$e^{2/w}$

Assignment

1. $\int_{1/50}^{2} \frac{e^{2/w}}{w^2}\, dw = \int_{1/50}^{2} e^{2/w}\, w^{-2}$

$e^{2/w} = x$

$-2e^{2/w}\, \cancel{xw}\, w^{-2}\, dw = dx$

$\int_{1/50}^{2} x\, dx = \left[\frac{x^2}{2}\right]_{1/50}^{2} = \left[\frac{4}{2} - \frac{1/2500}{2}\right] = 2 - \frac{1}{1250}$

$= \frac{2499}{1250} = 1\frac{1249}{1250}$

~~3t~~ 4t

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2. $\int \frac{2t^3+1}{(t^4+2t)^3} = \int$

$\cancel{3}t^4 = x \quad (t^4+2t)^3 = x$

$4t^3 \cancel{dt}\, dx \quad 3(t^4+2t)^2(4t^3+2)dt = dx$

$4t^3+2\, dt = \frac{dx}{3(t^4+2t)^2}$

$\int x +$

$\int \frac{1}{6x(t^4+2t)^2} = \int \frac{1}{6x\, x^{2/3}} = \frac{1}{6}\int x^{-5/3} = \frac{1}{6} \times \frac{-3}{2} x^{-2/3}$

$= -\frac{1}{4} x^{-2/3} = \frac{1}{4}\left[(t^4+2t)^3\right]^{-2/3}$

$= \frac{1}{4(t^4+2t)^2}$

3. $f(x) = x^4 + 3x - 9$

$$\int x^4 + 3x - 9 = \frac{x^5}{5} + \frac{3x^2}{2} - 9x$$

4. $F(x) = \begin{cases} 6 & x > 1 \\ 3x^2 & x \le 1 \end{cases}$

$$\int_{-2}^{3} f(x)\,dx = \int_{-2}^{1} 3x^2 + \int_{1}^{3} 6 = \left[\frac{3x^3}{3}\right]_{-2}^{1} + \left[6x\right]_{1}^{3} = \left[x^3\right]_{-2}^{1} + \left[6x\right]_{1}^{3}$$

$$= \left[1 - (-8)\right] + \left[18 - 6\right] = 9 + 12$$

$$= 21$$

5. $\int 3(8y - 1)e^{4y^2 - y}\,dy$

$4y^2 - y = x$

$8y - 1\,dy = dx$

$$\int 3e^x\,dx = 3xe^x = 3(4y^2 - y)e^{4y^2 - y}$$

6. $f''(x) = 15\sqrt{x} + 5x^3 + 6$

$$\int 15\sqrt{x} + 5x^3 + 6 = \int \frac{\cancel{15}^{5} \times 2x^{3/2}}{\cancel{3}_{1}} + \frac{5x^4}{4} + 6x = \int 10x^{3/2} + \frac{5x^4}{4} + 6x$$

$$= \cancel{4}\,\cancel{10}^{2}\,\cancel{x} \times 2\,\frac{x^{5/2}}{\cancel{5}} + \cancel{5}^{1}\frac{x^5}{\cancel{20}_{4}} + \cancel{6}^{3}\frac{x^2}{\cancel{2}_{1}}$$

$$= 4x^{5/2} + \frac{x^5}{4} + 3x^2$$

6. $\frac{dv}{dt} = 9.8 - 0.196v$

$\frac{dv}{dt} + 0.196v = 9.8$

$e^{\int P dx} = e^{\int 0.196\, dt} = e^{0.196t}$

$e^{0.196t} \frac{dv}{dt} + 0.196 e^{0.196t} v = 9.8 e^{0.196t}$

$\int \left(e^{0.196t} v\right)' dt = \int 9.8 e^{0.196t} dt$

$e^{0.196t} v + k = 50 e^{0.196t} + c$

$v(t) = 50 + c e^{-0.196t}$

7.

8.

10. $t y' + 2y = t^2 - t + 1$ $\quad y(1) = 1/2$

$$t \frac{dy}{dt} + 2y = t^2 - t + 1$$

$$\frac{dy}{dt} + \frac{2}{t} y = t - 1 + \frac{1}{t}$$

$$F(t) = e^{\int 2/t \, dt} = e^{2 \ln t} = t^2$$

$$t^2 y' + 2ty = t^3 - t^2 + t$$

$$\frac{d}{dt}\left[t^2 y\right] = t^3 - t^2 + t$$

$$t^2 y = \int t^3 - t^2 + t \, dt$$

$$= \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} + c$$

$$y = \frac{t^2}{4} - \frac{t}{3} + \frac{1}{2} + \frac{c}{t^2}$$

$$\frac{1}{2} = \frac{1}{4} - \frac{1}{3} + \frac{1}{2} + \frac{c}{1}$$

$$\frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{2} = c$$

$$c = \cancel{-3+4}\ \frac{1}{12}$$

$\int y^{-3} = \frac{y}{-2}$ $\quad \frac{1}{2\sqrt{1+x^2}} \; 2x$

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11. $\frac{dy}{dx} = \frac{x y^3}{\sqrt{1+x^2}}$ $\quad y(0) = -1$

$\int \frac{dy}{y^3} = \int \frac{x}{\sqrt{1+x^2}} dx$

$\frac{-1}{2y^2} = \sqrt{1+x^2} + c$

$\sqrt{\frac{-1}{2\sqrt{1+x^2}} + c} = \cancel{2} y$

$\sqrt{\frac{-1}{2\sqrt{1}} + c} = -1$

$-\frac{1}{2} + c = 1$

$c = \frac{1}{2}$

12. $F(x) = x^3+10x^2+6$ $\qquad x = 3$

$$f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{F'''(a)}{3!}(x-a)^3+\dots$$

$f(1)=17$

$F'(x) = x(3x+20)$

$f'(1) = 23$

$F''(x) = 6x+20$

$F''(1) = 26$

$f'''(x) = 6$

$f'''(1) = 6$

$$\therefore f(x) \approx \frac{17}{0!}(x-1)^0+\frac{23}{1!}(x-1)^1+\frac{26}{2!}(x-1)^2+\frac{6}{3!}(x-1)^3$$

$$\approx 17+23(x-1)+13(x-1)^2+(x-1)^3$$

14. $f(x) = \sqrt{1+x}$

$F'(x) = \dfrac{1}{2\sqrt{x+1}}$ $\qquad F''(x) = \dfrac{-1}{4(x+1)^{3/2}}$ $\qquad F'''(x) = \dfrac{3}{8(x+1)^{5/2}}$

$F'(0) = \dfrac{1}{2}$ $\qquad F''(0) = -\dfrac{1}{4}$ $\qquad F'''(0) = \dfrac{3}{8}$

$$F(x) \approx \frac{1}{0!}x^0+\frac{1/2}{1!}x^1+\frac{-1/4}{2!}x^2+\frac{3/8}{3!}x^3$$

$$\approx 1+\frac{1}{2}x-\frac{1}{8}x^2+\frac{1}{16}x^3$$

15. $f(x)=e^{-6x}$

$f'(x)=-6e^{-6x}$ $\qquad f''(x)=36e^{-6x}$ $\qquad f'''(x)=-216e^{-6x}$

$F'(-4)=-6e^{24}$ $\qquad f''(-4)=36e^{24}$ $\qquad F'''(-4)=-216e^{24}$

$$f(x) \approx \frac{e^{24}}{0!}(x-(-4))^0+\frac{-6e^{24}}{1!}(x-(-4))^1+\frac{36e^{24}}{2!}(x-(-4))^2$$

$$+\frac{-216e^{24}}{3!}(x-(-4))^3$$

$$= e^{24}-6e^{24}(x+4)+18e^{24}(x+4)^2-36e^{24}(x+4)^3$$

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16. $f(x) = \ln(3+4x)$

$f'(x) = \frac{4}{4x+3}$ $\quad f''(x) = -\frac{16}{(4x+3)^2}$ $\quad f'''(x) = \frac{128}{(4x+3)^3}$

$f'(0) = \frac{4}{3}$ $\quad f''(0) = -\frac{16}{9}$ $\quad f'''(0) = \frac{128}{27}$

$f(x) = \frac{\ln(3)}{0!} x^0 + \frac{4/3}{1!} x^1 + \frac{-16/9}{2!} x^2 + \frac{\frac{128}{27}}{3!} x^3$

$= \ln(3) + \frac{4}{3} x - \frac{8}{9} x^2 + \frac{64}{81} x^3$

13.