Assignment - 2

1) X = claim size on a home insurer

$X \sim N(800, 100^2)$

Y = Claim size on a car Insurer

$Y \sim N(1200, 300^2)$

$P(y_1 + y_2 + y_3 > x_1 + x_2 + x_3 + x_4 + 800)$

$P(y_1 + y_2 + y_3 - x_1 - x_2 - x_3 - x_4 > 800)$

$y_1 + y_2 + y_3 - x_1 - x_2 - x_3 - x_4 \sim N(3(1200) - 4(800), 3 \times 300^2 + 4 \times 100^2)$

$\therefore \sim N(400, 310000)$

$$P\left(z > \frac{800-400}{\sqrt{310000}}\right) = P(z > 0.718)$$

$$= 1 - P(z < 0.718)$$

$$= 0.23576$$

2) N = Number of claims

N ~ negative binomial dis. (3, 0.9)

X = Claim amount

X ~ Gamma (6, 2)

Y = Total claim amount

$M_y(t) = M_n\, \log y\, (M_x(t))$

$M_x(t) = \left(1 - \frac{t}{2}\right)^{-6}$

$M_y(t) = M_n\left(\log\left(1-\frac{t}{2}\right)^{-6}\right)$

$\left(M_n(t) = \left(\frac{pe^t}{1-qe^t}\right)^k\right)$

$M_y(t) = \left(\frac{pe^{\log(1-t/2)^{-6}}}{1-qe^{\log(1-t/2)^{-6}}}\right)^3$

$M_y(t) = \left(\frac{(0.9)(1-t/2)^{-6}}{1-(0.1)(1-t/2)^{-6}}\right)^3$

$E(N) = \frac{0.3}{0.9} = 3.33$

$V(N) = \frac{k(0.1)}{(0.9)^2} = 3.33 \times \frac{1}{9} = 0.37$

$$E(\lambda) = \frac{\alpha}{\lambda} = \frac{6}{2} = 3 \qquad v(X) = \frac{\alpha}{\lambda^2} = \frac{6}{4} = 1.5$$

$$v(Y) = E(N)\,v(X) + E(X)^2\,v(N)$$

$$= 5 + \frac{10}{3} = \frac{25}{3} = 8.333$$

$$SD(Y) = \sqrt{\frac{25}{3}} = 2.88$$

3) $f_X(x) = \lambda e^{-\lambda(x-\alpha)}$

$$M_X(t) = E\left(e^{tx}\right)$$
$$= \int_{\alpha}^{\infty} e^{tx} \lambda e^{-\lambda(x-\alpha)} \, dx$$
$$= \lambda e^{\lambda\alpha} \int_{\alpha}^{\infty} e^{x(t-\lambda)} \, dx$$
$$= \frac{\lambda e^{\lambda\alpha}}{t-\lambda} \left[e^{(t-\lambda)x} \right]_0^{\infty} = \frac{\lambda e^{\lambda\alpha}}{t-\lambda} \left[e^{(\lambda-t)x} \right]_0^{\infty}$$

$$M_X(t) = \frac{\lambda e^{\lambda\alpha}}{t-\lambda} \left(0 - e^{-\lambda\alpha} e^{t\alpha}\right)$$
$$= \frac{\lambda e^{t\alpha}}{\lambda t}$$

$$M'_X(0) = \frac{\lambda(1) + \lambda^{\alpha}}{\lambda^2 \lambda} = \frac{1 + \lambda^{\alpha}}{\lambda} = \frac{1 + \lambda\alpha}{\lambda}$$

$$M''_X(t) = \lambda e^{t\alpha}(-2)(\lambda - t)^{-3}(-1) + \lambda(\lambda-t)^{-2} e^{t\alpha}(\alpha)$$
$$+ \lambda\alpha e^{t\alpha}(\lambda-t)^{-2}(-1)(-1) + \lambda\alpha(\lambda-t)^{-1}(e^{t\alpha})(\alpha)$$
$$= \frac{2\lambda e^{t\alpha}}{(\lambda t)^3} + \frac{\lambda\alpha e^{t\alpha}}{(\lambda t)^2} + \frac{\lambda\alpha e^{t\alpha}}{(\lambda - t)^2} + \frac{\alpha^2 \lambda e^{t\alpha}}{\lambda - t}$$

$$E(X^2) = M''_X(0) = \frac{2\lambda(1)}{\lambda^2} + \frac{2\lambda\alpha}{\lambda} + \frac{\lambda\alpha^2}{\lambda}$$
$$= \frac{2}{\lambda} + \frac{2\alpha}{\lambda} + \alpha^2$$

$$V(x) = E(x^2) - [E(x)]^2$$

$$= \alpha^2 + \frac{2\alpha}{\lambda} + \frac{2}{\lambda^2} - \left(\frac{1 + \lambda^2\alpha^2 + 2\alpha\lambda}{\lambda^2}\right)$$

$$= \alpha^2 + \frac{2\alpha}{\lambda} + \frac{2}{\lambda^2} - \frac{1}{\lambda^2} - \alpha - \frac{2\alpha}{\lambda}$$

$$V(x) = \frac{1}{\lambda^2}$$

4) $G_U(t) = \left(\frac{P}{1-qt}\right)^K$, $P+q=1$

$\therefore$ Distribution is -ve binomial

$$P(U=4) = \frac{\sqrt{K+4}}{\sqrt{5}\sqrt{K}} p^4 q^4$$

5) $f(x,y) = ax^2 + axy$ $\qquad 0 < x < T$

$\qquad\qquad\qquad\qquad\qquad 10 < y < 20$

$$1 = \int_{10}^{20}\int_{0}^{5} ax^2 + axy \, dx\, dy$$

$$1 = \int_{10}^{20} \frac{125a}{3} + \frac{25}{2} ay \, dy$$

$$1 = \frac{125a}{3}\left(y\right)_{10}^{20} + \frac{25}{2} a \left(y^2\right)_{0}^{20}$$

$$a = \frac{3}{6875}$$

$$f_X(x) = a\int_{10}^{20} x^2 + xy \, dy$$

$$= \frac{3}{6875}\int_{10}^{20} x^2 + xy \, dy$$

$$= \frac{3}{6875}\left(10x^2 + 150x\right)$$

$$E(Y/X) = y f(Y/X) = y\int_{10}^{20} \frac{f(x,y)}{f_x(X)}$$

$$= \int_{10}^{20} \frac{y(3/6875)(x^2+xy)\, dy}{3/6875\,(10x^2+150x)}$$

$$= \left(\frac{1}{10(x+15)}\right)\left(\left[\frac{xy^2}{2} + \frac{y^3}{3}\right]_{10}^{20}\right)$$

$$= \left(\frac{1}{10(x+15)}\right)\left(150x + \frac{7000}{3}\right)$$

$$= \frac{45x + 7000}{3(x+15)}$$

$P(Y > 1/3\ x + 10)$

$$f_Y(y) = \int \frac{3}{6875} (x^2 + xy)\, dx$$

$$= \frac{1}{6875} \left(\frac{250 + 75y}{2} \right)$$

$$P\left(y > \frac{1}{3}x + 10\right) = \int_{\frac{1}{3}x+10}^{20} \frac{1}{6875} \left(\frac{250 + 75y}{2} \right) dy$$

$$= \frac{1}{6875(2)} \left[1000 - \frac{1780\cancel{x}}{3} - \frac{25x^2}{3} \right]$$

6) $X \to Poi(\lambda)$ $\quad Y \sim Poi(\mu)$

$$M_X(t) = e^{\lambda(e^t)} \qquad M_Y(t) = e^{\mu(e^t - 1)}$$

$$M_{X-Y}(t) = E(e^{t(X-Y)})$$

$$= \frac{E}{E}\left(\frac{e^{tx}}{e^{t(y-1)}}\right) = \frac{e^{\lambda}(e^{(t-1)})}{e^{\mu}(e^{(t-1)})} = e^{\lambda - \mu(e^{t}-1)}$$

H. P

$$M_{X+Y}(t) = E(e^{t(x+y)})$$

$$= E(e^{tx})\,E(e^{ty})$$

$$= e^{\lambda(et-1)}\;e^{\mu(et-1)}$$

$$= e^{\lambda+\mu(et-1)}$$

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7) (i) $Var(X/Y=2) = E(X^2/Y=2) - (E(X/Y=2))^2$

$$= \frac{\sum_x x^2 P(X=x, Y=y)}{P(Y=2)} - \left(\sum_x x \frac{P(X=x, Y=y)}{P(Y=y)}\right)^2$$

$$= \frac{1^2(0) + 2^2(0.3) + 3^2(0.1) + 4^2(0.2)}{0.6} - [1(0) + 2(0.3) + 3(0.1) + 4]$$

$$= \frac{53}{6} - \frac{289}{30} = \frac{29}{36}$$

(ii) $f(u,v) = \frac{48}{67}(2uv - v^2) \qquad 0<u<1 \qquad 0<v<2$

$$E(U/V=v) = \int_0^1 u f(u/V=v)\, du$$

$$= \int_0^1 \frac{u f(u, v=v)}{f(V=v)}\, du \quad \text{---} \ (1)$$

$$f_V(V=v) = \int_0^1 \frac{48}{67}(2uv - u^2)\, du$$

$$= \frac{48}{67}\left[\frac{2u^2 v}{2} - \frac{u^3}{3}\right]_0^1$$

$$= \frac{48}{67}\left(v - \frac{1}{3}\right) = \frac{48}{67}\frac{(3v-1)}{(3)} = \frac{16}{67}(3v-1)$$

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$$E(V/v=u) = \int_0^1 \frac{v\,(48/67)\,(2uv - v^2)\,dv}{16/67\,(3v-1)}$$

$$= \frac{3}{3v-1} \int_0^1 2v^2u - v^3\,dv$$

$$= \frac{3}{3v-1}\left[\frac{3v}{3} - \frac{1}{v}\right]$$

$$= \frac{8v-3}{4(3v-1)}$$

8) $\alpha = 3 \quad \lambda = 2$

$X \sim \text{Gamma}(3,2)$

$E(\cancel{(Y)}\, x = x) = 3x+1 \qquad Var(y \neq x = x) = 2x^2 + 5$

$E(y) = E(E(y/x = x))$

$= E(3x+1)$

$= 3E(x)+1$

$= 3(3/2)+1 = 11/2$

$V(y) = E(V(y/x=x) - V E(y/x=x)$

$= E(2x^2+5) + V(3x+1)$

$= 2(3)+5 + 9(3/4)$

$= 6+5+6.75$

$= \underline{\underline{17.75}}$

g) $Z = X + Y$

$E(X) = 3$ $\qquad$ $E(Y) = 4$

$\sigma_X = \sqrt{V(X)} = 2$ $\qquad$ $\sigma_y = \sqrt{V(y)} = 1$

$$\begin{aligned} Cov(X, Z) &= cov(X, X+Y) \\ &= cov(X, X) + cov(X, Y) \\ &= V(X) + (-0.6) \quad = 3.4 \end{aligned}$$

$$\begin{aligned} Var(Z) &= \cancel{cov}\ Var(X+Y) \\ &= V(X) + V(Y) + 2\,Cov(X+Y) \\ &= 4 + 1 + 2(0.6) \\ &= 3.8 \end{aligned}$$

10) $f(x,y) = K x^{-\alpha} e^{-y/\beta}$ $\qquad 1 < x < \infty$

$1 < y < \infty$

$$1 = k \int_1^{\infty}\int_1^{\infty} x^{-\alpha} e^{-y/\beta}\,dx\,dy$$

$$1 = k\left(\frac{1}{\alpha+1}\right)\left(-\beta e^{-1/\beta}\right)$$

$$k = \frac{-1+\alpha}{\beta e^{-1/\beta}}$$