

HW1

Lagrange's Interpolation

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	x_0	x_1	x_2	x_3
x	7	8	9	10
$y = f(x)$	3	1	1	9
	$f(x_0)$	$f(x_1)$	$f(x_2)$	$f(x_3)$

To Find $\rightarrow f(9.5)$

Lagrange's Interpolation Formula says $\rightarrow$ (here)

$$f(x_k) = \frac{(x_k - x_1)(x_k - x_2)(x_k - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)} \cdot f(x_0) \\ + \frac{(x_k - x_0)(x_k - x_2)(x_k - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)} \cdot f(x_1) \\ + \frac{(x_k - x_0)(x_k - x_1)(x_k - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)} \cdot f(x_2) \\ + \frac{(x_k - x_0)(x_k - x_1)(x_k - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)} \cdot f(x_3)$$

$$f(9.5) = \frac{(9.5 - 8)(9.5 - 9)(9.5 - 10)}{(7 - 8)(7 - 9)(7 - 10)} \times 3 = \frac{1.5 \times 0.5 \times -0.5}{-1 \times -2 \times -3} \times 3 \\ + \frac{(9.5 - 7)(9.5 - 9)(9.5 - 10)}{(8 - 7)(8 - 9)(8 - 10)} \times 1 = \frac{2.5 \times 0.5 \times -0.5}{1 \times -1 \times -2} \times 1 \\ + \frac{(9.5 - 7)(9.5 - 8)(9.5 - 10)}{(9 - 7)(9 - 8)(9 - 10)} \times 1 = \frac{2.5 \times 1.5 \times -0.5}{2 \times 1 \times -1} \times 1 \\ + \frac{(9.5 - 7)(9.5 - 8)(9.5 - 9)}{(10 - 7)(10 - 8)(10 - 9)} \times 9 = \frac{2.5 \times 1.5 \times 0.5}{3 \times 2 \times 1} \times 9$$

$$f(9.5) = (0.1875) + (-0.3125) + (0.9375) + (2.8125) \\ f(9.5) = 3.625$$

Ans $\rightarrow f(9.5) = 3.625$

2.

	x_0	x_1	x_2	x_3
x	3	7	9	10
$f(x) = y$	168	120	72	63
	y_0	y_1	y_2	y_3

To Find $f(6) = ?$

Lagrange's Interpolation Formula says,

$$f(x_k) = \frac{(x_k - x_1)(x_k - x_2)(x_k - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)} \times f(x_0) \\ + \frac{(x_k - x_0)(x_k - x_2)(x_k - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)} \times f(x_1) \\ + \frac{(x_k - x_0)(x_k - x_1)(x_k - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)} \times f(x_2) \\ + \frac{(x_k - x_0)(x_k - x_1)(x_k - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)} \times f(x_3)$$

$$f(6) = \frac{(6-7)(6-9)(6-10)}{(3-7)(3-9)(3-10)} \times 168 \\ + \frac{(6-3)(6-9)(6-10)}{(7-3)(7-9)(7-10)} \times 120 \\ + \frac{(6-3)(6-7)(6-10)}{(9-3)(9-7)(9-10)} \times 72 \\ + \frac{(6-3)(6-7)(6-9)}{(10-3)(10-7)(10-9)} \times 63$$

$$f(6) = \frac{(-1)(-3)(-4)}{(-4)(-6)(-7)} \times 168 \\ + \frac{3 \times -3 \times -4}{4 \times -2 \times -3} \times 120 \\ + \frac{3 \times -1 \times -4}{6 \times 2 \times -1} \times 72 \\ + \frac{3 \times -1 \times -3}{7 \times 3 \times 1} \times 63$$

$$f(6) = 12 + 180 + (-72) + 27 \\ f(6) = 147$$

Hence, $f(6) = 147$.

3.	x	1	2	7	8	To find
	$y=f(x)$	4	5	5	4	$f(6)=?$

Using Lagrange's Interpolation,

$$\begin{aligned}
 f(6) = & \frac{(6-2)(6-7)(6-8)}{(1-2)(1-7)(1-8)} \times 4 \\
 & + \frac{(6-1)(6-7)(6-8)}{(2-1)(2-7)(2-8)} \times 5 \\
 & + \frac{(6-1)(6-2)(6-8)}{(7-1)(7-2)(7-8)} \times 5 \\
 & + \frac{(6-1)(6-2)(6-7)}{(8-1)(8-2)(8-7)} \times 4
 \end{aligned}$$

$$\begin{aligned}
 f(6) = & \frac{4 \times -1 \times -2}{-1 \times -6 \times -7} \times 4 & \Rightarrow & \frac{-16}{21} \\
 & + \frac{5 \times -1 \times -2}{1 \times -5 \times -6} \times 5 & + & \frac{5}{3} \\
 & + \frac{5 \times 4 \times -2}{6 \times 5 \times -1} \times 5 & + & \frac{20}{3} \\
 & + \frac{5 \times 4 \times -1}{7 \times 6 \times 1} \times 4 & + & \left(\frac{-40}{21} \right)
 \end{aligned}$$

$$f(6) = \frac{17}{3} = 5.6667$$

Hence, $f(6) = 5.6667$

4.

x	1	2	3	4	7
$f(x)=y$	2	4	8	16	128

To Find $\Rightarrow f(5) = ?$

Using Lagrange's Interpolation Formula,

$$\begin{aligned}
 f(5) = & \frac{(5-2)(5-3)(5-4)(5-7)}{(1-2)(1-3)(1-4)(1-7)} \times 2 \\
 & + \frac{(5-1)(5-3)(5-4)(5-7)}{(2-1)(2-3)(2-4)(2-7)} \times 4 \\
 & + \frac{(5-1)(5-2)(5-4)(5-7)}{(3-1)(3-2)(3-4)(3-7)} \times 8 \\
 & + \frac{(5-1)(5-2)(5-3)(5-7)}{(4-1)(4-2)(4-3)(4-7)} \times 16 \\
 & + \frac{(5-1)(5-2)(5-3)(5-4)}{(7-1)(7-2)(7-3)(7-4)} \times 128
 \end{aligned}$$

$$\begin{aligned}
 f(5) = & \frac{3 \times 2 \times 1 \times -2}{-1 \times -2 \times -3 \times -6} \times 2 \Rightarrow -\frac{2}{3} \\
 & + \frac{4 \times 2 \times 1 \times -2}{1 \times -1 \times -2 \times -5} \times 4 \Rightarrow +\frac{32}{5} \\
 & + \frac{4 \times 3 \times 1 \times -2}{2 \times 1 \times -1 \times -4} \times 8 \Rightarrow +(-24) \\
 & + \frac{4 \times 3 \times 2 \times -2}{3 \times 2 \times 1 \times -3} \times 16 \Rightarrow +\frac{128}{3} \\
 & + \frac{4 \times 3 \times 2 \times 1}{6 \times 5 \times 4 \times 3} \times 128 \Rightarrow +\frac{128}{15}
 \end{aligned}$$

$$f(5) = \frac{494}{15} = 32.933$$

Ans.

5.

x	10	14	16	20
$f(x)=y$	0.25	0.15	0.10	0.2

To find $f(15)=?$

Using Lagrange's Interpolation Formula,

$$\begin{aligned}
 f(15) = & \frac{(15-14)(15-16)(15-20)}{(10-14)(10-16)(10-20)} \times 0.25 \\
 & + \frac{(15-10)(15-16)(15-20)}{(14-10)(14-16)(14-20)} \times 0.15 \\
 & + \frac{(15-10)(15-14)(15-20)}{(16-10)(16-14)(16-20)} \times 0.10 \\
 & + \frac{(15-10)(15-14)(15-16)}{(20-10)(20-14)(20-16)} \times 0.2
 \end{aligned}$$

$$\begin{aligned}
 f(15) = & \frac{1x-1x-5}{-4x-6x-10} \times 0.25 & \Rightarrow -\frac{1}{192} \\
 & + \frac{5x-1x-5}{4x-2x-6} \times 0.15 & + \frac{5}{64} \\
 & + \frac{5x1x-5}{6x2x-4} \times 0.10 & + \frac{5}{96} \\
 & + \frac{5x1x-1}{10x6x4} \times 0.2 & + \left(\frac{-1}{240}\right)
 \end{aligned}$$

$$f(15) = \frac{29}{240} = 0.1208333$$

Hence, $f(15) = 0.1208333$

M.W.
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Regula Falsi Method / False Position Method

1. $x^3 + x - 1 = 0$ b/w $(0.5, 1)$

x	y	$x = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$
0.5	-0.375	
0.7	0.043	
0.8	0.312	
1	1	

$x = 0.5$ $y = -0.375$

$x = 0.7$ $y = 0.043$

$\bar{x} = \frac{(0.5 \times 0.043) - (0.7 \times -0.375)}{0.043 + 0.375}$

$\bar{x} = 0.679426$

$\bar{y} = -0.0069379$

x	y
0.6794	-0.007
0.7	0.043

$\bar{x} = 0.682284$

$\bar{y} = -0.0001044$

Hence, $x = 0.682284$

2. $x^2 - 2x + 0.5 = 0$ b/w (0, 1)

x	y
0	0.5
$x_1 [0.2$	$0.108] y_1$
$x_2 [0.4$	$-0.236] y_2$
0.8	-0.588
1	-0.5

$$\bar{x} = \frac{(0.2 \times -0.236) - (0.4 \times 0.108)}{-0.236 - 0.108}$$

$$\bar{x} = 0.2628$$

$$\bar{y} = -0.0074333$$

x	y
0.2	0.108
0.2628	-0.00743

$$\bar{x} = \frac{(0.2 \times -0.00743) - (0.2628 \times 0.108)}{-0.00743 - 0.108}$$

$$\bar{x} = 0.25875$$

$$\bar{y} = -0.0001714$$

Hence, $x = 0.2587$,

3. $x^3 + 2x - 20 = 0$ b/w (2, 2.5)

x	y
2	-8
2.2	-4.952
2.4	-1.376
2.5	0.625

$$\bar{x} = \frac{(2.4 \times 0.625) - (2.5 \times -1.376)}{0.625 + 1.376}$$

$$\bar{x} = 2.4687 \quad \bar{y} = -0.01583$$

x	y
2.4687	-0.01583
2.5	0.625

$$\bar{x} = \frac{(2.4687 \times 0.625) - (2.5 \times -0.01583)}{0.625 + 0.01583}$$

$$\bar{x} = 2.46954, \quad \bar{y} = -0.000178$$

Hence, $x = 2.469537$,

4. $3x - \cos x - 1 = 0$ b/w $(0, 1)$

x	y	
0.2 0	-2	$\bar{x} = \frac{(0.6 \times 0.4000974) - (0.8 \times -1.99945)}{0.4000974 + 0.199945}$
0.2	-1.399..	
0.6	-0.199945	$\bar{x} = 0.666644$
0.8	0.4000974	$\bar{y} = -0.00000144$
1	1.000152	

Hence, $x = 0.666644$

5. $x^3 - 3x - 5$ b/w $(2, 2.5)$

x	y	
2	-3	$\bar{x} = \frac{(2.1 \times 0.267) - (2.3 \times -2.039)}{0.267 + 2.039}$
2.1	-2.039	
2.3	0.267	$\bar{x} = 2.277$
2.5	3.125	$\bar{y} = -0.0273$
2.8		
$\bar{x}$	$\bar{y}$	$\bar{x} = \frac{(2.277 \times 0.267) - (2.3 \times -0.0273)}{0.267 + 0.0273}$
2.277	-0.0273	$\bar{x} = 2.2791$ $\bar{y} = 0.001444$
2.3	0.267	

x	y	
2.277	-0.0273	$\bar{x} = \frac{(2.277 \times 0.00144) - (2.2791 \times -0.0273)}{0.00144 + 0.0273}$
2.2791	0.00144	
		$\bar{x} = 2.27901865$ $\bar{y} = -0.000001708$

Hence, $x = 2.27901865$

6. ~~$x \log 10$~~ $x \log_{10} x - 1.2 = 0$ b/w (2,3)

x	y
2	-0.598
2.3	-0.368
[2.6	-0.12107]
[2.8	0.052]
3	0.2314

$$\bar{x} = \frac{(2.6 \times 0.052) - (2.8 \times -0.12107)}{0.052 + 0.12107}$$

$$\bar{x} = 2.74$$

$$\bar{y} = -0.000643$$

x	y
2.74	-0.000643
2.8	0.052

$$\bar{x} = \frac{(2.74 \times 0.052) - (2.8 \times -0.000643)}{0.052 + 0.000643}$$

$$\bar{x} = 2.74064$$

$$\bar{y} = -0.000003$$

Hence, $\bar{x} \equiv x = 2.74064$

7. $x^3 - 4x + 1 = 0$

x	y
0	1
[1	-2]
[2	1]
3	16
4	49

$$\bar{x} = \frac{(1 \times 1) - (-2 \times 2)}{1 + 2} = \frac{1 + 4}{3} = \frac{5}{3}$$

$$\bar{x} = 1.6667$$

$$\bar{y} = -1.03704$$

x	y
[1.6667	-1.03704]
[2	1]

$$\bar{x} = \frac{(1.6667 \times 1) - (-1.03704 \times 2)}{1 + 1.03704}$$

$$\bar{x} = 1.8363$$

$$\bar{y} = -0.1528$$

x	y
[1.8363	-0.1528]
[2	1]

$$\bar{x} = \frac{(1.8363 \times 1) - (-0.1528 \times 2)}{1 + 0.1528}$$

$$\bar{x} = 1.858$$

$$\bar{y} = -0.0179$$

$$\begin{array}{c} x \\ \begin{bmatrix} 1.858 \\ 2 \end{bmatrix} \end{array} \quad \begin{array}{c} y \\ \begin{bmatrix} -0.0179 \\ 1 \end{bmatrix} \end{array}$$

$$\bar{x} = \frac{(1.858 \times 1) - (-0.0179 \times 2)}{1 + 0.0179}$$

$$\bar{x} = 1.8605 \quad \bar{y} = -0.002$$

$$\begin{array}{c} x \\ \begin{bmatrix} 1.8605 \\ 2 \end{bmatrix} \end{array} \quad \begin{array}{c} y \\ \begin{bmatrix} -0.002 \\ 1 \end{bmatrix} \end{array}$$

$$\bar{x} = \frac{(1.8605 \times 1) - (-0.002 \times 2)}{1 + 0.002}$$

$$\bar{x} = 1.8608 \quad \bar{y} = -0.0000374$$

Hence, $x = 1.8608$,

8. $x^3 + 2x^2 - 8 = 0$

$$\begin{array}{c} x \\ 0 \end{array} \quad \begin{array}{c} y \\ -8 \end{array}$$

$$\begin{array}{c} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} \quad \begin{bmatrix} -5 \\ 8 \\ 37 \\ 88 \\ 167 \end{bmatrix} \end{array}$$

$$\bar{x} = \frac{(2 \times 1) - (-8 \times 3)}{1 + 8}$$

$$\bar{x} = 2.8889 \quad \bar{y} = 3$$

$$\bar{x} = \frac{(1 \times 8) - (-5 \times 2)}{8 + 5}$$

$$\bar{x} = 1.38462 \quad \bar{y} = -1.51115$$

$$\begin{array}{c} x \\ \begin{bmatrix} 1.38462 \\ 2 \end{bmatrix} \end{array} \quad \begin{array}{c} y \\ \begin{bmatrix} -1.51115 \\ 8 \end{bmatrix} \end{array}$$

$$\bar{x} = \frac{(1.38462 \times 8) - (-1.51115 \times 2)}{8 + 1.51115}$$

$$\bar{x} = 1.4824 \quad \bar{y} = -0.34753$$

$$\begin{array}{c} x \\ \begin{bmatrix} 1.4824 \\ 2 \end{bmatrix} \end{array} \quad \begin{array}{c} y \\ \begin{bmatrix} -0.34753 \\ 8 \end{bmatrix} \end{array}$$

$$\bar{x} = \frac{(1.4824 \times 8) - (-0.34753 \times 2)}{8 + 0.34753}$$

$$\bar{x} = 1.50394 \quad \bar{y} = -0.0747$$

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$$\begin{array}{c} x \\ 1.50394 \\ 2 \end{array} \quad \begin{array}{c} y \\ -0.0747 \\ 8 \end{array} \quad \begin{aligned} \bar{x} &= \frac{(1.50394 \times 8) - (-0.0747 \times 2)}{8 + 0.0747} \\ \bar{x} &= 1.508527 \end{aligned} \quad \bar{y} = -0.01581$$

$$\begin{array}{c} x \\ 1.508527 \\ 2 \end{array} \quad \begin{array}{c} y \\ -0.01581 \\ 8 \end{array} \quad \begin{aligned} \bar{x} &= \frac{(1.508527 \times 8) - (-0.01581 \times 2)}{8 + 0.01581} \\ \bar{x} &= 1.5095 \end{aligned} \quad \bar{y} = -0.00333$$

Hence, $x = 1.5095$

9. $x^3 - 9x + 1 = 0$

$$\begin{array}{c} x \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \quad \begin{array}{c} y \\ 1 \\ -7 \\ -9 \\ 1 \\ 29 \\ 81 \end{array} \quad \begin{aligned} \bar{x} &= \frac{(2 \times 1) - (-9 \times 3)}{1 + 9} \\ \bar{x} &= 2.9 \end{aligned} \quad \bar{y} = -0.711$$

$$\begin{array}{c} x \\ 2.9 \\ 3 \end{array} \quad \begin{array}{c} y \\ -0.711 \\ 1 \end{array} \quad \begin{aligned} \bar{x} &= \frac{(2.9 \times 1) - (-0.711 \times 3)}{1 + 0.711} \\ \bar{x} &= 2.94155 \end{aligned} \quad \bar{y} = -0.0215$$

$$\begin{array}{c} x \\ 2.94155 \\ 3 \end{array} \quad \begin{array}{c} y \\ -0.0215 \\ 1 \end{array} \quad \begin{aligned} \bar{x} &= \frac{(2.94155 \times 1) - (-0.0215 \times 3)}{1 + 0.0215} \\ \bar{x} &= 2.9428 \end{aligned} \quad \bar{y} = -0.00034$$

Hence, $x = 2.9428$

10. $x^3 - x - 4 = 0$

x	y
0	-4
1	-4
2	2
3	20
4	56

$$\bar{x} = \frac{(1 \times 2) - (-4 \times 2)}{2 + 4}$$

$$\bar{x} = 1.6667$$

$$\bar{y} = -1.037$$

x	y
1.6667	-1.037
2	2

$$\bar{x} = \frac{(1.6667 \times 2) - (-1.037 \times 2)}{2 + 1.037}$$

$$\bar{x} = 1.7805$$

$$\bar{y} = -0.13601$$

x	y
1.7805	-0.13601
2	2

$$\bar{x} = \frac{(1.7805 \times 2) - (-0.13601 \times 2)}{2 + 0.13601}$$

$$\bar{x} = 1.7945$$

$$\bar{y} = -0.01603$$

x	y
1.7945	-0.01603
2	2

$$\bar{x} = \frac{(1.7945 \times 2) - (-0.01603 \times 2)}{2 + 0.01603}$$

$$\bar{x} = 1.7961$$

$$\bar{y} = -0.001862$$

x	y
1.7961	-0.001862
2	2

$$\bar{x} = \frac{(1.7961 \times 2) - (-0.001862 \times 2)}{2 + 0.001862}$$

$$\bar{x} = 1.7963$$

$$\bar{y} = -0.00019$$

Hence, $x = 1.7963$