3/12 Assignment (Prob Stats) :

1. (i) $4, 7, 9, 9, 11, 15, 18, 18, 18, 25 = x$

mean $\mu = \frac{\Sigma x}{N} = \frac{134}{10} = \boxed{13.4}$

median $= \left(\frac{n+1}{2}\right)th = 5.5th = \frac{5th + 6th}{2}$

$= \frac{11 + 15}{2} = \frac{26}{2} = \boxed{13}$

mode → highest frequency value

$= \boxed{18}$

$Q_1 = \left(\frac{n+1}{4}\right)th = 2.5th = \frac{2nd + 3rd}{2}$

$= \frac{16}{2} = \boxed{8}$

$Q_3 = 3\left(\frac{n+1}{4}\right)th = 7.5th = \frac{7th + 8th}{2}$

$= \frac{36}{2} = \boxed{18}$

$IQR = Q_3 - Q_1 = 18 - 8 = \boxed{10}$

$\sigma = \sqrt{\frac{\Sigma (x - \mu)^2}{N}} = \sqrt{\frac{374.4}{10}}$

$= \sqrt{37.44} = \boxed{6.118823}$

(ii)

No. of house	0	1	2	3	4
No. of memb.	5	11	26	15	3

$$(\text{mean})\ \mu = \frac{\sum_{i=0}^{N} f_i x_i}{\sum_{i=1}^{4} f_i} = \frac{120}{60} = \boxed{2}$$

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} = 30.5^{th} = \boxed{2}$$

$$\text{Mode} = \boxed{2}$$

$$\text{S.d.} = \sqrt{\frac{1}{n-1} \sum f x^2 - \bar{x}^2 n}$$

$$= \sqrt{\frac{1}{n-1} \sum f (x-\bar{x})^2}$$

$$= \sqrt{\frac{58}{59}} = \boxed{0.9915}$$

$$\text{IQR} = Q_3 - Q_1$$

$$= \frac{3}{4}(n+1)^{th} - \frac{1}{4}(n+1)^{th}$$

$$= 45.75^{th} - 15.25^{th}$$

$$= 3 - 1 = \boxed{2}$$

Page No.:
Date:

2. $X \sim Poi(0.5)$

(i) $P(X=0) = \dfrac{e^{-\lambda} \cdot \lambda^{x}}{x!}$

$= \dfrac{e^{-0.5} \cdot 0.5^{0}}{0!}$

$= e^{-0.5} = \boxed{0.60653}$

(ii) $P(X_1 = 1, X_2 = 0, X_3 = 0) \times 3$

$= (P(X=0))^2 \times (1 - P(X \leq 0)) \times 3$

$= [((0.60653)^2 \times (1 - 0.60653))] \times 3$

$= \boxed{0.43425}$

(iii) doubt $P(X \geq 2) = 1 - P(X \leq 2)$

$= 1 - (1 - e^{-\lambda x})$

$= e^{-\lambda x} = e^{-(2)(0.5)}$

$= \boxed{0.367879}$

3. $\Gamma(2, 1/2)$ $\quad 0 < x < \infty$

(i) $E(X) = \frac{\alpha}{\lambda} = \frac{2}{1/2} = \boxed{4}$ mean

$V(X) = \frac{\alpha}{\lambda^2} = \frac{2}{(1/2)^2} = \boxed{8}$

$S.D.(X) = \sigma = \sqrt{V(X)} = \sqrt{8}$
$= \boxed{2.8284}$

(ii) The distribution is skewed since it has a high variance and ~~mea~~ comparitively lower mean. The it seems to be positively skewed.

(iii) $F_X(x) = \int_0^\infty f_X(x)\,dx$

$= \int_0^\infty \frac{0.5^2}{1!} \cdot x^1 \cdot e^{-x/2}$

$$= 0.25 \int_0^\infty x \cdot e^{-x/2}$$

$$= 0.25 \left[x \cdot e^{-x/2}(-2) - \frac{e^{-x/2}}{-1/2 x - 1/2} \right]_0^\infty$$

$$= 1 - \frac{0.5(\infty)}{\sqrt{e^\infty}} - \frac{1}{\sqrt{e^\infty}}$$

$$= \boxed{1}$$

4. $M = 10 \quad F = 8 \quad P(M) = 10/18 \quad P(F) = \frac{8}{18}$

$P(A/M) = \frac{6}{10} \quad P(A/F) = 7/8$

$P(2AF$

(6)

Page No.:
Date:

5. $P(L1) = \frac{4}{10}$ $P(L2) = \frac{5}{10}$

$P(L3) = \frac{1}{10}$ $P(L/L1) = \frac{2}{10}$

$P(L/L2) = \frac{4}{10}$ $P(L/L3) = \frac{1}{10}$

$$P(L2/L) = \frac{P(L/L2) \times P(L2)}{\sum_{i=1}^{3} P(L/Li) \cdot P(Li)}$$

$$= \frac{0.4 \times 0.5}{(0.4 \times 0.2)(0.1 \times 0.1)(0.5 \times 0.4)}$$

$$= \boxed{0.689655}$$

6. $X \sim U(1, 2)$ $Y = \frac{3}{6-x}$

$$F_Y(y) = P(Y \leq y) = P\left(\frac{3}{6-x} \leq y\right)$$

$$= P\left(x \leq 6 - \frac{3}{y}\right)$$

Page No.:

Date:

$\therefore F_Y(y) = F_X\left(6 - \frac{3}{y}\right)$

To find PDF we differentiate the function.

$F_Y'(y) = f_Y(y) = \frac{6y-3}{y}$

$= F_X'(g^{-1}(y))$

$x = 6 - 3/y$

$\frac{dx}{dy} = -\left(-\frac{3}{y^2}\right) = \frac{3}{y^2}$

$f_X(x) = \frac{1}{2-1} = 1$

$f_Y(y) = 1 \cdot \frac{3}{y^2} = \boxed{\frac{3}{y^2}}$

7\. (i) $F(3) = P(X=1) + P(X=2) + P(X=3)$

$= 0.05 + 0.15 + 0.35$

$= \boxed{0.55}$

(ii) $E(X) = \sum_{x=1}^{5} x \cdot P(X=x) = \boxed{3.25}$

(iii) $V(X) = E(X^2) - (E(X))^2$

$= 11.45 - (3.25)^2$

$= \boxed{0.8875}$

(iv) coefficient of skewness

$= \dfrac{3(\text{mean} - \text{median})}{\sigma}$

$= \dfrac{3(3.25 - 3)}{\sqrt{0.8875}}$

$= \boxed{0.796117}$

8. $X \sim Bin(15, 0.2)$

$P(X < 3) = P(X=0) + P(X=1) + P(X=2)$

$= {}^{15}C_0 \, (0.2)^0 \, (0.8)^{15}$

$= 0.035184 + 0.131941$

$+ 0.230897$

$= \boxed{0.398022}$

9. $X \sim NBin(3, 0.01)$ 7 trials

$P(X=7) = {}^6C_2 \cdot (0.01)^3 \cdot (0.99)^4$

$= \boxed{0.000014409}$

10. $\lambda = 2$ per working week

$\lambda = \frac{2}{5}$ for a day

$P(X > 2) = 1 - P(X \leq 2)$

$= 1 - 0.99207$

$= \boxed{0.00793}$

11. P(next 3 are boys) $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \boxed{\frac{1}{8}}$

$\because$ having a girl or a boy is independent of previously outcomes.

Page No.:
Date:

12. $X \sim Poi(10)$ per day

$X \sim Exp(\lambda = 10)$

$P(X < 0.1) = 1 - e^{-\lambda x}$

$= 1 - e^{-10 \times 0.1} = 1 - e^{-1}$

$= \boxed{0.63212}$

13. $X \sim U(75, 120)$ $f(x)$ is constant

$P(80 < x < 100) = F(100) - F(80)$

$= \frac{100 - 75}{120 - 75} - \frac{80 - 75}{120 - 75}$

$= \boxed{0.2\bar{2}}$

14. $X \sim \Gamma(5, 0.4)$

$P(X < 22.89)$

$= P(2\lambda x < 22.89 \times 2 \times 0.4)$

$= P(\chi^2_{10} < 18.312)$

$= \boxed{0.94974}$

(11)

15. (i) $\int_0^\infty \frac{R}{(x+5)^4}\,dx = 1 \Rightarrow R\int_0^\infty \frac{1}{(x+5)^4}\,dx = 1$

$\Rightarrow R\left[\frac{(x+5)^{-3}}{-3}\right]_0^\infty = 1 \Rightarrow R\left(\frac{1}{375}\right) = 1$

$R = \boxed{375}$

(ii) $\int_0^{10} \frac{375}{(x+5)^4}\,dx = \frac{375}{-3}\left[(x+5)^{-3}\right]_0^{10}$

$= -125\left((15)^{-3} - (5)^{-3}\right)$

$= \boxed{0.96296}$

(iii) $E(X) = \int_0^\infty x \cdot \frac{375}{(x+5)^4}\,dx$ $x+5 = t$, $x = t-5$, $dx = 1\,dt$

$= \int_0^\infty 375 \cdot \frac{t-5}{t^4}\,dt = 375\int_0^\infty \frac{1}{t^3} - \frac{5}{t^4}\,dt$

$= 375\left[\frac{t^{-2}}{-2} - \frac{5t^{-3}}{-3}\right]_0^\infty = \boxed{2.5}$