Assignment - 1

Q.14 (i) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

$n = 10$

$\bar{X} \rightarrow \frac{\Sigma x}{n} = \frac{134}{10} = 13.4.$

$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{ term} = 5.5^{th} \text{ term}$

$= \frac{11 + 15}{2} = 13$

→ Mode = 18

→ $S.D = \sigma = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}} = \sqrt{\frac{374.4}{10}}$

$= 6.118823416$

x_i	$(x_i - \bar{x})^2$
4	88.36
7	40.96
9	19.36
9	~~9.76~~ 19.36
11	~~2.56~~ 5.76
15	~~21.16~~ 2.56
18	2.. 21.96
18	21.16
18	21.16
25	134.56
	374.4

Q_1 is the $\left(\frac{n}{4}\right)^{th}$ term which is

2.5^{th} term which is $= \frac{7+9}{2} = 8$

Q_3 is the $\left(\frac{3n}{4}\right)^{th}$ term which is the

7.5^{th} term which is $= \frac{18+18}{2} = 18$

Interquartile range $= Q_3 - Q_1$

$= 18 - 8 = 10$.

ii)

x_i	f_i	cf
0	5	5
1	11	16
2	26	42
3	15	57
4	3	60

$n = 60$.

→ Mean $\bar{x} = \frac{\sum f_i x_i}{n}$

$= \frac{120}{60}$

$= 2$

→ Median $= \left(\frac{n+1}{2}\right)^{th}$ term

$= (30.5)^{th}$ term lies in $x_i = 2$

thus Median $= 2$.

→ Mode $= 2$

x_i	f_i	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
0	5	9	0
1	11	81	81
2	26	576	1152
3	15	169	507
4	3	1	4

x_i	f_i	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
0	5	4	20
1	11	1	11
2	26	0	0
3	15	1	15
4	3	4	12

$$\sigma^2 = \frac{58}{60} =$$

$\sigma = 0.98$.

Q_1 is $\left(\frac{n}{4}\right)^{th}$ term $= 15^{th}$ term.

1^{st} term $= 1$

Q_3 is $\left(\frac{3n}{4}\right)^{th}$ term $= 45^{th}$ term.

3^{rd} term $= 3$.

$$Q_3 - Q_1 = 3 - 1 = 2$$

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Q.24 $\lambda = 0.5$.

(i) Poisson Distribution is used as Poi(0.5).

P(No claim arising) = P(X=0)

$$P(X=0) = \frac{\lambda^x e^{-x}}{x!}$$

$$= \frac{(0.5)^0 e^{-0.5}}{0!}$$

$$= \frac{1 \cdot e^{-0.5}}{0!}$$

$$= 0.60653066.$$

(ii) This will be a binomial distribution
Let success be one or more than one claim aris
~~Future~~ Faliure is no claiming arising

$$P(S) = P(X>0) = 1 - P(X=0)$$
$$= 1 - 0.60653066$$
$$= 0.39346934.$$

$$P(f) = P(X=0) \Rightarrow 0.60653066$$

Thus, $X \sim (3, 0.39346934)$

$P(X=1) = {}^3C_1 (0.39346934)^1 (0.60653066)^2$
$= 0.434247843$
$= 0.434$

(iii) Since $\alpha = 1$

Let time be, T

$P(x>2) = 1 - P(X \leq 2)$
$= 1 - F(2)$
$= 1 - (1 - e^{-\lambda x})$
$= e^{-\lambda x}$
$= e^{-0.5 \times 2}$
$= e^{-1}$
$P(x>2) = 0.367879441$
$= 0.368$

Q.34 i) $\alpha = 2$ $\lambda = 1/2$
$\rightarrow E(X) = \frac{\alpha}{\lambda}$
$= \frac{2}{1/2}$
$= 4$
$\rightarrow V(X) = \frac{\alpha}{\lambda^2}$
$= \frac{2}{1/4} = 8$

S.D $\rightarrow \sqrt{8}$
$= 2\sqrt{2}$
$= 2.83.$

ii) Coefficent of Skewness
$= \frac{2}{\sqrt{\alpha}}$
$= 1.1414$
Hence the data ~~at~~ is +vely skewed

iii) $f_x(x) = \frac{\lambda^{\alpha}}{(\alpha-1)!} x^{\alpha-1} e^{-\lambda x}$

$= \frac{1}{4}\left(xe^{-x/2}\right)$

$F_x(x) = \frac{1}{4}\int_0^x x \cdot e^{-x/2}$

$= \frac{1}{2}\left(e^{-x/2} - e^{-x/2} - 1\right)$

Q.4) P(choosing 2 members from the grp) = ${}^{18}C_2$ = P(

P(atleast one female) = 1 - P(both are male)

$= 1 - \frac{{}^{10}C_2}{{}^{18}C_2}$

$= 0.70588 2353$

P(2 Qualified females) = $\frac{{}^{7}C_2}{{}^{18}C_2}$

$P(F \cap Q)$ $= 0.1372 54902$

$P(Q|F) = \frac{P(F \cap Q)}{P(F)}$

$= \frac{0.137254902}{0.70588 2353}$

$= 0.1945$

Q.54 $P($~~Dot..~~ vial level - 1$) = P(L_1) = 0.4$.

$P(L_2) = 0.5$
$P(L_3) = 0.1$

$P($arriving late using Level - 1$) = 0.2$
$P(L|L_2) = 0.4$.

$P(L|L_3) = 0.1$

$$P(L_2|L) = \frac{P(L_2)(P(L|L_2))}{P(L_1)(P(L|L_1)) + P(L_2)P(L|L_2) + P(L_3)(P(L|L_3))}$$

$$= \frac{0.5 \times 0.4}{(0.4 \times 0.2) + (0.5 \times 0.4) + (0.1 \times 0.1)}$$

$$= \frac{0.2}{0.08 + 0.2 + 0.1}$$

$$= \frac{0.2}{0.29}$$

$$= 0.69.$$

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Q.6) $X \sim U(1,2)$

$f_X(x) = \frac{1}{\beta - \alpha} = \frac{1}{2-1}$
$= 1$

$Y = \frac{3}{6-x}$.

$Y(6-x) = 3$.

$6 - x = \frac{3}{Y}$.

$x = 6 - \frac{3}{Y}$

$g^{-1}(y) = 6 - \frac{3}{Y}$.

$\frac{d}{dy} g^{-1}(y) = -3\left(\frac{-1}{Y^2}\right)$

$= \frac{3}{Y^2}$

$f_X g^{-1}(y) = 1$ {As R.V X can ~~still~~ only take val b/w 0 and 1}.

$f_Y(y) = f_X g^{-1}(y) = 1 \times \frac{3}{y^2} = 3y^{-2}$

Q.7 (i) $F(3) = P(X \leq 3)$
$= 0.05 + 0.15 + 0.35$
$= 0.55$

ii) $E(X) = 1\cdot(0.05) + 2(0.15) + 3(0.35) + 4(0.4) + 5(0.05)$
$= 3.25$

(iii) $V(X) = 1^2(0.05) + 2^2(0.15) + 3^2(0.35) + 4^2(0.4) + 5^2(0.05)$
$= 11.45$

(iv) Coefficient of skewness $= \frac{\text{Skewness}}{\sigma^3}$

$= \frac{E(x^3) - 3\mu\sigma^2 - \mu^3}{\sigma^2}$

$= 1(0.05) + 8(0.15) + 27(0.35) + 64(0.4) + 125(0.05)$
$= 42.55$

Coefficient of skewness $=$ ~~-2.67~~
$= \frac{42.55 - 3(3.25)(11.45) - (3.2}{(5)^{3/2}}$
$= -2.67$

Q.8) Let Success be defined claim amount exceeding 1000

$P(S) = 0.2$
$P(f) = 0.8$

It is a binomial distribution

$P(=X<3) = P(X \le 2)$
$= P(X=0) + P(X=1) + P(X=2)$
$= 0.035184372 + 0.131941395 +$
0.230897442
$= 0.398$

Q.9) Let success be defined as winning as t[illegible] prize

$P(S) = 0.01$
$P(f) = 0.99$

$P(X=3) = {}^{7}C_{3} (0.01)^3 (0.99)^4$
$= 0.00003362$|

Q.10) It will follow Poisson Distribution

$\lambda = \frac{2}{5} = 0.4$

$P(X>2) = 1 - P(X \leq 2)$

$P(X=0) = \frac{\lambda^{x} e^{-\lambda}}{x!} = \frac{(0.4)^{0} e^{-0.4}}{0!}$

$= e^{-0.4}$

$= 0.670320046$

$P(X=1) = \frac{(0.4)^{1} e^{-0.4}}{1!}$

$= 0.4 e^{-0.4}$

$= 0.268128018$

$P(X=2) = \frac{(0.4)^{2} e^{-0.4}}{2!}$

$= 0.053625604$

$P(X \leq 2) = 0.992073668$

$P(X>2) = 1 - P(X \leq 2)$

$= \cancel{0.0079263}\ 0.007926332$

Q.11) $P(b) = P(g)$
$P(b) + P(g) = 1$

$2P(b) = \frac{1}{2}$

$2P(g) = 1$

$P(g) = P(b) = \frac{1}{2}$

Let E be the event of having 3 boys.

$P(E) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$P(E) = \frac{1}{8}$

Q.12) It is following exponential distribution as $Exp(10)$

$\lambda = 10$
$\alpha = 1$
$t = 0.1$

$P(T < 0.1) = F(0.1)$
$= 1 - e^{-\lambda t}$
$= 1 - e^{-1}$
$= 0.632120559$

Q.13) $X \sim U(75, 120)$

$$f_X(x) = \frac{1}{120-75}$$

$$= \frac{1}{45}$$

$$P(80 < X < 100) = \frac{100-80}{45}$$

$$= \frac{20}{45}$$

$$= 0.444.$$

Q.14) $X \sim \text{Gamma}(5, 0.4)$

Thus, $2\lambda X \sim \chi^2_{10}$.

We need to calculate $P(X < 22.89)$

$$P(X < 22.89) = P(0.8X < 18.312)$$

$$= P(\chi^2_{10} < 18.312)$$

$$P(\chi^2_{10} < 18) = 0.9450$$

$$P(\chi^2_{10} < 18.5) = 0.9529$$

0.5 causes a change of 0.0079 in probability.

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$0.5 = 0.0079$

$0.312 = \frac{0.0079 \times 0.312}{0.5}$

$= 0.0049296.$

Thus, $P(\chi^2_{10} < 18.312) = 0.9450 + 0.0049296.$

$= 0.9499296$

Q.15) $f_X(x) = \frac{k}{(x+5)^4}, \quad x > 0$

i) $\int_0^\infty f_X(x) = 1.$

$k\int_0^\infty \frac{1}{(x+5)^4}\, dx = 1$

$x+5 = t.$
$dx = dt.$

$k\int_5^\infty t^{-4}\, dt = 1.$

$k\int_5^\infty \frac{t^{-3}}{-3}$

$k\left[\frac{t^{-3}}{-3}\right]_5^\infty$

$$k\left[\frac{1}{-3t^3}\right]_5^\infty = 1$$

$$\frac{k}{375} = 1$$

$$k = 375$$

ii) $F(10) = P(X \le 10)$

$$375\int_0^{10} \frac{1}{(x+5)^4}.$$

$x+5 = t$.

$dx = dt$

$$= 375\int_5^{15} t^{-4}$$

$$= 375\left[\frac{-1}{3t^3}\right]_5^{15}$$

$$= 375\left[\frac{1}{375} - \frac{1}{10125}\right]$$

$$= 0.963$$

iii) $E(X) = \int_0^\infty x \cdot f_x(x)\, dx$.

$$= \int_0^\infty \frac{x\,k}{(x+5)^4}.$$

$$= 375\int_0^\infty \frac{x}{(x+5)^4}$$

Let $x+5 = t$.

$dx = dt$.

$$375\int_0^\infty \frac{t-5}{t^4}$$

$$= 375\left[\frac{5}{3t^3} - \frac{1}{2t^2}\right]_5^\infty$$

$$= \frac{375}{50} - 5$$

$$= 7.5 - 5$$

~~E(X) = 2.5~~ $E(X) = 2.5$