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(1) X = claim size on a home insurance

$X \sim N(800, 100^2)$

Y = claim size on a car insurance

$Y \sim N(1200, 300^2)$

$P(y_1 + y_2 + y_3 > x_1 + x_2 + x_3 + x_4 + 800)$

$P(y_1 + y_2 + y_3 - x_1 - x_2 - x_3 - x_4 > 800)$

$y_1 + y_2 + y_3 - x_1 - x_2 - x_3 - x_4 \sim N(3(1200) - 4(800), 3 \times 300^2 + 4 \times 100^2)$

$\therefore \sim N(400, 310000)$

$P\left(Z > \frac{800-400}{\sqrt{310000}}\right) = P(Z > 0.718)$

$= 1 - P(Z < 0.718)$

$= 0.23576$

(2) N = Number of claims

$N \sim$ negative binomial dist $(3, 0.9)$

X = Claim Amount

$X \sim$ Gamma $(6, 2)$

Y = total claim amount

$M_Y(t) = M_N \log(M_X(t))$

$M_X(t) = \left(1 - \frac{t}{2}\right)^{-6}$

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$$M_y(t) = M_N\left(\log\left(1-\frac{t}{2}\right)^{-6}\right)$$

$$\left(M_N(t) = \left(\frac{pe^t}{1-qe^t}\right)^k\right)$$

$$M_y(t) = \left(\frac{pe^{\log(1-t/2)^{-6}}}{1-q\,e^{\log(1-t/2)^{-6}}}\right)^3$$

$$M_y(t) = \left(\frac{(0.9)\left(1-\frac{t}{2}\right)^{-6}}{1-(0.1)\left(1-\frac{t}{2}\right)^{-6}}\right)^3$$

$$E(N) = \frac{0.3}{0.9} = \frac{10}{3} = 3.33$$

$$V(N) = \frac{K(0.1)}{(0.9)^2} = 3.33 \times \frac{1}{9} = 0.37$$

$$E(X) = \frac{\alpha}{\lambda} = \frac{6}{2} = 3 \qquad V(X) = \frac{\alpha}{\lambda^2} = \frac{6}{4} = 1.5$$

$$V(Y) = E(N)V(X) + E(X)^2 V(N)$$
$$= 5 + \frac{10}{3} = \frac{25}{3} = 8.333$$

$$SD(Y) = \sqrt{\frac{25}{3}} = 2.88$$

(3) $f_X(x) = \lambda e^{-\lambda(x-\alpha)}$.

$$M_X(t) = E(e^{tx})$$

$$= \int_{\alpha}^{\infty} e^{tx} \lambda e^{-\lambda(x-\alpha)} \, dx$$

$$= \lambda e^{\lambda\alpha} \int_{\alpha}^{\infty} e^{x(t-\lambda)} \, dx$$

$$= \frac{\lambda e^{\lambda\alpha}}{t-\lambda} \left[e^{(t-\lambda)x} \right]_{\alpha}^{\infty} = \frac{\lambda e^{\lambda\alpha}}{t-\lambda} \left[e^{-(\lambda-t)x} \right]_{\alpha}^{\infty}$$

$$M_X(t) = \frac{\lambda e^{\lambda\alpha}}{t-\lambda} \left(0 - e^{-\lambda\alpha} e^{t\alpha} \right)$$

$$= \frac{\lambda e^{t\alpha}}{\lambda - t}$$

ii) $$M'_X(t) = \lambda(e^{t\alpha})(\lambda-t)^{-2}(-1)(-1) + \lambda(\lambda-t)^{-1}(e^{t\alpha})(\alpha)$$

$$= \frac{\lambda e^{t\alpha}}{(\lambda-t)^{2}} + \frac{\lambda e^{t\alpha}\alpha}{\lambda-t}$$

$$E(X) = M'_X(0) = \frac{\lambda(1)}{\lambda^2} + \frac{\lambda\alpha}{\lambda} = \frac{1+\lambda\alpha}{\lambda} = \frac{1+\lambda\alpha}{\lambda}$$

$$M''_X(t) = \lambda e^{t\alpha}(-2)(\lambda-t)^{-3}(-1) + \lambda(\lambda-t)^{-2} e^{t\alpha}(\alpha)$$
$$+ \lambda\alpha e^{t\alpha}(\lambda-t)^{-2}(-1)(-1) + \lambda\alpha(\lambda-t)^{-1}(e^{t\alpha})(\alpha)$$

$$= \frac{2\lambda e^{t\alpha}}{(\lambda-t)^3} + \frac{\lambda\alpha e^{t\alpha}}{(\lambda-t)^2} + \frac{\lambda\alpha e^{t\alpha}}{(\lambda-t)^2} + \frac{\alpha^2\lambda e^{t\alpha}}{\lambda-t}$$

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$$E(X^2) = M''_X(0) = \frac{2\lambda(1)}{\lambda^2} + \frac{2\lambda\alpha}{\lambda^2} + \frac{\lambda\alpha^2}{\lambda}$$

$$= \boxed{\frac{2}{\lambda} + \frac{2\alpha}{\lambda} + \alpha^2}$$

$$V(X) = E(X^2) - (E(X))^2$$

$$= \alpha^2 + \frac{2\alpha}{\lambda} + \frac{2}{\lambda^2} - \left(\frac{1 + \lambda^2\alpha^2 + 2\alpha\lambda}{\lambda^2}\right)$$

$$= \alpha^2 + \frac{2\alpha}{\lambda} + \frac{2}{\lambda^2} - \frac{1}{\lambda^2} - \alpha^2 - \frac{2\alpha}{\lambda}$$

$$\boxed{V(X) = \frac{1}{\lambda^2}}$$

Q4) $G_X(t) = \left(\frac{p}{1-qt}\right)^K$, $p+q=1$

$\therefore$ Distribution is -ve binomial

$$P(X=4) = \frac{\sqrt{K+4}}{\sqrt{5}\sqrt{K}} p^4 q^4$$

Q5) $f(x,y) = ax^2 + axy$ $\quad 0 < x < 5$

$10 < y < 20$

$$1 = \int_{10}^{20}\int_{0}^{5} ax^2 + axy \, dx \, dy$$

$$1 = \int_{10}^{20} \frac{125a}{3} + \frac{25}{2}ay \, dy$$

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$$1 = \frac{125a}{3}\left(y\right)_{10}^{20} + \frac{25}{2}a\left(\frac{y^2}{2}\right)_{10}^{20}$$

$$a = \frac{3}{6875}$$

$$f_x(x) = a\int_{10}^{20} x^2 + xy \, dy$$

$$= \frac{3}{6875}\int_{10}^{20} x^2 + xy \, dy$$

$$= \frac{3}{6875}\left(10x^2 + 150x\right)$$

$$E(Y/X) = y f(Y/x) = y\int_{10} \frac{f(x,y)}{f_x(x)}$$

$$= \int_{10}^{20} \frac{y\left(\frac{3}{6875}\right)\left(x^2+xy\right) dy}{\frac{3}{6875}\left(10x^2+150x\right)}$$

$$= \left(\frac{1}{10(x+15)}\right)\left(\left[\frac{xy^2}{2}+\frac{y^3}{3}\right]_{10}^{20}\right)$$

$$= \frac{1}{10(x+15)}\left(150x + \frac{7000}{3}\right)$$

$$E(Y/x) = \frac{45x + 7000}{3(x+15)}$$

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$P(Y > \frac{1}{3}x + 10)$

$$f_y(y) = \int_0 \frac{3}{6875}(x^2 + xy)\,dx$$

$$= \frac{1}{6875}\left(250 + \frac{75y}{2}\right)$$

$$P(y > \tfrac{1}{3}x + 10) = \int_{\frac{1}{3}x+10}^{20} \frac{1}{6875}\left(250 + \frac{75y}{2}\right)dy$$

$$= \frac{1}{6875(2)}\left[10000 - \frac{1750x}{3} - \frac{25x^2}{3}\right]$$

(b) $X \sim Poi(\lambda)$ $\quad Y \sim Poi(\mu)$

$$M_X(t) = e^{\lambda(e^t - 1)} \qquad M_Y(t) = e^{\mu(e^t-1)}$$

$$M_{X-Y}(t) = E(e^{t(X-Y)})$$

$$= E(e^{tX - xY})$$

$$= \frac{E(e^{tX})}{E(e^{t(Y)})} = \frac{e^{\lambda(e^t-1)}}{e^{\mu(e^t-1)}} = e^{\lambda-\mu(e^t-1)}$$

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$$M_{X+Y}(t) = E(e^{t(X+Y)})$$

$$= E(e^{tx})E(e^{ty})$$

$$= e^{\lambda(e^t-1)} e^{\mu(e^t-1)}$$

$$= e^{\lambda+\mu(e^t-1)}$$

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Q7) i) $Var(X/y=2) = E(X^2/y=2) - (E(X/y=2))^2$

$$= \sum_x \frac{x^2 P(X=x, y=y)}{P(y=2)} - \left(\sum_x x \frac{P(X=x, y=y)}{P(y=y)}\right)^2$$

$$= \frac{1^2(0) + 2^2(0.3) + 3^2(0.1) + 4^2(0.2)}{0.6} - [1(0) + 2(0.3) + 3(0.1) + 4($$

$$= \frac{53}{6} - \frac{289}{30} = \frac{29}{36}$$

ii) $f(u,v) = \frac{48}{67}(2uv - u^2) \quad 0<u<1 \quad \frac{0<v<2}{2}$

$$E(U|V=v) = \int_0^1 u f(U/V=v)\,du$$

$$= \int_0^1 u \frac{f(U, V=v)}{f(V=v)}\,du \quad (1)$$

$$f_V(V=v) = \int_0^1 \frac{48}{67}(2uv - u^2)\,du$$

$$= \frac{48}{67}\left[\frac{2u^2v}{2} - \frac{u^3}{3}\right]_0^1$$

$$= \frac{48}{67}\left(v - \frac{1}{3}\right) = \frac{48}{67}\frac{(3v-1)}{(3)} = \frac{16}{67}(3v-$$

from (1)

$$E(U/V=v) = \int_0^1 \frac{u\left(\frac{48}{67}\right)(2uv - u^2)\,du}{\frac{16}{67}(3v-1)}$$

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$$= \frac{3}{3v-1}\int_{0}^{1} 2v^2 v - v^3 \, d\theta$$

$$= \frac{3}{3v-1}\left[\frac{3v}{3} - \frac{1}{v}\right]$$

$$= \boxed{\frac{8v-3}{4(3v-1)}}$$

(18) $\alpha = 3 \quad \lambda = 2$

$X \sim \text{Gamma}(3,2)$

$E(Y \mid X=x) = 3x+1 \qquad Var(Y \mid X=x) = 2x^2+5$

$E(Y) = E(E(Y \mid X=x)) \qquad V(Y) = E($

$= E(3x+1)$

$= 3E(x)+1$

$= 3\left(3/2\right)+1 = 11/2$

$V(Y) = E(V(Y \mid X=x)) - V\,E(Y \mid X=x)$

$= E(2x^2+5) + V(3x+1)$

$= 2(3)+5+9\left(3/4\right) = 6+5+6.75$

$= 17.75$

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Q9) $Z = X + Y$

$E(X) = 3$ $\quad E(Y) = 4$

$\sigma_X = \sqrt{V(X)} = 2$ $\quad \sigma_Y = \sqrt{V(Y)} = 1$

$\operatorname{cov}(X, Z) = \operatorname{cov}(X, X+Y)$

$= \operatorname{cov}(X, X) + \operatorname{cov}(X, Y)$

$= V(X) + (-0.6)$ $\quad = 3.4$

$\operatorname{Var}(Z) = \operatorname{var}(X+Y)$

$= V(X) + V(Y) + 2\operatorname{cov}(X+Y)$

$= 4 + 1 + 2(-0.6)$

$= 3.8$

Q10) $f(x, y) = k x^{\alpha} e^{-y/\beta}$ $\quad 1 < x < \infty$

$1 < y < \infty$

$1 = k \int_1^{\infty} \int_1^{\infty} x^{-\alpha} e^{-y/\beta} \, dx\, dy$ $\quad \alpha > 1, \beta > 0$

$1 = k \left(\frac{1}{\alpha + 1} \right] \left(-\beta e^{-1/\beta} \right)$

$$\boxed{k = \frac{-1 + \alpha}{\beta e^{-1/\beta}}}$$

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