2 + 6x = 0$$

$$x^2 + 2x = 0$$

$$x(x+2) = 0$$

$$x = 0$$

$$\underline{\underline{09}} \quad x = -2$$

$$f''(x) = 6x + 6$$

$$\therefore x = 0$$

$$f''(0) = 0 + 6 \\ = 6$$

$\therefore$ ⁱⁿⁱ Maximum at $x = 0$

$$\therefore x = -2$$

$$f''(-2) = -12 + 6 \\ = -6$$

$\therefore$ ^{axi} Minimum at $x = -2$

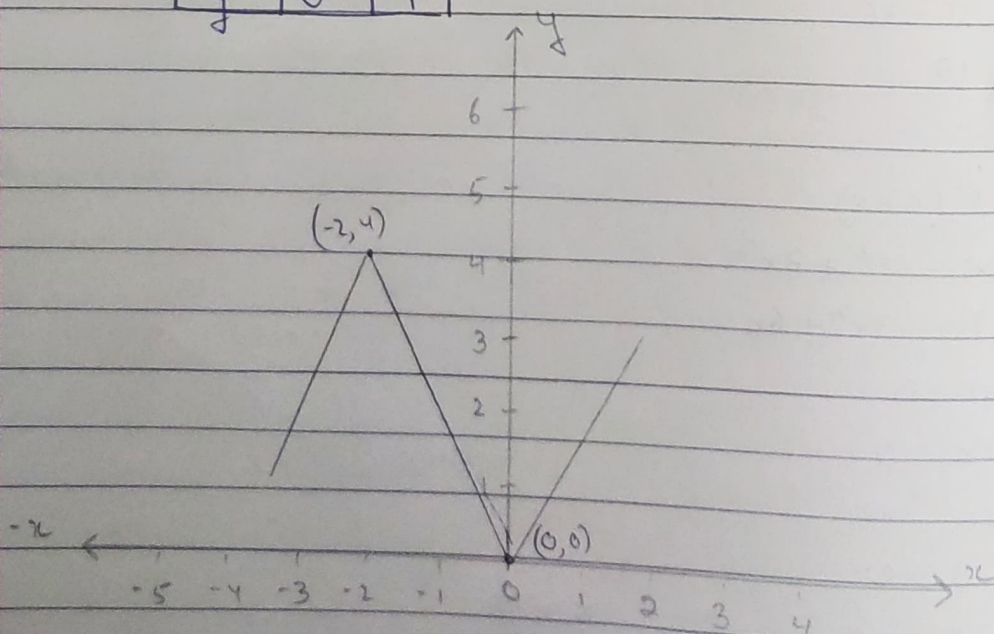
Put $x = 0$ in $f(x)$

$$\text{Minimum value} = f(0) = 0$$

Put $x = -2$ in $f(x)$

$$\text{Maximum value} = f(-2) = 4(-2+3) = 4$$

x	0	-2
y	0	4



16. $f(x) = 3x^2 - 6x$

i.e. $y = 3x^2 - 6x$

Put $x = -3$

$$\begin{aligned} y &= 3 \times (-3)^2 - 6(-3) \\ &= 27 + 18 \\ &= 45 \end{aligned}$$

Put $x = -2$

$$\begin{aligned} y &= 3 \times (-2)^2 - 6(-2) \\ &= 12 + 12 \\ &= 24 \end{aligned}$$

Put $x = -1$

$$\begin{aligned} y &= 3 \times (-1)^2 - 6(-1) \\ &= 3 + 6 \\ &= 9 \end{aligned}$$

Put $x = 0$

$$\begin{aligned} y &= 3 \times (0)^2 - 6 \times (0) \\ &= 0 \end{aligned}$$

Put $x = 1$

$$\begin{aligned} y &= 3 \times (1)^2 - 6 \times (1) \\ &= -3 \end{aligned}$$

Put $x = 2$

$$\begin{aligned} y &= 3 \times 4 - 12 \\ &= 12 - 12 \\ &= 0 \end{aligned}$$

Put $x = 3$

$$\begin{aligned} y &= 3(9) - 18 \\ &= 27 - 18 \\ &= 9 \end{aligned}$$

x	-3	-2	-1	0	1	2	3
y	45	24	9	0	-3	0	9

