NMA Assignment 1

1) $R_{measured} = 12.65$
$R_{actual} = 12.5$

Now,
Actual Area of plate = ~~Area~~ A_{actual}

$A_{actual} = \pi (R_{actual})^2$
$= \pi (12.5)^2$
$= 156.25 \times 3.14$
$= 490.625$

Measured Area of plate = $A_{measured}$
$A_{measured} = \pi (R_{measured})^2$
$= 3.14 \times (12.65)^2$
$= 3.14 \times 160.0225$
$= 502.47065$

i) Absolute Error = larger value − Smaller value
$= 502.47065 - 490.625$
$= 11.84565$

ii) Relative Error = $\frac{\text{Absolute Error}}{\text{Measured}}$
$= \frac{11.84565}{502.47065}$
$= 0.023575$

iii) percentage Error = 0.023575×100
$= 0.00023575$

3) $x^4 - x - 10 = 0$

$f(x) = x^4 - x - 10$

x	y
0	-10
2	4

i) $x_0 = 0$ $\quad y_0 = -10$

$x_1 = 2$ $\quad y_1 = 4$

$x_2 = \frac{0+2}{2} = 1$

$y_2 = \frac{-10}{2} = -5$

ii) $x_2 = 1$; $y_2 = -5$

$x_1 = 2$; $y_1 = 4$

$x_3 = \frac{3}{2} = 1.5$

$y_3 = \frac{-1}{2} = -0.5$

iii) $x_3 = 1.5$; $y_3 = -0.5$

$x_1 = 2$; $y_1 = 4$

$x_4 = \frac{3.5}{2} = 1.75$

$x_4 = \frac{3.5}{2} = 1.75$

x_5

iv) $x_3 = 1.5$; $y_3 = -0.5$

$x_4 = 1.75$; $y_4 = 1.475$

$x_5 = \frac{3.25}{2} = 1.625$

$y_5 = \frac{1.70}{2} = 0.85$

v) $x_3 = 1.5$; $y_3 = -0.5$

$x_5 = 1.625$; $y_5 = 0.85$

$\boxed{x_6 = 1.5625 ; y_6 = 0.35}$

4) $f(x) = x^3 - x - 1$

$\therefore f'(x) = 3x^2 - 1$

x	$f(x)$
0	-1
$x_0 \to 1$	-1
$x_1 \to 2$	5

Root here is between $x=1, x=2$

$\boxed{x_0 = 1.5} \to x_0 = \frac{x_0 + x_1}{2}$

1st Iteration

$f(1.5) = (1.5)^3 - 1.5 - 1$

$f(1.5) = 0.875$

$f'(1.5) = 3 \times (1.5)^2 - 1$

$= \cancel{4.5} = 3(2.25) - 1$

$= \cancel{3.5}\ 6.75 - 1$

$f'(1.5) = 5.75$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 1.5 - \frac{0.875}{5.75}$

$x_1 = 1.34783$

2nd iteration

$f(x_1) = f(1.34783)$

$= (1.3478)^3 - 1.3478 - 1$

$= 0.10068$

$f'(1.34783)$

$= 3 \times (1.3478)^2 - 1$

$= 4.44991$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$x_2 = 1.34783 - \frac{0.10068}{4.44991}$

$x_2 = 1.3252$

3^{rd} iteration (x_2)

$f(1.3252) = (1.3252)^3 - (1.3252) - 1$

$= 0.00206$

$f'(1.3252) = 3 \times (1.3252)^2 - 1$

$= 4.26847$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 1.3252 - \frac{0.00206}{4.26847}$

$x_3 = 1.32472$

4^{th} iteration (x_3)

$f(x_3) = f(1.32472)$

$= (1.32472)^3 - (1.32472) - 1$

$= 0$

$f'(1.32472) = 3(1.32472)^2 - 1$

$= 4.26463$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$

$\boxed{x_4 = 1.32472} \rightarrow$ root

5) $A^2 = A$

A is a squarematrix i.e $m \times n$

$7A - (I+A)^3 \rightarrow$ I is Identity Matrix i.e

$7A - (I^3 + A^3 + 3IA(I+A))$

$\rightarrow \because (a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$7A - I^3 - A^3 - 3I^2A + 3IA$

$7A - I^3 - A^3 - \cancel{3A} + \cancel{3A}$

$\rightarrow \because I^n \cdot A = A$

$7A - I^3 - (A^2)A$

$7A - I^3 - (A)A \rightarrow \because A^2 = A$

$7A - I^3 - A^2$

$7A - I^3 - A \rightarrow \because A^2 = A$

$6A - I^3$

$6A$

$\Rightarrow A - I = A$

Q) Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$|A| = 1(-6) - 1(-4-0) + 2(4)$
$= -6 + 4 + 8$
$|A| = 6$

A is invertible

Minor are as follows

$M_1 = -6$, $M_2 = -4$, $M_3 = 4$
$M_4 = -1$, $M_5 = -1$, $M_6 = 1$
$M_7 = -6$, $M_8 = -2$, $M_9 = 4$

New matrix.

$A_n = \begin{bmatrix} -6 & -4 & 4 \\ -1 & -1 & 1 \\ -6 & -2 & 4 \end{bmatrix}$

Cofactor matrix.

$A_c = \begin{bmatrix} -6 & 4 & 4 \\ 1 & -1 & -1 \\ -6 & 2 & 4 \end{bmatrix}$

$\text{Adj } A_c = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$

7) $A = 8i - 9j$

$B = 7i - 9j$

$\theta = ?$

$\because$ It scalar product

$\therefore \bar{A} \cdot \bar{B} = |\bar{A}| |\bar{B}| \cdot \cos\theta$

$\therefore \theta = \cos^{-1} \dfrac{\bar{A} \cdot \bar{B}}{|\bar{A}| |\bar{B}|}$

$\bar{A} \cdot \bar{B} = (8i - 9j)(7i - 9j)$

$= 56 + 81$

$\underline{\bar{A} \cdot \bar{B} = 137}$

$|\bar{A}| = \sqrt{64 + 81}$

$= \sqrt{145}$

$\underline{|\bar{A}| = 12.042}$

$|\bar{B}| = \sqrt{49 + 81}$

$\underline{|\bar{B}| = 11.402}$

$\theta = \cos^{-1} \dfrac{132}{137}$

$\theta = \cos^{-1} (0.96)$

$\boxed{\theta \approx 1}$

9) Sum of all 3 digit no. which remainder 2 when divided by 6

Series,

302, 308, 314 998

The series is in AP

$\therefore a = 302$

$d = 6 \quad T_n = 998$

formula

$$S_n = \frac{n}{2}(a + T_n)$$

$$\cancel{S_n} = T_n = a + (n-1)d$$

$$\therefore 998 = 302 + (n-1)6$$

$$998 = 302 + 6n - 6$$

$$\boxed{n = 117}$$

$$\therefore S_n = \frac{117}{2}(302 + 998)$$

$$\boxed{S_n = 76050}$$

$$\therefore \boxed{S_{117} = 76050}$$

10) for a G.P

$T_6 = 32$

$T_8 = 128$

$r = ?$

formula

$T_n = ar^{n-1}$

$\therefore T_6 = a \cdot r^5$ ~~= 32~~

~~$T_8 =$~~

$32 = a \cdot r^5 \longrightarrow (1)$

$\therefore T_8 = a \cdot r^7$

$128 = a \cdot r^7 \longrightarrow (2)$

~~from (2) and (1)~~

dividing (2) by (1)

$$\frac{128}{32} = \frac{\cancel{a} \cdot r^7}{\cancel{a} \cdot r^5}$$

$4 = r^2$

$\boxed{r = 2}$

11) $(4+3x)^5$

$(3x+4)^5$

$(3x+4)^5 = {}^5C_0(3x)^5 + {}^5C_1(3x)^4(4) + {}^5C_2(3x)^3(4)^2 + {}^5C_3(3x)^2(4)^3 + {}^5C_4(3x)(4)^4 + {}^5C_5(4)^5$

$= 1\cdot 243x^5 + 5\cdot 81x^4\cdot 4 + 10\cdot 27x^3\cdot 16 + 10\cdot 9x^2\cdot 64 + 5\cdot 3x\, 256 + 1024$

$= 243x^5 + 1620x^4 + 4320x^3 + 5760x^2 + 3840x + 1024$

eigen value of

$$A = \begin{pmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$AX - \lambda I = 0$$

$$\begin{pmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{pmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} = 0$$

$$\begin{vmatrix} (2-\lambda) & -3 & 0 \\ 2 & (-5-\lambda) & 0 \\ 0 & 0 & 3-\lambda \end{vmatrix} = 0$$

$$2 - \lambda - 3 = 0$$

$$\boxed{\lambda = -1}$$

$$2 - \lambda - \lambda = 0$$

$$\boxed{\lambda = 3}$$

$$3 - \lambda = 0$$

$$\lambda = 3$$

Hence the eigen values ... are ... $\lambda = 3$