FM - Assignment

1) $i^{(4)} = 8\%$

a) $i = \left(1 + \frac{i^{(4)}}{4}\right)^4 - 1$

$i = (1 + 0.02)^4 - 1$

$i = 8.243216\%$

$1 + i = e^{\delta}$

$\delta = \ln(1+i)$

$\delta = \ln(1.08243216)$

$\delta = 7.921050918\%$

b) $i = 8.243216\%$

c) $d^{(12)}$, $d = \frac{i}{1+i} = 7.615457\%$

$1 - d = \left(1 - \frac{d^{(12)}}{12}\right)^{12}$

$d^{(12)} = 12\left[1 - (1-d)^{1/12}\right]$

$= 12\left[1 - (1 - 7.615457\%)^{1/12}\right]$

$= 7.89496\%$

2) $\delta(t) = 0.004t + 0.0002t^2$

i) $PV = C \cdot V(0,10)$

$$= \frac{1000}{A(0,10)}$$

$$= \frac{1000}{e^{\int_0^{10} 0.004t + 0.0002t^2 \, dt}}$$

$$= \frac{1000}{e^{\left[0.002t^2 + \frac{0.0002}{3} t^3\right]_0^{10}}}$$

$$\frac{1000}{e^{[0.2 + 0.2/3]}} = \frac{1000}{e^{0.8/3}}$$

$$= 765.9283384$$

ii) $\cancel{1+i = e}$ $1000 = 765.9283 \times (1+i)^{10}$

$$(1.30560)^{1/10} = 1+i$$

$$i = 2.7025\%$$

3) i) Interest earned on an investment of 1 paid at the beginning of the period is d. Interest earned on a investmend of 1 paid at the end of period is i. Therefore, if we discount i from the end of the period with the discount factor v, we obtain d.

4) ii) $i = 0.08$

a) $d^{(12)}$

$$d^{(p)} = p\left[1-(1+i)^{-1/p}\right]$$

$$d^{(12)} = 12\left[1-(1.08)^{-1/12}\right]$$

$$= 7.671477\%$$

b) i^{365}

$$i^{(p)} = p\left[(1+i)^{1/p}-1\right]$$

$$i^{365} = 365\left[(1.08)^{1/365}-1\right]$$

$$= 7.696915\%$$

c) $\delta = \cancel{\log(1} \ln(1+i)$

$$= \ln(1.08)$$

$$= 7.696104\%$$

d) $i^{0.5}$

$$i^{0.5} = 0.5\left[(1.08)^{1/0.5}-1\right]$$

$$i^{0.5} = 8.32\%$$

5) $d^{(4)} = 6\%$

a) $i^{(2)} = ?$

$$\frac{1}{1+i} = \left(1 - \frac{d^{(4)}}{4}\right)^4$$

$$1+i = (1 - 1.5\%)^{-4}$$

$$i = 6.23193\%$$

$$i^{(2)} = 2\left((1+i)^{1/2} - 1\right)$$
$$= 6.13775\%$$

b) $d^{(12)}$

$$\cancel{1-d = \left(1 - \frac{d^{(4)}}{4}\right)^4}$$

$$\cancel{1-d} = d = \frac{i}{1+i} = \frac{0.0623193}{1.0623193}$$
$$= 5.86634\%$$

$$1-d = \left(1 - \frac{d^{(12)}}{12}\right)^{12}$$

$$d^{(12)} = \left(1 - (1-d)^{1/12}\right) \times 12$$
$$= 6.03024\%$$

17 Months

ii) 15/4/05 — 17/7/05 — 15/4/06

2L — 2.2L

a) $i = 10\%$

$\frac{i}{4} = 2.5\%$ (duration reduced ~~by~~ to 1/4th as 3 months is 1/4th of year)

$2{,}00{,}000 \times 2.5\%$

2,05,000

6) a) Cash payment → 30%

~~$70 = 100 \times (1-d)$~~

7) 17 Months

20,000 $\quad\quad$ 26000

$$i = \frac{26000 - 20000}{20000} \times \frac{12}{17}$$

$$= \frac{6}{20} \times \frac{12}{17}$$

$$= 21.1764\%$$

i) $i^{(4)} = 4\left[(1+i)^{1/4} - 1\right]$

$$= 19.6763\%$$

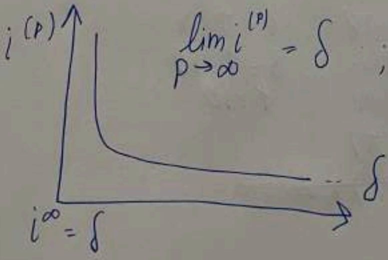
ii) $d^{(2)} = 2\left(1 - (1+i)^{-1/2}\right)$

$$= 18.3142\%$$

iii) $i = 21.1764\%$

iv) As i increase, $d^{(p)}$ & $i^{(p)}$ starts to have a larger difference.

8) $\lim_{p \to \infty} i^{(p)} = \delta \; ; \; p \uparrow \rightarrow i^{(p)} \downarrow$



9) $i^{(2)} = 0.0775$

i) Force of Interest is the instantaneous change in the fund value expressed as an annualised percentage of the current fund value

ii) $i = \left(1 + \frac{i^{(2)}}{2}\right)^2 - 1$

$= \left(1 + \frac{0.0775}{2}\right)^2 - 1$

$= 7.90\%$

iii) $\delta = \cancel{\log(1+}\ \ln(1+i)$

$= 7.60\%$

iv) $d^{(12)} = 12\left[1 - (1+i)^{-1/12}\right]$

$= 7.579\%$

v) $i^{(0.5)} = 0.5\left[(1+i)^{1/0.5} - 1\right]$

$= 8.21205\%$

10) $\delta(t) = \begin{cases} 0.08 & ; 0 \le t \le 5 \\ 0.06 & ; 5 \le t \le 10 \\ 0.04 & ; t \ge 10 \end{cases}$

$PV = C \times V(0,t)$

$= \cancel{1 \times V(0,5) \times V(5,10}$

Assuming $0 \le t < 5$

$$PV = V(0,t)$$

$$= \frac{1}{A(0,t)}$$

$$= \frac{1}{e^{\int_0^t 0.08s \, ds}} = \frac{1}{e^{[0.08s]_0^t}}$$

$$= \frac{1}{e^{0.08t}}$$

Assuming $5 \le t < 10$

$$PV = V(0,5) \times V(5,t)$$

$$= \frac{1}{e^{0.08 \times 5}} \times V(5,t)$$

$$V(5,t) = \frac{1}{e^{\int_5^t 0.06s \, ds}} = \frac{1}{e^{[0.06s]_5^t}}$$

$$= \frac{1}{e^{0.06t - 0.06 \times 5}} = \frac{1}{e^{0.06t - 0.3}}$$

$$PV = \frac{1}{e^{0.4}} \times \frac{1}{e^{0.06t - 0.3}} = \frac{1}{e^{0.1 - 0.06t}}$$

Assuming $t \ge 10$

$$PV = V(0,5) \times V(5,10) \times V(10,t)$$

$$= \frac{1}{e^{0.1 - 0.06 \times 10}} \times V(10,t)$$

$$V(10,t) = \frac{1}{e^{\int_{10}^t 0.04s \, ds}} = \frac{1}{e^{[0.04s]_{10}^t}} = \frac{1}{e^{0.04t - 0.4}}$$

$$PV = \frac{1}{e^{0.01-0.6}} \times \frac{1}{e^{0.04t-0.4}}$$

$$= \frac{1}{e^{0.04t-0.9}}$$

$$P(0,t) = \begin{cases} e^{-0.06t} & ; 0 \le t \le 5 \\ e^{0.06t-0.1} & ; 5 \le t \le 10 \\ e^{0.9-0.04t} & ; t \ge 10 \end{cases}$$

11) b) i) $\delta = 6\%$

$\cancel{\frac{1}{1-d} = e^{\delta}}$ $\quad d = 1 - e^{-\delta}$

$d = 1 - e^{-0.06}$

$\delta = \ln\left(\frac{1}{1-d}\right)$ $\quad d = 5.82354\%$

$$d^{(12)} = 12\left[1 - (1-d)^{1/12}\right]$$

$$= 5.9850\%$$

ii) $d^{(4)} = 6\%$

$i^{(12)}, i^{(4)} = ?$

$$d^{(4)} = 4\left[1 - (1+i)^{-1/4}\right]$$

$$(1+i) = \left(1 - \frac{6\%}{4}\right)^{-4}$$

$i = 6.2319\%$

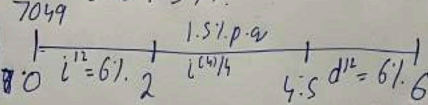
$i^{(12)} = 12\left[(1+i)^{1/12} - 1\right]$

$= 6.0606\%$

$i^{(4)} = 4\left[(1+i)^{1/4} - 1\right]$

$= 6.09134\%$

12)



$V_0 = 7049 \times A(0,2) \times A(2,4.5) \times A(4.5,6)$

~~$= 7049 \times$~~

$A(0,2)$ $\qquad \dfrac{i^{(12)}}{12} = 0.5\%$

$A(0,24)$

$(1+i)^{24} = (1 + 0.5\%)^{24} = (1.005)^{24}$

$= 1.12716$

$A(2,4.5) = A(8,18) = (1+i)^{10} = (1.015)^{10}$

$= 1.16054$

$A(4.5,6) = A(54,72) = \dfrac{1}{V(54,72)} = \dfrac{1}{\left(1 - \frac{d^{(12)}}{12}\right)^{18}}$

$= \dfrac{1}{(1-0.005)^{18}} = 1.094421$

$AV_6 = 7049 \times 1.12716 \times 1.16054 \times 1.094421$

$= 10091.55$

13) a) $i = 10\%$ p.a

$\cancel{200 = 100 \times (1+i)^n}$

$(1+i)^n = 2$

$(1.1)^n = 2$

$n \log(1.1) = \log 2$

$n = \dfrac{\log 2}{\log(1.1)}$

$= 7.27254$ yrs

b) $d^{(12)} = 10\%$ p.a

$d = 1 - \left(1 - \dfrac{d^{(12)}}{12}\right)^{12}$

$= 9.554162\%$

$2 = \dfrac{1}{(1-d)^n}$

$-n \log(1-d) = \log 2$

$n = \dfrac{-\log 2}{\log(1-d)} = 6.90255$ Yrs