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FM Assignment.
Krishangi Trivedi 82

Q.1. $d^{(4)} = 8\%$

(i) $\delta = \ln\left(\frac{1}{(1-d)}\right)$

$$\left(1-\frac{d^{(4)}}{4}\right)^4 = (1-d)$$

$$\left(1-\frac{8\%}{4}\right)^4 = 1-d$$

$\therefore d = 0.077632$

$$\delta = \ln\left(\frac{1}{(1-d)}\right)$$

$$= \ln\left[\frac{1}{(1-0.077632)}\right]$$

$\therefore \delta = \underline{0.080811} = 8.0811\%$

(ii) $$(1+i) = \frac{1}{\left(1-\frac{d^{(4)}}{4}\right)^4} = \frac{1}{0.922368}$$

$1+i = 1.084166$

$\therefore i = \underline{0.084166} = 8.4166\%$

(iii) $$\left(1-\frac{d^4}{4}\right)^4 = \left(1-\frac{d^{(12)}}{12}\right)^{12}$$

$$(0.922368)^{1/12} = \left(1-\frac{d^{12}}{12}\right)$$

$$\left(1 - \frac{d^{(12)}}{12}\right) = 0.993288$$

$$\frac{d^{12}}{12} = 0.006712$$

$$\therefore d^{(12)} = 0.080539 = 8.0539\%$$

(2.) (i) $\delta(t) = 0.004(t) + 0.0002t^2$

$n = 10$ years $C = 1000$

$$PV_0 = C \times e^{-\int_0^n \delta(t)\,dt}$$

$$PV_0 = 1000 \times e^{-\int_0^{10} 0.004(t) + 0.0002 t^2\,dt}$$

$$= 1000 \times e^{-\int_0^{10} 0.004(t) + 0.0002 t^2\,dt}$$

$$PV_0 = 1000 \times e^{-\left[0.002(t^2) + 0.000067(t)^3\right]_0^{10}}$$

$$PV_0 = 1000 \times \cancel{e^{-[0.2 + 0.067]}}\ e^{-[0.2 + 0.66667]}$$

$$PV_0 = 1000 \times \cancel{e^{-[0.267]}}\ e^{-[0.26666]}$$

$$PV_0 = 1000 \times \cancel{0.7657307}1\ 0.765928$$

$$PV_0 = \cancel{765.7307}1\ 765.928$$

Present value is ₹ 765.75071

(ii) We know that

$$e^{\delta(t)} = (1+i)^t$$

$$\therefore e^{\int_0^{10} 0.004t + 0.0002t^2\,dt} = (1+i)^{10}$$

(ii) $\therefore \; e^{[0.266667]} = (1+i)^{10}$

$(1.305605)^{1/10} = (1+i)$

$\therefore \; i = 0.027025$

$= 2.7025\%$

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3 (i) We know that.

$$(1-d) = \frac{1}{1+i}$$

$$\therefore d = 1 - \frac{1}{1+i}$$

$$= \frac{1+i-1}{1+i}$$

$$= \frac{i}{1+i} = i \times \frac{1}{1+i} = iv.$$

$$\left[\because v = \frac{1}{1+i}\right]$$

(ii) (a) $\frac{d^{(4)}}{4} = 2.25\%$

$$\therefore \left(1 - \frac{d^{(2)}}{2}\right)^2 = \left(1 - \frac{d^{(4)}}{4}\right)^4$$

$$\therefore \left(1 - \frac{d^{(2)}}{2}\right)^2 = (1 - 2.25\%)^4$$

$$1 - \frac{d^{(2)}}{2} = (0.912992)^{1/2}$$

$$\frac{d^{(2)}}{2} = 1 - 0.955506$$

$$d^{(2)} = 2 \times 0.04494$$

$$= 0.089988$$

$$d^{(2)} = 8.8988\%$$

Ans. ~~Rate of~~ $d^{(2)} = 8.8987\%$.

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(ii) (b) $d^{(1/2)} = 5\%$

$$\left[1 - \frac{d^{1/2}}{1/2}\right]^{1/2} = \cancel{(1-d)} \left(1 - \frac{d^{(2)}}{2}\right)^2$$

$$\left[1 - \frac{5\%}{1/2}\right]^{1/2} = \left[1 - \frac{d^{(2)}}{2}\right]^2$$

$$(0.948683)^{1/2} = 1 - \frac{d^{(2)}}{2}$$

$$\therefore d^{(2)} = 0.025996 \times 2$$

$$= 0.051993$$

$$d^{(2)} = 5.1993\%$$

iii) $n = 91$ days. ; Rate of return is 5%.

Let d be the discount rate of the government bill.
Let the payout of the ~~govt~~ government bill be ₹100.

Assume the present value of the bill to be X such that

$$X = 100 \times (1 + 5\%)^{-91/365}$$
$$= 100 \times (1.05)^{-91/365}$$

$$X = 98.790$$

Now, for simple annual discount rate ; →

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$$100 \times \left[1 - \frac{91}{365}(d)\right] = 98.790$$

$$\therefore 1 - \frac{91}{365}(d) = \frac{98.790}{100}$$

$$\therefore d = (1 - 0.98790) \times \frac{365}{91}$$

$$= 0.012090 \times \frac{365}{91}$$

$$\therefore d = 0.048495$$

$$d = 4.8495\%$$

4(i) $$(1-d) = \frac{1}{1+i}$$

$$\therefore d = 1 - \frac{1}{1+i} = \frac{i}{1+i} = i \frac{1}{1+i} = iv.$$

hence $d = iv$.

(ii) $i = 0.08$

(a) $d^{(12)}$

$$\frac{1}{(1+i)} = \left(1 - \frac{d^{(12)}}{12}\right)^{12}$$

$$\therefore (1+i)^{-1/12} = \left(1 - \frac{d^{(12)}}{12}\right)$$

$$\left(1 - (1.08)^{-1/12}\right) \times 12 = d^{(12)}$$

$$\therefore d^{(12)} = 0.076715$$

$$= 7.6715\%$$

(b) = $i^{(365)}$

$$\left(1+\frac{i^{(365)}}{365}\right)^{365} = (1+i)$$

$$\left(1+\frac{i^{365}}{365}\right) = (1+i)^{1/365}$$

$$i^{(365)} = \left[(1.08)^{1/365} - 1\right] \times 365$$

$$i^{(365)} = 0.076969$$

$$= 7.6969\%$$

(c) δ =

$$\delta = \ln(1+i)$$

$$= \ln(1.08)$$

$$\delta = 0.076961$$

(d) = $i^{(0.5)}$

$$\left(1+\frac{i^{0.5}}{0.5}\right)^{0.5} = (1+i)$$

$$i^{(0.5)} = \left[\left[(1+i)^{1/0.5}\right] - 1\right] \times 0.5$$

$$i^{(0.5)} = 0.0832$$

$$i^{(0.5)} = 8.32\%$$

5(i) $\quad d^{(4)} = 6\%$

$$\left(1+\frac{i^{(2)}}{2}\right)^2 = \frac{1}{\left(1-\frac{d^{(4)}}{4}\right)^4} = \frac{1}{\left(1-\frac{0.06}{4}\right)^4}$$

$$= \cancel{\frac{1}{(0.941337)^{1/2}}}$$

$$\therefore \left(1+\frac{i^{(2)}}{2}\right) = \frac{1}{\left(1-\frac{0.06}{4}\right)^2} = \quad 1+\frac{i^{(2)}}{2} = 1.0306887$$

$$\therefore i^{(2)} = 0.061377$$

(b) $i^{(12)} =$

$$i^{(12)} = 1 - \left[\left(1 - \frac{d^{(4)}}{4}\right)^{4/12}\right] \times 12$$

$$i^{(12)} = 0.060303$$

$$i^{(12)} = 6.0303\%$$

(ii) Amount Borrowed by amit = 200,000

Amount to be repaid by Amit = 2,20,000

$n = 1$ year.

$\therefore$ Accumulation factor $= \frac{2,20,000}{200000}$

$$(1+i)^1 = 1.1$$

$$\therefore i = 0.1$$

Amit Repays The amount in just ~~93~~ 93 days

Hence he pays it 272 days prior.

$\therefore$ The amount to be subtracted $= \frac{272}{365} \times 200000$ ₹

$= 149044.10959$.

Therefore; The amount to be paid after 93 days.

$= 220000 - 149044.10959$

$= 205905.8904$

(6) (i) Assuming that the retail price is ₹100.
The cash price is ₹70 ~~and~~ or
₹75 to be paid in 6-month's time.

$$\left(1 - \frac{d^{(2)}}{2}\right)^2 = (1-d)$$

$$\therefore \left(1 - \frac{d^{(2)}}{2}\right) = (1-d)^{1/2}$$

$$\frac{70}{75} = (1-d)^{1/2}.$$

$$(0.93333)^2 = (1-d)$$

$$\therefore d = 0.1289.$$

$$= 12.89\%.$$

ii) $d^{(12)}$

$$\left[1-(1-d)^{1/12}\right] \times 12 = d^{(12)}$$

$$\left[1-(1-0.1289)^{1/12}\right] \times 12 = d^{12}$$

$$\therefore d^{(12)} = \cancel{1387}\ 0.137208$$

$$= 13.7208\%$$

$i^{(2)}$

$$i = \frac{d}{1-d} = 0.147842$$

$$i^{(2)} = \left[(1+i)^{1/2} - 1\right]2$$

$$= \left[(1.147842)^{1/2} - 1\right]2$$

$$= 0.142748$$

$$= 14.2748\%$$

$$\delta = \ln(1+i)$$

$$= \ln(1.147842)$$

$$= 0.137884$$

(7.) ₹ 20000 to ₹ 26,000

in 14 months.

i.e. ~~152.5 days.~~ 517.5 days.

$\therefore$ Accumulation factor $= \left(\cancel{(1+i)^{\frac{152.5}{365}}}\right)$

$$= (1+i)^{\frac{517.5}{365}} = \frac{26000}{20000}$$

$$(1+i) = (1.3)^{365/517.5}$$

~~$i = 0.203273\%$~~

$$i = 0.203495.$$

(i) $i^{(4)}$

$$\left(1+\frac{i^{(4)}}{4}\right)^4 = 1+i$$

$$\therefore i^{(4)} = \left[(1+i)^{1/4} - 1\right] 4$$

$$= 0.189584.$$

(ii) $d^{(2)}$.

$$d = \frac{i}{1+i}$$

$$d = 0.169083$$

$$\left[1 - (1-d)^{1/2}\right] \times 2$$

$$\therefore d^{(2)} = 0.176907$$

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iii. Simple interest (i)

$$\left(1 + \frac{517}{365} i\right) = 1.3$$

$$\therefore \frac{517}{365} i = 0.3$$

$$i = 0.211799$$

7(iv.) Interest rates keep on decreasing.
But, d has the lowest value and
i has the highest value.
$d < d^{(12)} < i^{(4)} < i$

Simple ~~rat~~ rate of interest ~~has~~ is higher than
the compound discount rate to maintain the
same amount

(8) $i = 8\%$

We know that.

$$(1+i) = \left(1+\frac{i^{(p)}}{p}\right)^{p}$$

$$\therefore \; i^{(p)} = \left[(1+i)^{1/p} - 1\right] \times p$$

$$\therefore \; i^{(p)} = \left[(1.08)^{1/p} - 1\right] \times p$$

When $p = 2$

$i^{(p)} = \left[(1.08)^{1/2} - 1\right] \times 2$

$i^{(p)} = 0.078461$

$= 7.8\%$

When $p = 4$

$i^{(4)} = 0.077706$

$= 7.7\cancel{8}7\%$

When $p = 5$

$i^{(5)} = 7.755\%$

$p = 10$

$i^{(p)} = 0.07725$

$= 7.725$

$p = 30$

$i^{(30)} = 7.7060\%$

$p = 50$

$i^{(50)} = 7.7020\%$

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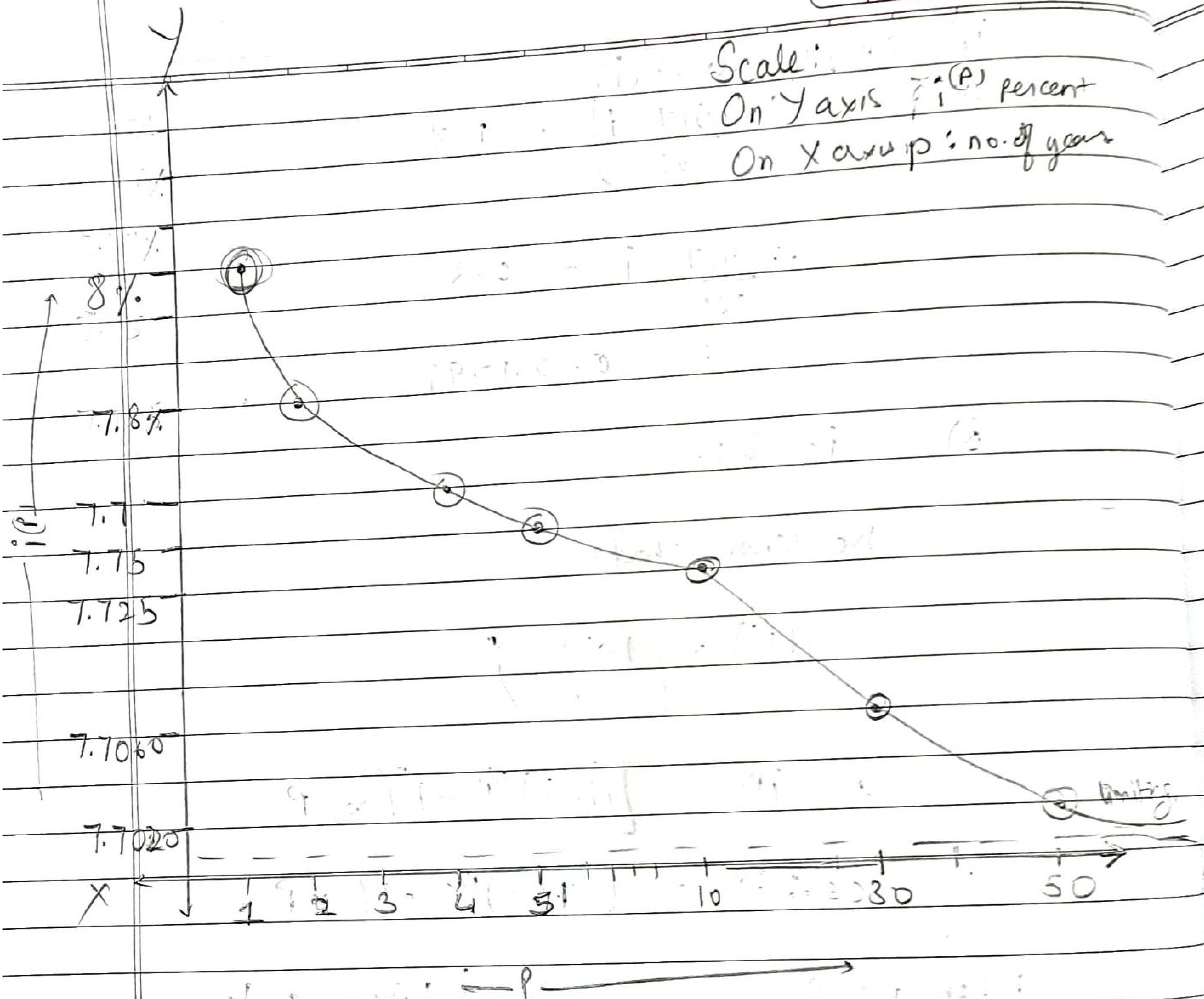
Scale:
On Y axis $i^{(p)}$ percent
On X axis p : no. of years



→ Hence we can conclude that;
as 'p' (no. of years) increases;

$i^{(p)}$ goes on decreasing.
This change is less significant as time goes on.
There is a limiting factor to the decrease of $i^{(p)}$

This is known as 'force of interest' (δ)
When value of 'p' ~~reaches~~ tends to ∞ ($p \to \infty$) there is an instantaneous change.

(9) (i) Force of interest is the nominal rate of interest or ~~discount~~ nominal rate of discounted convertible infinitely.

- It is the rate of interest compounded continuously.
- As 'p' goes on increasing ; $j^{(p)}$ decreases.

There is a limiting factor to this decrease of $j^{(p)}$

Hence, ~~when~~ ;

$$\lim_{p \to \infty} j^{(p)} = \cancel{\lim} \; j^{(\infty)} = \delta$$

This 'δ' is the force of interest.

(ii) $j^{(2)} = 0.0775$

$$i = \left(1 + \frac{i^{(2)}}{2}\right)^2 - 1$$

$$= \left[1 + \frac{0.0775}{2}\right]^2 - 1$$

$i = 0.079002$ $\therefore$ $i = 7.9002\%$

$\Rightarrow \delta = \ln(1+i)$

$\therefore \delta = 0.076036$

$\rightarrow d^{(12)}$

$\cancel{(1+i)} = \frac{1}{(1+i)} = \left(1 - \frac{d^{(12)}}{12}\right)^{12}$

$\therefore d^{(12)} = 1 - \left[(1+i)^{-1/12}\right] \times 12$

$= 1 - \left[\left[(1.079002)\right]^{-1/12}\right] \times 12$

$d^{(12)} = 0.075796$

$= 7.5796\%$

$\rightarrow i^{(0.5)}$

$i^{(0.5)} = \left[\left[1 + 0.079002\right]^{1/0.5} - 1\right] \times 0.5$

$i^{(0.5)} = 0.082128.$

$= 8.2123\%.$

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(10 E)

$$PV_0 = 1 \times V(t)$$

$$PV_0 = 1 \times V(0,5) \times V(5,10) \times V(10,t)$$

$$\therefore PV_0 = e^{-\int_0^5 \delta(s)\,ds} \times e^{-\int_5^{10} \delta(s)\,ds} \times e^{-\int_{10}^{t} \delta(s)\,ds}$$

$$PV_0 = e^{-\int_0^5 0.08\,ds} \times e^{-\int_5^{10} 0.06\,ds} \times e^{-\int_{10}^{t} 0.04\,ds}$$

$$PV_0 = e^{-[0.08 \times (5)]} \times e^{-[0.06(10) - 0.06(5)]} \times e^{-[0.04(t) - 0.04(10)]}$$

$$PV_0 = e^{-0.4} \times e^{-[0.6 - 0.3]} \times e^{-[0.04t - 0.4]}$$

$$= e^{-0.4} \times e^{-0.3} \times e^{-0.04t + 0.4}$$

$$PV_0 = e^{-0.4} \times e^{-0.3} \times e^{0.4} \times e^{-0.04t}$$

$$= 0.670320 \times 0.740818 \times 1.491825 \times e^{-0.04t}$$

$\therefore PV_0 = 0.74081 \times e^{-0.04(t)}$

$\therefore \boxed{v(t) = 0.74081 \times e^{-0.04(t)}}$

11. (a)

$d = 18\%$ p.a.

$n = 9$ months. $= 274$ days.

$PV_0 = 50000 \times (1-d)^{274/365}$

$= 50000 \times (1 - 18\%)^{274/365}$

$= 43079.5702$

(b) (i) $\delta = 6\%$

$e^{\delta} = 1+i$

$\therefore i = 0.061837.$

$d^{(12)} = 1 - \left[[1+i]^{-1/12}\right] \times 12$

$= 0.059850$

(ii) $d^{(4)} = 6\%$. $i^{(12)} = ?$

$\left(1 + \frac{i^{(12)}}{12}\right)^{12} = \left(\frac{1}{\left(1 - \frac{d^{(4)}}{4}\right)}\right)^{4}$

$\therefore \left(1 + \frac{i^{(12)}}{12}\right) = \left(1 - \frac{d^{(4)}}{4}\right)^{-1/3} = 1.005050$

$\therefore i^{(12)} = 0.060670$

(c) Ascending order:

$$d < \delta < i^{(4)} < i$$

This order is due to the rate of discount that is paid during the beginning of period. And rate of interest is paid during the end of the period. Hence value of 'd' is a lot smaller than 'i'. Hence, $i^{(4)}$ which is ~~interest~~ nominal rate of interest (d) payable quarterly is also less than 'i'.

(d) 'd' is the discount rate hence it is the amount paid at the beginning of the year where interest is paid at the start of the period. hence d is the present value paid at the start of the year. and 'i' is the interest for the p.v. $\therefore d = iv$.

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(12) initial amount 7049.

$i^{(12)} = 6\%$ for 2 years.

i simple per quarter $= 1.5\%$ for 2 and half

$d^{(12)} = 6\%$ $1\frac{1}{2}$ years.

$n = 6$ year

$\cancel{AV_{(0,6)} = 7049 \times A(0,2) \times A(2, 4\tfrac{1}{2}) \times A(4\tfrac{1}{2}, 6)}$

$\cancel{= 7049 \times (1+i)^2 \times (1+i)^{30/12} \times (1+i)^{18/12}}$

$\rightarrow i^{(12)} = 6\%$

$\therefore \left(1+\dfrac{i^{(12)}}{12}\right)^{12} = 1+i$

$\therefore (1+i) = 1.06167$

$\rightarrow$ i Simple interest per quarter $= 1.5\%$.

$\therefore$ simple interest per year $= 6\%$.

$A(2, 4\tfrac{1}{2}) = \left(1+\dfrac{30}{12}\,i\right)$

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$\rightarrow d^{(12)} = 6\%$.

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = (1-d)$$

$$\therefore (1-d) = 0.941623.$$

$$\therefore i = \frac{d}{1-d} = \frac{0.058377}{0.941625}$$

$$\therefore i = 0.061996.$$

$$AV_{(0,6)} = 7049 \times A(0,2) \times A(2,4\tfrac{1}{2}) \times A(4\tfrac{1}{2},6)$$

$$= 7049 \times (1+i)^2 \times \left(1 + \frac{30}{12} i\right) \times (1+i)^{18/12}$$

$$= 7049 \times (1.06167)^2 \times \left(1 + \frac{30}{12} \times 6\%\right) \times (1.0618996)^{18/12}$$

$$= 7049 \times 1.127143 \times 1.15 \times 1.094421$$

$$= 9999.739596.$$

13. (i) $i = 10\%$ p.a.

Let 'x' be the initial amount
hence $2x$ is accumulated value.

$$2x = x(1+i)^n$$
$$(1+i)^n = 2$$
$$(1+10\%)^n = 2$$
$$(1.1)^n = 2$$
$$n \log(1.1) = \log(2)$$

$n \log(1.1) = \log(2)$

$n\ (0.041393) = 0.301030$

$n = 7.272489$ years.

~~$n \simeq 7$ years.~~

(ii) $d^{(12)} = 10\%$

$$1 - \left(1 - \frac{d^{(12)}}{12}\right)^{12} = d$$

$\therefore d = 0.095542$

$X = 2X\ (1-d)^n$

$\therefore (1-d)^n = \frac{1}{2}$

$\therefore n \log(0.904458) = \log\left(\frac{1}{2}\right)$

$n(-0.043611) = -0.301030$

$\therefore n = 6.902616$ years.

$n \simeq 7$ years.