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PRLI Assignment

Q1.

i. Constant force of mortality is an assumption to model fractional ages. It says the force of mortality between the age x & x+1 is constant

ii. Under CFM

$p_x = e^{-\mu x}$

$\mu_x = -\ln(p_x)$

$\therefore \mu_{55} = -\ln(1-q_{55})$

$\therefore \mu_{55} = 0.004701032 55$

Q2.

i. A select mortality is death rate of individual who have recently undergone some medical test. It is generally lower mortality rates as the individual has passed the test.

ii. $_{20}p_{[25]:[30]} = {}_{20}p_{[25]} \times {}_{2}p_{[30]}$

$= \dfrac{9801.3123}{9951.9913} \times \dfrac{9712.0728}{9923.7497}$

$= 0.9638520314$

Q3. UDD

$_{1.4}q_{54.5} = 1 - {}_{1.4}p_{54.5}$

$= 1 - {}_{0.5}p_{54.5} \times {}_{0.9}p_{55}$

$= \left[1 - \left(\dfrac{1 - 0.5\times0.00714}{1-0.5\times0.00714}\right)\left(1-0.9\times0.00797\right)\right]$

$= 0.01073009121$

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Q4. $P.\ddot{a}_{35:\overline{30}|} = 5000000\ \ddot{A}^{1}_{35:\overline{30}|} + 30P\ A_{35:\overline{30}|}^{\ \ 1}$

from tables:

$\ddot{a}_{35:\overline{30}|} = 14.35$

$A_{35:\overline{30}|} = A_{35} - v^{30}\cdot {}_{30}p_{35}\ A_{65}$

$= 0.09488 - A \times 0.40177$

$= 0.03251$ —— (B)

$A_{35:\overline{30}|}^{\ \ 1} = v^{30}\cdot {}_{30}p_{35}$

$= (1.06)^{-30}\left(\frac{8821.2612}{9894.4299}\right)$

$= 0.1552258147$ —— (A)

$P(14.35 - 30A) = 5000,000 \times B$

$\therefore P = 16771.98381$

Q5.

i. $\bar{a}_x$

ii. $P.V = a_{\overline{T_x}|}$, $T_x > 0$

$EPV = \bar{a}_x = \int_0^\infty v^t\ {}_tp_x\ dt$

Q6.

i. $P = 5,000,000\ A^{1}_{45:\overline{20}|}$

$A^{1}_{45:\overline{20}|} = A_{45:\overline{20}|} - A_{45:\overline{20}|}^{\ \ 1}$

$= 0.46998 - (1.04)^{-20} \left(\frac{8821.2612}{9801.3123}\right)$

0.0592280

$P = 296140.0863$

Total single premium $= 10000 \times P$

$= 2961400863$

ii. $V = 500,000\, A'_{46:\overline{19|}} - 296140.0863$

$= 5000000 \left[0.34599 - (1.06)^{-19} \left(\frac{8821.2612}{9786.9534}\right)\right]$

$- 296140.0863$

$= -55694.26417$

DSAR $= 5000000 - (-55694.26417)$

$= 50,55,694.26417$

EDS $= 10000\, q_{45} \times$ DSAR

$= 74065920.97$

ADS $= 10 \times$ DSAR

$= 50556942.6417$

Mortality Profit $=$ EDS $-$ ADS

$= 23508978.33$

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Q7. $EPV = 100{,}000\, A^{1}_{40:\overline{20}|} + 200000\, A_{60}\, v^{20} \cdot {}_{20}p_{40}$

$= 100{,}000 \left[A_{40} - v^{20}\, {}_{20}p_{40}\, A_{60} \right] + 200{,}000\, v^{20}\, {}_{20}p_{40}\, A_{60}$

$v^{20} \cdot {}_{20}p_{40} = 1.06^{-20} \cdot \frac{9287.2164}{9856.2863}$

$= 0.29380$

$A_{40} = 0.12313$

$A_{60} = 0.32692$

$\therefore EPV = \cancel{29}\ 21917.97945$

$EPV^2 = 1798358788$

$\therefore V(PV) = 1798358788 - (21917.97945)^2$

$= (36303.73211)^2$

$\therefore S.d(PV) = 36303.73211$

Q8. $A_{65} = 0.5412$

$A_{66} = 0.559,$

$A_{65} = q_{65} + A_{66} v - A_{66} \cdot v \cdot q_{65}$

$q_{65} = \frac{0.5412 - 0.5591(1.05)^{-1}}{1 - 0.5591(1.05)^{-1}}$

$\therefore q_{65} = 0.048754$

$$A[65] = 0.75\, q_{65} + \frac{(1-0.75\, q_{65}) \times 0.5591}{1.05}$$

$$A[65] = 0.535321$$

$$\therefore P = 100{,}000 \times 0.535321 2963$$
$$= 53532.12963$$

Q9

i. $P = 120{,}000\, a_{70}$

$= 120{,}000\, (\ddot{a}_{70} - 1)$

$= 120{,}000\, (10.562)$

$= 1267{,}440$

ii. $P(L_0 > 0) = 1$

$P(L \leq 0 \mid 9 \cdot 4n) = P(K_{70} > n)$

Q10 (i) $PV = \bar{a}_{\overline{T_x}|}$

$$EPV = \int_0^\infty a_{\overline{t}|} \cdot {}_tP_x \cdot \mu_{x+t}\, dt = \bar{a}_x.$$

(ii.) $$\int_0^\infty e^{-(\mu+\delta)t}\, dt = 16$$

$$= \frac{1}{-(\mu+0.04)} \cdot \left[e^{-(\mu+\delta)t} \right]_0^\infty = 16$$

$$= \frac{-1}{\mu+0.04}(0-1) = 16$$

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$1 = 16\mu + 0.64$

$\mu = \frac{1-0.64}{16}$

$\therefore \mu = 0.0225$

$V(PV) = \frac{1}{\delta^2}\left({}^2A_x - \bar{A}_x^{\,2}\right)$

$\bar{a}_x = 1 - \delta \bar{a}_x$

$\bar{a}_x = 1 - 0.64$

$= 0.36$

$^2\bar{A}_x = \int_0^\infty e^{-2\delta t} \cdot e^{-\mu t} \cdot \mu \, dt$

$0.0225 \left[\frac{e^{-(2\delta+\mu)t}}{-(2\delta+\mu)} \right]_0^\infty$

$= \frac{-0.0225}{(2 \times 0.0225 + 0.04)} \quad [0 - 1]$

$= \frac{9}{34}$

$SD = \sqrt{\frac{1}{0.04^2}\left(\frac{9}{34} - 0.36^2\right)}$

$= 9.1891882$

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Q11.

(i) In term assurance the premiums are paid annually in advance where if the policy holder fails to pay the premium, he or she will no longer be in force with the contract. In the case of an annuity contract, the prem is a single premium, the policy holder would have paid the premium at the start & the policy is in force till he/she dies.

(ii) Term Assurance

$$P \cdot \ddot{a}_{40:\overline{25|}} = 2,500,000\ A^{1}_{40:\overline{25|}}$$

$$A^{1}_{40:\overline{25|}} = A_{40:\overline{25|}} - A_{40:\overline{25|}}^{\ \ \ 1}$$

$$= 0.05334485019 \quad \text{—— (1)}$$

$$P = \frac{2500000 \times A}{15.884}$$

$$= 8396.002296 \quad \text{—— (2)}$$

$$_{10}V = 2500,000\ A^{1}_{50:\overline{15|}} - P\ddot{a}_{50:\overline{15|}}$$

$$A^{1}_{50:\overline{15|}} = 0.062855$$

$$\therefore\ _{10}V = 87334.42964 \quad \text{—— (3)}$$

Annuity

$_{10}V = 25000\ \ddot{a}_{60}$

$= 3908000$ —— (4)

Total portfolio reserve $= \overset{8500 \times 4}{\cancel{8500 \times 3}} + 9900 \times 3$

$= \cancel{393}$

39431542650

(iii.) Premium of term assurance
Investments in different securities.

Q12.

(i) $s_{55:\overline{10}|} = a_{55:\overline{10}|}\,(1+i)^{10} \cdot {}_{10}p_{55}$

$= 12.57810568$

(ii.) $\ddot{a}^{(4)}_{[40]:\overline{20}|} = \ddot{a}^{(4)}_{[40]} \cdot v^{20} \cdot {}_{20}p_{[40]} \cdot \ddot{a}^{(4)}_{60}$

$= \left(15.494 - \frac{3}{8}\right) - (1.06)^{-20}\left(11.891 - \frac{3}{8}\right)\left(\frac{9287.2164}{9854.3036}\right)$

$= 14.67350635$

Q.13 $P\ddot{a}_{45:\overline{15|}} = 100{,}000\, A_{45:\overline{20|}}^{1} + 100{,}000\, A_{45:\overline{20|}}$

$\ddot{a}_{45:\overline{15|}} = 11.866$

$A_{45:\overline{20|}} = 0.46991$

$A_{45:\overline{20|}}^{\ \ 1} = (1.04)^{-20}\ {}_{20}p_{45}$

$= 0.4107519827$

$P = 7734.608748$

${}_{13}V = 100{,}000\, A_{58:\overline{7|}}^{1} + 100{,}000\, A_{58:\overline{7|}}^{\ \ 1} - P\ddot{a}_{58:\overline{2|}}$

$A_{58:\overline{7|}} = 0.76576$

$A_{85:\overline{7|}}^{\ \ 1} = (1.04)^{-7} \times {}_{7}p_{88}$

$= 0.712085$

$\ddot{a}_{58:\overline{2|}} = 1.955$

${}_{13}V = 132603.4289$

$DSAR = 10000 - {}_{13}V$

$= -32603.42895$

$EDS = 199 \times 0.00535 \times DSAR$

$= -36657.66534$

$ADS = 4 \times DSAR$

$= -130413.7158$

$M.P = EDS - ADS$

$= 93756.05046$

Q14

i. $5800 \bar{a}_{\overline{T_{58}-2|}} v^2$, $T_{63} > 2$

0 , $T_{63} \leq 2$

ii. $5800 \times 100 \times {}_2p_{63} \times v^2 \bar{a}_{65}$

$= 580000 \, (14.871 - 0.5)(1.04)^{-2} \left(\frac{9703.708}{9775.888}\right)$

EPV $= 6594347.317$

iii. $var(PV) = 5800^2 \times 100 \times \frac{1}{\delta^2} \left[v^{2n} \cdot {}_nq_x + \delta \cdot v^{2n} \cdot {}_np_x \, {}^2\bar{A}_{x:n} - \right.$

$\left. v^n \cdot {}_nq_x + v^n \cdot {}_np_x \cdot (\bar{A}_{x+n})^2 \right]$

$\Rightarrow \bar{A}_{65} = 1 - \delta \bar{a}_{65}$

$\bar{A}_{65} = 0.4363559$

$2\bar{A}_{65} = (1.04)^{0.5} (0.20847)$

$= 0.2125985$

${}_2p_{63} = \frac{9703.708}{9775.888}$

$= 0.9926$

${}_2q_{63} = 1 - 0.9926$

$= 0.007383$

$var(PV) = 5800^2 \times 100 \times 13.53$

$= (183937.6604)^2$

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Q.15 (a) $({}_tV + P)(1+i) \times (1+i)^{11/12} - (1+i)^{10/12} - S\,q_{x+t} = {}_{t+1}V_{1,x+1}$

(b) $({}_tV + P)(1+i) - S\,q_{x+t} - R \cdot {}_{0.5}p_{x+t} = {}_{t+1}V \cdot p_{x+t}$.