Financial Mathematics
Assignment - 1

Q1] $d^{(4)} = 8\%$

$\Rightarrow$ $d = 1 - \left(1 - \frac{d^{(n)}}{4}\right)^4$

$= 1 - (1 - 2\%)^4$

$d = 7.763184\%$

i) $\delta = -\ln(1-d)$

$= 8.081083\%$

ii) $i = \frac{d}{1-d}$

$= 8.416578\%$

iii) $d^{(12)} = 12\left[1-(1-d)^{1/12}\right]$

$= 8.053934\%$

Q2] $\delta(t) = 0.004t + 0.0002t^2$

$\Rightarrow$ i) Amt. due $= C = ₹1000$

$PV_0 = C \times v(t)$

$= 1000 \times e^{-\int_0^{10} 0.004t + 0.0002t^2}$

$= 1000 \times e^{-[0.002t^2 + 0.0002\frac{t^3}{3}]_0^{10}}$

$= 1000 \times e^{-0.266666667}$

$= 765.93$

ii) $(1+i)^{10} = e^{0.266666667}$

$i = 2.7025403\%$

Q3] i) The relationship $d = iv$, has an interesting verbal interpretation. Interest earned on an investment, paid at the beginning of the period is d. Interest earned on an investment, paid at the end of the period is i. Therefore, if we discount i from the end of the period with the discounting factor v, we obtain d.

ii) (a) $\frac{d^{(4)}}{4} = 2.25\%$

$d = 1-(1-2.25\%)^4$

$d = 8.7007806\%$

$d^{(2)} = 2(1-(1-d)^{1/2})$

$d^{(2)} = 8.89875\%$

(b) $d^{(1/2)} = 5\%$

$d = \cancel{(1+} \; 1-\left(1-\frac{d^{(1/2)}}{(1/2)}\right)^{1/2}$

$d = 1-(1-10\%)^{1/2}$

$d = 5.1316702\%$

$d^{(2)} = 2(1-(1-d)^{1/2})$

$d^{(2)} = 5.1992507\%$

iii) $v\left(0; \frac{91}{365}\right) = \frac{1}{(1+i)^{(91/365)}} = \left(1-\frac{91}{365}d\right)$

$\frac{1}{(1.05)^{91/365}} = \left(1-\frac{91}{365}d\right)$

$d = \frac{365}{91}\left(1-\frac{1}{(1.05)^{91/365}}\right)$

$d = 4.849462\%$

Q4] i) The relationship $d=iv$, has an interesting verbal interpretation. Interest earned on an investment, paid at the begining of the period is d. Interest earned on an investment, paid at the end of the period is i. Therefore, if we discount i, from the end of the period with the discounting factor v, we obtain d.

ii) $i = 0.08$

(a) $d = \frac{i}{1+i} = 7.407407407\%$

$d^{(12)} = 12\left[1-(1-d)^{1/12}\right]$

$d^{(12)} = 7.671478\%$

(b) $i^{(365)} = 365\left((1+i)^{1/365} - 1\right)$
$= 7.696915\%$

(c) $= \delta = \ln(1+i)$
$= 7.696104\%$

(d) $i^{(0.5)} = \frac{1}{2}\left((1+i)^{1/1/2} - 1\right)$
$= \frac{1}{2}\left((1+i)^2 - 1\right)$
$= 8.32\%$

Q5) i) $d^{(4)} = 6\%$
$d = 1 - (1 - 1.5\%)^4$
$d = 5.866344937\%$

(a) $i = \frac{d}{1-d} = 6.231931538\%$
$i^{(2)} = 2\left((1+i)^{1/2} - 1\right)$
$i^{(2)} = 6.137752\%$

(b) $d^{(12)} = 12\left[1 - (1-d)^{1/12}\right]$
$d^{(12)} = 6.030253\%$

ii) $i = \frac{\cancel{20,000}}{\cancel{2,00,000}} \times 1\cancel{0}0 = 10\%$

No. of days $= 93$

$AV_{93\,days} = 2,00,000 \times (1.1)^{93/365}$
$= 2,04,916.3564$

Q6) Let the retail price be x

If retailer pays cash $= 70\%(x)$
then he has to pay
$PV = 0.7x$

If retailer uses the 6 month credit $= 75\%(x)$
mode, then he has to pay
$AV = 0.75x$ in 6 months.

We need to find the discounting done on 6 mnths credit made for the retailers who pay in cash.

Time = 6 mnths = $\frac{1}{2}$ yrs

$$PV_0 = AV(1-d)^n$$

$$0.7x = 0.75x(1-d)^{1/2}$$

$$d = 1-\left(\frac{0.7}{0.75}\right)^2$$

$$d = 12.888889\%$$

ii) (a) $d^{(12)} = 12[1-(1-d)^{1/12}]$

$d^{(12)} = 13.719544\%$

(b) $i = \frac{d}{1-d} = 14.79591837\%$

$i^{(2)} = 2[(1+i)^{1/2}-1]$

$i^{(2)} = 14.285714\%$

(c) $\delta = -\ln(1-d)$

$\delta = 13.798574\%$

Q7] Amt inv: $C = 20000$

$AV_{17 mnths} = 26000$

17 mnths $= 1+\frac{5}{12}$ yrs

$= 1.416666667$ yrs

$$AV = C(1+i)^n$$

$$\cancel{25000} = \cancel{20000}(1+i)^{1.416666667}$$

$$i = 20.34570672\%$$

i) $i^{(4)} = 4((1+i)^{1/4}-1)$

$i^{(4)} = 18.955255\%$

ii) $d = \frac{i}{1+i} = 16.9060511 4\%$

$d^{(2)} = 2(1-(1-d)^{1/2})$

$d^{(2)} = 17.688235\%$

iii) $26000 = 20000(1 + 1.416666667\, i_s)$

$i_s = 21.1764706\%$.

Q9) i) Force of interest: The measure of interest at individual moments of time or at any interest of time is known as force of interest.

ii) $i^{(2)} = 0.0775$

(a) $i = \left(1 + \frac{i^{(2)}}{2}\right)^2 - 1$

$i = 7.900156\%$.

(b) $\delta = \ln(1+i)$

$= 7.603813\%$.

(c) $d = \frac{i}{1+i}$

$= 7.321728276\%$.

$d^{(12)} = 12\left(1-(1-d)^{1/12}\right]$

$d^{(12)} = 7.579579\%$.

(d) $i^{(1/2)} = \frac{1}{2}\left((1+i)^2 - 1\right)$

$= 8.212219\%$.

Q10) $\delta(t) = \begin{cases} 0.08 & 0 \leq t \leq 5 \\ 0.06 & 5 \leq t \leq 10 \\ 0.04 & t \geq 10 \end{cases}$

When $0 \leq t \leq 5$

$v(t) = e^{-\int_0^t 0.08}$

$e^{-0.08t}$

When, $5 \leq t \leq 10$

$v(t) = e^{-\int_0^5 0.08 - \int_5^t 0.06}$

$= e^{-[(0.08t)_0^5 + (0.06t)_5^t]}$

$= e^{-0.4 + 0.3 - 0.06t}$

$= e^{-0.06t - 0.1}$

When $t \geq 10$

$$v(t) = e^{-\int_0^5 0.08 - \int_5^{10} 0.06 - \int_{10}^t 0.04}$$

$$= e^{-[\int_0^5 (0.08t) + \int_5^{10} (0.06t)] + \int_{10}^t (0.04)}$$

$$= e^{-0.4 + 0.3 - 0.6 + 0.4 - 0.04t}$$

$$= e^{-0.07t - 0.3}$$

Q12) Amt invested $= C = 7049$

$$AV_6 = C \cdot A(0,6)$$

$$= 7049 \times A(0,2) \times A(2,4.5) \times A(4.5,6)$$

$\rightarrow$ For $A(0,2)$

$$i^{(12)} = 6\%$$

$$i^{(12)}/12 = 0.5\%$$

$$A(0,2) = (1.005)^{24}$$

$\rightarrow$ For $A(2,4.5)$

i per quarter $= 1.5\%$

$$A(2,4.5) = (1 + 10(0.015)) = (1.15)$$

$\rightarrow$ For $A(4.5,6)$

$$d^{(12)} = 6\%$$

$$d^{(12)}/12 = 0.5\%$$

$$A(4.5,6) = \frac{1}{(1-0.005)^{18}}$$

$$AV_6 = 7049 \times A(0,2) \times A(2,4.5) \times A(4.5,6)$$

$$= 7049 \times (1.005)^{24} \times (1.15) \times \frac{1}{(1-0.005)^{18}}$$

$$AV_6 = 9999.894$$