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Assignment - 1
Financial Mathematics

Ques: (i) $d^4 = 8\%$

$\delta = ?$

$$\left(1 - \frac{d^4}{4}\right)^4 = e^{-\delta(1)}$$

$$(1 - 0.02)^4 = e^{-\delta}$$

$$\ln(0.08) = e^{-\delta} - \delta$$

$$-\delta = \ln(0.922368160)$$

$$\delta = 0.08081$$

$$\boxed{\delta = 8.081083\%}$$

(ii) $1 + i = e^{\delta}$

$i = e^{\delta} - 1$

$$\boxed{i = 8.41657\%}$$

(iii) $$\left(1 - \frac{d^4}{4}\right)^4 = \left(1 - \frac{d^{12}}{12}\right)^{12}$$

$$[0.9223682]^{1/12} = \left(1 - \frac{d^{12}}{12}\right)$$

$$\boxed{d^{12} = 8.053929}$$

$= -\left[0.002t^2 + \frac{0.0002t^3}{3}\right]_0^{10}$

$= 1000 \times e^{-\left[0.002(100) + \frac{0.002(1000)}{3}\right]}$

$=$

Qus 2:

① $f(t) = 0.004t + 0.0002t^2$

$PV_0 = 1000 \times e^{-\int_0^{10} f(t)\,dt}$

$= 1000 \times e^{-\int_0^{100} (0.004t + 0.0002t^2)\,dt}$.

$= 1000 \times e^{-\left[0.002t^2 + 0.0002\frac{t^3}{3}\right]_0^{10}}$

$= 1000 \times e^{-\left[0.002(100) + \frac{0.0002}{3}(1000)\right]}$

$PV_0 = 765.9283384$

② $1000 \cdot AV_{10} = 765.9283384 \times (1+i)^{10}$

$\left[\frac{1000}{765.928}\right]^{1/10} = 1 + i$

$i = 2.702540\%$

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Qu3: (i) $d = \frac{i}{1+i}$

and $\frac{1}{1+i} = v$ (d is discounting factor).

$$\frac{1}{i}\left(\frac{i}{1+i}\right) = d \times \frac{1}{i}$$

$$v = \frac{d}{i}$$

$$\boxed{d = iv}$$

(ii) (a) $(1-d)^4 = \left(1 - \frac{d^2}{2}\right)^2$

$$(1 - 0.0225)^2 = \left(1 - d^2/2\right)$$

$$\boxed{d^2 = 8.89875\%}$$

(b) $(1-d)^{1/2} = \left(1 - d^2/2\right)^2$

$$\left[1 - 0.210005\right]^{1/4} = \left(1 - \frac{d^2}{2}\right)$$

$$\boxed{d^2 = 5.19925\%}$$

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(iii) $i = 0.05$

$$(1+i)^{91/365} = \left(1 - \frac{91}{365} d\right)^{-1}$$

$$(1.05)^{-91/365} = 1 - \frac{91}{365} d$$

$$\boxed{d = 4.84946\%}$$

Ques 4 (i) Same as Q3 (i).

(ii) $i = 0.08$

(a) $$(1+i) = \left(1 - \frac{d^{12}}{12}\right)^{-12}$$

$$(1+0.08)^{-1/12} = 1 - \frac{d^{12}}{12}$$

$$\boxed{d^{12} = 7.671477\%}$$

(b) i^{365}

$$1+i = \left[1 + \frac{i^{365}}{365}\right]^{365}$$

$$(1+0.08)^{1/365} = 1 + \frac{i^{365}}{365}$$

$$\boxed{i^{365} = 7.6969\%}$$

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(w) $\delta = ?$

$$\ln(1+i) = \delta$$
$$\ln(1.08) = \delta$$

$$\boxed{\delta = 7.6961\%}$$

(w) $i^{0.5}$

$$1+i = \left(1 + \frac{i^{0.5}}{0.5}\right)^{0.5}$$

$$(1.08)^{1/0.5} = 1 + \frac{i^{0.5}}{0.5}$$

$$\boxed{i^{0.5} = 8.32\%}$$

Jawab: ① $d^4 = 6\%$

(a) $\left(1 + \frac{i^2}{2}\right)^2 = \left(1 - \frac{d^4}{4}\right)^{-4}$

$$\left(1 + \frac{i^2}{2}\right) = \left(1 - \frac{0.06}{4}\right)^{-2}$$

$$\boxed{i^2 = 6.13775\%}$$

(b) $\left(1 - \frac{d^{12}}{12}\right)^{12} = \left(1 - \frac{d^4}{4}\right)^4$

$$\left(1 - \frac{d^{12}}{12}\right)^{3} = \left(1 - \frac{d^4}{4}\right)^{1/3}$$

50000

(ii) @ $d^{12} = [1 - (1-d)^{1/12}] \times 12$

$= [1 - (1 - 0.06667)^{1/12}] \times 12$

$= [1 - (0.93333)^{1/12}] \times 12$

$= [1 - 0.99426] \times 12$

$= 0.068888$

$\boxed{d^{12} = 6.88\%}$

(b) $i^2 = [(1-d)^{1/2} - 1] \times 2$

$= [(1 - 0.06667)^{1/2} - 1] \times 2$

$= [(0.93333)^{1/2} - 1] \times 2$

$= 0.066609$

$\boxed{i^2 = 7.02003\%}$

(c) $\delta = \ln(1-d)$

$= \ln(1 - 0.06667)$

$= \ln 0.93333$

$\delta = 0.068996$

$\boxed{\delta = 6.8996\%}$

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Ques 7: (i) $i^4 = ?$

$$26000 = 20{,}000 \times \left(1 + \frac{i^4}{4}\right)^{4 \times 17 / 4 \times 3}$$

$$\frac{26000}{20000} = \left(1 + \frac{i^4}{4}\right)^{17/3}$$

$$(1.3)^{3/17} = 1 + \frac{i^4}{4}$$

$$1.047388 = 1 + \frac{i^4}{4}$$

$$0.047388 = \frac{i^4}{4}$$

$$i^4 = 0.189525$$

$$\boxed{i^4 = 18.95525\%}$$

(ii) $$26000 = 20000 \left(1 - \frac{d^2}{2}\right)^{-2 \times 17 / 2 \times 3}$$

$$(1.3)^{-6/17} = \left(1 - \frac{d^2}{2}\right)$$

$$0.9115588 = 1 - \frac{d^2}{2}$$

$$\frac{d^2}{2} = 0.0884411$$

$$d^2 = 0.1768823$$

$$\boxed{d^2 = 17.68823\%}$$

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(iii) $26000 = 20000\left(1 + \frac{17}{12} i\right)$

$\frac{26000}{20000} = 1 + \frac{17}{12} i$

$1.3 = 1 + \frac{17}{12} i$

$\frac{0.3 \times 12}{17 \times 10} = i$

$0.211764 = i$

$\boxed{i = 21.17647\%}$

Ques9:

(1) It refers to nominal interest rate or a discount rate compounded infinite number of times per time period.

(2) $i^2 = 0.0775$

$\left(1 + \frac{i^2}{2}\right)^2 = 1 + i$

$\left[1 + \frac{0.0775}{2}\right]^2 = 1 + i$

$\left[\frac{1.038875}{2}\right]^2 = 1 + i$

$1.079001875 = 1 + i$

$i = 0.079001563$

$\boxed{i = 7.9001563\%}$

(2) $\ln(1+i) = \delta$

$\ln(1+0.079001563) = \delta$

$\delta = \ln 1.079001563$

$\delta = 0.076036134$

$\boxed{\delta = 7.603\,6134\%}$

(3) $1+i = \left(1-\frac{d^{12}}{12}\right)^{-12}$

$(1+i)^{-1/12} = 1-\frac{d^{12}}{12}$

$(1+0.079001563)^{-1/12} = 1-\frac{d^{12}}{12}$

$(1.079001563)^{-1/12} = 1-\frac{d^{12}}{12}$

$0.993683687 = 1-\frac{d^{12}}{12}$

$\frac{d^{12}}{12} = 1-0.9936837$

$\frac{d^{12}}{12} = 0.0063163$

$d^{12} = 0.757956$

$\boxed{d^{12} = 7.57\,957\%}$

(4). $i^{1/2} \Rightarrow \left(1 + \frac{i^{1/2}}{1/2}\right)^{1/2} = 1+i$

$$1 + 2i^{1/2} = (1+i)^2$$

$$2i^{1/2} = (1+0.079001563)^2$$

$$2i^{1/2} = (1.079001563)^2$$

$$2i^{1/2} = 1.164244373$$

$$\frac{2}{2}i^{1/2} = 0.582122$$

$$\boxed{i^{1/2} = 5.82122\%}$$

Ques 10. when $0 < t < 5$

$$V(t) = e^{-\int_0^t (0.08)\,ds}$$

$$= e^{-[0.08s]_0^t}$$

$$\boxed{v(t) = e^{-0.08t}}$$

when $5 \leq t \leq 10$.

$$V(t) = e^{-\int_0^5 0.08\,ds} \times e^{-\int_5^t (0.06)\,ds}$$

$$= e^{-0.08(5)} \times e^{-[0.06t]_5^t}$$

$$= e^{-0.4} \times e^{-[0.06t - 0.3]}$$

$$= e^{-0.4+0.3-0.06t}$$

$$\boxed{v(t) = e^{-0.1-0.06t}}$$

when $t > 10$

$$v(t) = e^{-0.1-0.06(10)} \times e^{-\int_{10}^{t} (0.04)\, ds}$$

$$= e^{-0.1-0.6} \times e^{-[0.04s]_{10}^{t}}$$

$$= e^{-0.7} \times e^{-0.04t+0.4}$$

$$\boxed{v(t) = e^{-0.7-0.04t}}.$$

Qus 11: (a) $d = 18\%$

$t = 9/12$

$PV = \text{Amt. due} \times (1-d)^{9/12}$

$= 50,000 \times (1-0.18)^{9/12}$

$$\boxed{PV = 43085.42623}.$$

(b) (i) $\delta = 6\%$; $d^{12} = ?$

$$e^{-0.06/12} = \left(1 - \frac{d^{12}}{12}\right)$$

$$\boxed{d^{12} = 5.9850250\%}$$

(ii) $d^4 = 6\%$ $\quad i^{12} = ?$

$$\left(1 - \frac{d^4}{4}\right)^{-4} = \left(1 + \frac{i^{12}}{12}\right)^{12 \times 3}$$

$$\left(1 - \frac{0.06}{4}\right)^{-12} = 1 + \frac{i^{12}}{12}$$

$$\boxed{i^{12} = 6.0607089\%}$$

(c) $\quad d < \delta < i^{(4)} < i$.

(d) Same as Qus 3(i).

Qu12 $$AV_6 = 7049 \times \left(1 + \frac{i^{12}}{12}\right)^{24} \times [1 + 10 \times 0.015] \times \left[1 - \frac{d^{12}}{12}\right]^{-18}$$

$$= 7049 \left(1 + \frac{0.06}{12}\right)^{24} \times [1 + 10 \times 0.015] \times \left[1 - \frac{0.06}{12}\right]^{8}$$

$$= 7049 \times 1.12715978 \times 1.15 \times 1.09442123$$

$$\boxed{AV_6 = 9999.883762}$$

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Q13 (i) $2P = P + (1+i)^t$

$2 = [1 + 0.1]^t$

$\log 2 = t \log [1.1]$

$t = \dfrac{\log 2}{\log 1.1}$

$\boxed{t = 7.272541 \text{ years}}$

(ii) $2 = \left(1 - \dfrac{d^{12}}{12}\right)^{-12t}$

$2 = \left(1 - \dfrac{0.1}{12}\right)^{-12t}$

$\ln 2 = -12t \ln \left(1 - \dfrac{0.1}{12}\right)$

$\boxed{t = 6.90255 \text{ yrs.}}$