Calculus assignment

1) $\int_{1/50}^{2} \frac{e^{2/w}}{w^2} \, dw$

let $t = \frac{2}{w}$

$\frac{dt}{-2} = \frac{dw}{w^2}$

$-\frac{1}{2} \int_{1/50}^{2} e^t \, dt$

$= -\frac{1}{2} [e^t]_{1/50}^{2}$

$= -\frac{1}{2} [e^2 - e^{1/50}]$

$= -3.1844$

2) $\int \frac{2t^3 + 1}{(t^4+2t)^3} \, dt$

let $u = t^4 + 2t$

$\frac{du}{dt} = 4t^3 + 2$

$\frac{du}{2} = (2t^3 + 1) \, dt$

$\frac{1}{2} \int \frac{du}{u^3}$

$= \frac{1}{2} \times \frac{u^{-2}}{-2} + C$

$= -\frac{u^{-2}}{4} + C$ Ans

Q3) $f'(x) = x^4 + 3x - 9$ $\quad f(x) = \int f'(x)\,dx$

$$f(x) = \int (x^4 + 3x - 9)\,dx$$

$$= \frac{x^5}{5} + \frac{3x^2}{2} - 9x + C$$

Q4) $f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \le 1 \end{cases}$

$$\int_{-2}^{3} f(x)\,dx = \int_{-2}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx$$

$$\int_{-2}^{1} 3x^2\,dx + \int_{1}^{3} 6\,dx$$

$$[x^3]_{-2}^{1} + [6x]_{1}^{3}$$

$$1 + 8 + 18 - 6 = 21$$

6) $y''(x) = 15\sqrt{x} + 5x^3 + 6$

$y'(x) = \cancel{15}^{5} \times \frac{2}{\cancel{3}_1} x^{3/2} + 5\frac{x^4}{4} + 6x + C_1$

$y(x) = 10 \times \frac{2}{5} \times x^{5/2} + \frac{5}{4} \times \frac{x^5}{5} + \frac{6x^2}{2} + C_1 x + C_2$

$y(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 + C_1 x + C_2$

$y(1) = -5/4 = 4 + \frac{1}{4} + 3 + C_1 + C_2$

$-5/4 = 7 + \frac{1}{4} + C_1 + C_2$

$C_1 + C_2 = \frac{-17}{2}$ — ①

$y(4) = 404 = 128 + 256 + 48 + 4C_1 + C_2$

$$
\begin{aligned}
4C_1 + \cancel{C_2} &= -28 \quad —② \\
-\ C_1 + \cancel{C_2} &= -17/2 \\
- \qquad - \qquad & \quad + \\
\hline
3C_1 &= \frac{-39}{2}
\end{aligned}
$$

$C_2 = -2$ Ans

$C_1 = -\frac{\cancel{13}}{\cancel{3}} - \frac{13}{2}$ Ans

7) $f(x+iy)+g(x-iy)$

$\Rightarrow f(x)+g(x)+if(y)-ig(y)$

8) $\frac{dr}{d\theta} = \frac{r^2}{\theta}$

$r = \frac{-1}{\log(\theta)}$

5) $\int 3(8y-1)e^{4y^2-y}\,dy$

$\int 3e^{x}\,dx$ let $4y^2-y = x$

$\frac{dx}{dy} = 8y-1$

$= 3e^{x}+C$

$= 3e^{4y^2-y}+C$ Ans

Q9) $$\frac{dv}{dt} = 9.8 - 0.196v$$

$$\int \frac{dv}{9.8 - 0.196V} = \int dt$$

$$\log(9.8 - 0.196v) = -0.196t + C$$

Q12) Talor series for $f(x) = x^3 - 10x^2 + 6$

$f(x) = x^3 - 10x^2 + 6$ $\quad$ $f(3) = -57$

$f'(x) = 3x^2 - 20x$ $\quad$ $f'(3) = -33$

$f''(x) = 6x - 20$ $\quad$ $f''(3) = -2$

$f'''(x) = 6$ $\quad$ $f'''(3) = 6$

$$f(x) = f^{0}(3) + (x-3)f'(x) + \frac{(x-3)^2}{\lfloor 2} f''(3)$$
$$\frac{(x-3)^3}{\lfloor 3} f'''(3) \ldots\ldots$$

$$x^3 - 10x^2 + 6 = -57 + (x-3) - 33 + \frac{(x-3)^2}{2}(-2)$$
$$\frac{6(x-3)^3}{\lfloor 3} \ldots\ldots$$

Q 13) Maclaurin series for $(1+x)$

$$f'(x) = u(1+x)^{u-1}$$

$$f''(x) = (u)(u-1)(1+x)^{u-2}$$

$$f'''(x) = (u)(u-1)(u-2)(1+x)^{u-3}$$

$$f(x) = f(0) + \frac{x^1}{\angle 1} f'(0) + \frac{x^2}{\angle 2} f''(0) + \frac{x^3}{\angle 3} f'''(0) \ldots$$

$$\cancel{f(x) = f(0) + \frac{x^1}{\,}}$$

$$(1+x)^{u} = 1 + u + (u-1)(u) + (u)(u-1)(u-2) \ldots$$

Q 14) $f(x) = \sqrt{1+x}$

$$f'(x) = \frac{1}{2}(1+x)^{-1/2} \qquad f'(0) = 1/2$$

$$f''(x) = -\frac{1}{4}(1+x)^{-3/2} \qquad f'(0) = -1/4$$

$$f'''(x) = -\frac{3}{8}(1+x)^{-5/2} \qquad f''(0) = -3/8$$

$$\sqrt{1+x} = \sqrt{1+x} \quad + \frac{x^1}{\angle 1}\left(\frac{1}{2}\right) + \frac{x^2}{\angle 2}\left(\frac{-1}{4}\right) + \frac{x^3}{\angle 3}\left(\frac{-3}{8}\right) \ldots$$

Q15) Taylor Series

$f(x) = e^{-6x}$ $\qquad x = -4$

$f(x) = e^{-6x}$ $\qquad f(-4) = e^{+24}$

$f'(x) = e^{-6x}(-6)$ $\qquad f'(4) = e^{24}(6)^1$

$f''(x) = (-6)e^{-6x}(-6)$ $\qquad f''(-4) = e^{24}(6)^2$

$f'''(x) = (-6)^3 e^{-6x}$ $\qquad f'''(-4) = e^{24}(6)^3$

$$e^{-6x} = e^{+24} + \frac{(x-4)}{L1}(e^{24})(6) + \frac{(x-4)^2}{L2}(e^{24})(6)^2 + \frac{(x-4)^3}{L3}(e^{24})(6)^3 \ldots\ldots$$

Q16)

$f(x) = \ln(3+4x)$ $\qquad x = 0$ $\qquad \ln(3)$

$f'(x) = \frac{4}{3+4x}$ $\qquad f'(0) = 4/3$

$f''(x) = (-4)(4x+3)^{-2}(4)$ $\qquad f''(0) = (-1)(4)^2(3)^{-2}$

~~$f''(x) = (-4)(4x+3)^{-2}$~~ (4)

$f'''(x) = (+8)(4x+3)^{-3}(4)^2$ $\qquad f'''(0) = (-1)(-2)(3)^{-2}(4)^3$

$$\ln(3+4x) = \ln(3) + \frac{x}{L1}4/3 + \frac{x^2}{L2}(-1)(4)^2(3)^{-2} + \frac{x^3}{L3}(4)^3(2)(3^{-2})\ldots\ldots$$