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2. X_i ; $i = 1, 2, \ldots n$ are n IID Random variables.
mean μ & variance σ^2.

Let $\bar{x} = \sum_{1}^{n} x_i$ be sample mean &

$S^2 = \frac{1}{n-1} \sum_{1}^{n} (x_i - \bar{x})^2$ be sample variance

(i) $E(\bar{x}) = E\left(\frac{\sum x_i}{n}\right)$

$= \frac{\sum [E(x)]}{n}$

$= \frac{\sum \mu}{n} = \frac{n\mu}{n} = \mu$

$V(\bar{x}) = V\left(\frac{\sum x_i}{n}\right)$

$= \frac{1}{n^2} \sum x_i$

$= \frac{1}{n^2} \sum V(x_i)$

$= \frac{1}{\cancel{n^2}} \times \cancel{n}\sigma^2$

$= \frac{\sigma^2}{n}$

(ii) $E(S^2) = \sigma^2$

$$E(S^2) = E\left[\frac{1}{n-1}\left(\Sigma x_i^2 - n\bar{x}^2\right)\right]$$

$$= \frac{1}{n-1}\left[E\left(\Sigma x_i^2 - n\bar{x}^2\right)\right]$$

$$= \frac{1}{n-1}\left[\Sigma\left(E(X_i)^2 - nE(\bar{x})^2\right)\right]$$

$$= \frac{1}{n-1}\left[\Sigma(\mu^2 + \sigma^2) - n\left(\mu^2 + \frac{\sigma^2}{n}\right)\right]$$

$$= \frac{1}{n-1}\left[n\mu^2 + n\sigma^2 - n\mu^2 - \sigma^2\right]$$

$$= \frac{1}{n-1}\left[n\sigma^2 - \sigma^2\right]$$

$$= \frac{1}{n-1}(\sigma^2)(n-1) = \underline{\underline{\sigma^2}}$$

(iii) To show variance is $S^2 = \frac{2\sigma^4}{(n-1)}$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$

$$V\left[\frac{(n-1)S^2}{\sigma^2}\right] = \frac{(n-1)^2}{\sigma^2} V(S^2)$$

$$V\left[\chi^2_{n-1}\right] = \frac{(n-1)^2}{\sigma^2} V(S)^2$$

$$V(S^2) = \underline{\underline{\frac{2\sigma^4}{n-1}}}$$

(iv) given $n = 10$

$\sigma^2 = 100$

$P(0.5\sigma^2 < s^2 < 1.5\sigma^2)$

$= P(s^2 < 1.5\sigma^2) - P(s^2 < 0.5\sigma^2)$

(v) $P(s^2 < 1.5\sigma^2) = P(s^2 < 150)$

$= P\left[\frac{9s^2}{\sigma^2} < \frac{150 \times 9}{100}\right]$

$= P(\chi^2_9 < 13.5)$

$= 0.8587$

1. $X \sim Poi(\theta)$

$\frac{X - \theta}{\sqrt{\theta}} \sim N(0,1)$

$\Rightarrow X = 200$

$\therefore \theta = X \pm 1.96\left[\sqrt{\frac{\theta}{N}}\right]$

3.

No. of claim	0	1	2	3	4
No. of Polic	86	75	16	2	1

follows binomial with $n=4$ & p

$N=180$

(i) obtain MLE of p

$$L(P) = [(1-P)^4]^{86} \cdot [4P(1-P)^3]^{75} \cdot [6P^2(1-P)^2]^{16} \cdot [4P^3(1-P)]^2 (P^4)^1$$

$$= (1-P)^{344} p^{75}(1-p)^{225} p^{32}(1-p)^{32} p^6 (1-p)^2 p^4$$

$$= (1-p)^{603} p^{117}$$

Taking ln

$$\ln L(P) = 603 \ln(1-p) + 117 \ln p$$

$$\frac{d \ln L(P)}{dp} = \frac{603}{1-p} + \frac{117}{P}$$

$$= 117 - 117P = 603P$$

$$P = 0.1625$$

5.

(i) MoM estimate of a $U(a,b)$ is $\hat{a} = \bar{x} - \sqrt{3}s$ where $\bar{x}$ is sample mean & s is sample SD.

$X \sim U(a,b)$

$E(X) = \frac{a+b}{2}$ $\qquad V(X) = \frac{(b-a)^2}{12}$

$a = 2\bar{x} - b$ $\qquad a = b - 2\sqrt{3}s$

$b = \bar{x} + \sqrt{3}s$ $\qquad \underline{\underline{a = \bar{x} - \sqrt{3}s}}$

Sample = 2, 4, 6, 8, 10

$\bar{x} = 6$

$s = 3.16$

$a = 6 - \sqrt{3}(3.16)$ $\qquad b = 6 + \sqrt{3}(3.16) = 11.47$

$= 0.527$

$U(0,b)$ & $x = \frac{1}{b^n}$

$\log L = -n \log b$

$\frac{d \log L}{dL} = \frac{-n}{b}$

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7.

	1	2	3	4	5	6	7	8	9	10
Public	24	45	29	33	20	40	26.5	25	27.5	
Pvt.	30	54.5	30	40	28.5	36	30.5	30.5	35.5	

$\bar{X}_1$ for public = 30 $\bar{X}_2$ for private = 35.056
S_1^2 for public = 64.3125 S_2^2 for private = 67.403

$$\frac{(\bar{X}_1-\bar{X}_2)-(\mu_{pub}-\mu_{pri})}{SP\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \sim t_{n_1+n_2-2}$$

(ii) $H_0 : \mu_{pub} = \mu_{pri}$ $H_1 : \mu_{pub} \neq \mu_{pri}$

$$\frac{(30-35.056)}{\sqrt{58.123}\sqrt{\frac{2}{9}}} = -1.407$$

(iii) Public hospital result in higher claims

6. H_0 $P=0.1$ H_1 $P>0.1$
12 missiles fixed otherwise, H_0 3 or more.
24 miss. fixed H_0 : 5 or more hits

8.

(i) An unbiased estimator of a parameter is an estimator whose expected value is equal to the parameter.

Here, if $\hat{\theta}$ is the estimator of parameter θ then $E(\hat{\theta}) = \theta$

(ii) Bias refers to the tendency of a statistic to overestimate or underestimate a parameter.

(iii) $MSE(\hat{\theta}) = E[(\hat{\theta}-\theta)^2]$

9. (i) $t_k \sim N(0,1)$

$\Rightarrow \frac{\bar{x}-\mu}{s/\sqrt{n}} \sim t_{n-1}$

(ii) $\mu(t_k) = 0$

$Var(t_k)$ for $k>2$ is $\frac{k}{k-2}$

(iii) $n = 10$

$\bar{x} = 50$

$s^2 = 48.667$

$CI = 99\%$

(a) $t_{0.005, 9} = 3.25$

$\mu = \bar{x} \pm t_{0.5\%, 9} \frac{s}{\sqrt{n}}$

$= 50 \pm 3.25 \sqrt{\frac{48.667}{10}}$

$= 57.17 \mid 42.83$

10. claims $\sim$ Poi(λ) Poi(3)

$N = 1000$

$X \sim (3000, 3000)$

if $P(X < 3100)$ then $X \sim$ Poi 3

else does not follow Poi(3

(i) Type I error $= P(H_0$ is reject $\mid H_0$ is true$)$

$: \cancel{P(X)} = P(X > 3100, \lambda = 3)$

$= P\left(Z > \frac{3100 - 3000}{\sqrt{3000}}\right)$

$= P(Z > 1.823) = 3.34$

11

$N = 12$

$\Sigma x_i = 213200$

$\Sigma x_i^2 = 4919860000$

$\bar{x} = 17767$

$S^2 = 10144$

$x_1 = 11000$ $\quad x_1' = 27500$

$x_2 = 48000$ $\quad x_2' \; 31500$

$\Sigma x = 213200$

$= A + 11000 + 48000$

$A = 154200$

$\Sigma x' = A + x_1' + x_2' = 213200$

$\bar{x}' = 17767$

$\Sigma x^2 = 4919862000$

$B = 11000^2 + 48000^2$ $\quad B = 249486000$

$\Sigma x'^2 = B + x_1'^2 + x_2'^2 = 4243360000$

$$S = \frac{1}{\sqrt{11}} \sqrt{4243360000 - 12(17767)^2} = 6234.03$$