Calculus Assignment 2

Q1] $\int_{1/50}^{2} \frac{e^{2/w}}{w^2}\, dw$

Let $\frac{2}{w} = t$

$\therefore \frac{-2}{w^2} = dt$

at $w = 2$, $t = 1$ at $w = 1/50 \Rightarrow t = 100$

$\therefore \int_{100}^{1} \frac{e^t\, dt}{-2} = \left[\frac{e^t}{-2}\right]_{100}^{1}$

$= \left[\frac{e^1 - e^{100}}{-2}\right]$

$=$

Q2] $\int \frac{2t^3 + 1}{(t^4 + \cancel{3}2t)^3}\, dt$

$\frac{d}{dt}(t^4 + 2t)^{\cancel{3}} = 4t^3 + 2$
$= 2[2t^3 + 1] = du$

$\therefore = \int \frac{du}{2} \frac{1}{u^5} = \frac{u^{-2}}{-2} \cdot \frac{1}{2} + C$

$= \frac{-1}{4} \cdot \frac{1}{u^2} + c$

$= -\frac{1}{4(t^4 + 2t)^2} + c$

Q3] $f'(x) = x^4 + 3x - 9$

$f(x) = \int f'(x) = \int x^4 + 3x - 9 \, dx$

$= \frac{x^5}{5} + \frac{3x^2}{2} - 9x + c$

Q4] $= \int_{-2}^{3} f(x) \, dx = \int_{1}^{3} 6 \, dx + \int_{-2}^{1} 3x^2 \, dx$

$= [6x]_1^3 + [x^3]_{-2}^{1}$

$= [18 - 6] + [1 + 8]$

$= 12 + 9 = 21$

Q5] $= \int 3(8y-1) e^{4y^2 - y} \, dy$

Let $4y^2 - y = t$

$(8y-1) \cdot dy = dt$

$= \int 3e^t \, dt = 3e^t + c$

$= 3e^{4y^2-y} + c$

Q6] $f''(x) = 15\sqrt{x} + 5x^3 + 6$ $\quad f(1) = -5/4$

$\int f''(x) = \int 15\sqrt{x} + 5x^3 + 6 \, dx = f'(x)$ $\quad f(4) = 404$

$\therefore = \cancel{15}^{5} \times \frac{2}{\cancel{3}} (x^{3/2}) + \frac{5x^4}{4} + 6x + c$

$f'(x) = 10x^{3/2} + \frac{5}{4}x^4 + 6x + c_1$

$\therefore \int f'(x) = f(x)$

$= \int 10x^{3/2} + 5/4x^4 + 6x + c_1\,dx$

$= \frac{10\,x^{3/2+1}}{3/2+1} + \frac{5}{4}\cdot\frac{x^5}{5} + \frac{\overset{3}{\cancel{6}}x^2}{\cancel{2}} + c_1x + c_2$

$= \frac{2\times\overset{2}{\cancel{10}}}{\cancel{5}}x^{5/2} + x^5/4 + 3x^2 + c_1x + c_2$

$= 4x^{5/2} + x^5/4 + 3x^2 + c_1x + c_2$

Putting $x = 1$

$-5/9 = 4 + \frac{1}{4} + 3 + c_1 + c_2$

$c_1 + c_2 = -5/9 - 7 - 1/4$

$= -17/2$

$\therefore c_2 = -17/2 - c_1$

Putting $x = 4$

$404 = 4(4)^{5/2} + \frac{(4)^5}{4} + 3(4)^2 + c_1(4) + c_2$

$404 = 432 + 4c_1 - c_1 - \frac{17}{2}$

$c_1 = -6.5 \qquad \therefore c_2 = -2$

$f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 - 6.5x - 2$

Q7]

Q8] $= \frac{dr}{d\theta} = \frac{r^2}{\theta}$ $\quad r(1) = 2$

$\frac{dr}{r} = \frac{1}{\theta} d\theta$

Integrating Both sides

$\int \frac{1}{r} dr = \int \frac{1}{\theta} d\theta$

$\therefore -\frac{1}{r} = \log \theta + c$

Putting $r(1) = 2$

$-\frac{1}{(1)} = \log (2) + c$

$c = -3.3219$

$\therefore -\frac{1}{r} = \log \theta - 3.3219$

Q9] $\frac{dv}{dt} = 9.8 - 0.196v$

$$\left(\frac{1}{9.8-0.196v}\right) dv = 1\,dt$$

$$\int \frac{1}{9.8-0.196v}\,dv = \int 1\,dt$$

$$\int \frac{1}{\left(\frac{7\sqrt{5}}{5}\right)^2 - \left(\frac{7\sqrt{10v}}{50}\right)^2}\,dv = \int 1\,dt$$

$$= \frac{1}{2\times 7\cancel{\sqrt{5}}/\cancel{5}\sqrt{5}} \log\left|\frac{7\sqrt{5}/5 + 7\sqrt{10v}/50}{7\sqrt{5}/5 - 7\sqrt{10v}/50}\right| = t + c$$

$$\boxed{= \frac{\sqrt{5}}{14} \ln\left(\frac{\sqrt{50}+\sqrt{v}}{\sqrt{50}-\sqrt{v}}\right) = t + c}$$

$= \cancel{\ln \sqrt{3}}$

Q10] $t \cdot \frac{dy}{dt} + 2y = t^2 - t + 1$

$$\frac{dy}{dt} + 2/t \cdot y = t^2 - t + 1$$

$$y(IF) = \int Q(IF)\,dt = \ldots$$

Camlin

Q11] $\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$ $\quad y(0) = -1$

$$\int \frac{1}{y^3}\,dy = \int \frac{x}{\sqrt{1+x^2}}\,dx$$

Let $1+x^2 = t$

$dx = dt/2$

$$\frac{-1}{2y^2} = \frac{1}{2}\int \frac{1}{\sqrt{t}}\,dt$$

$$-\frac{1}{2}\cdot\frac{1}{y^2} = \cancel{2} \times \frac{1}{\cancel{2}}\sqrt{1+x^2} + c$$

$$\frac{-1}{2y^2} = \sqrt{1+x^2} + c$$

$$\frac{-1}{2(1)} = \sqrt{1+(0)^2} + c$$

$$\frac{-1}{2} = c + 1$$

$$c = -3/2$$

$$-1/2y^2 = \sqrt{1+x^2} - 3/2$$

Q12] $f(x) = x^3 - 10x^2 + 6$ for $x = 3$

$f'(x) = 3x^2 - 20x$

$f(3) = 27 - 9(10) + 6$
$= -57$

$f''(x) = 6x - 20$

$f'(3) = -33$

$f'''(x) = 6$

$f''(3) = -2$

$f''''(x) = 0$

$f'''(3) = 6$
$f''''(3) = 0$

$\therefore$ Taylor series

$$f(x) = f(a) + (x-a) f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a) + \dots$$

$a = 3$

$$= -57 + (x-3)(-33) + \frac{(x-3)^2}{2}(-2) + \frac{(x-3)^3}{3\times 2} \times (6) + 0$$

$$f(x) = -57 + (x-3)(-33) - (x-3)^2 + (x-3)^3$$

Q13] $f(x) = (1+x)^{u}$

Maclaurian series

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''$$

$+ \ldots$

$f(x) = (1+x)^{u}$

$f'(x) = u(1+x)^{u-1}$

$f''(x) = u(u-1)(1+x)^{u-2}$

$f'''(x) = u(u-1)(u-2)\cancel{(u-3)}(1+x)^{u-3}$

$f(0) = (1)^{u} = 1$

$f'(0) = u(1)^{u-1} = u$

$f''(0) = u(u-1)$

$f'''(0) = u(u-1)(u-2)$

Series

$$= 1 + x u + \frac{x^2}{2}(u(u-1)) + \frac{x^3}{3!}(u)(u-1)(u-2$$

$+ \ldots$

Q14] $f(x) = \sqrt{\cancel{0}1+x}$ $\quad f(0) = \sqrt{1} = 1$

Maclaurian series

$f'(x) = \cancel{0}\ \frac{1}{2\sqrt{1+x}}$ $\quad f'(0) = \frac{1}{2}$

$f''(x) = -\frac{1}{4}(1+x)^{-3/2}$ $\quad f''(0) = -\frac{1}{4}$

$f'''(x) = \frac{3}{8}(1+x)^{-5/2}$ $\quad f'''(0) = \frac{3}{8}$

∴ Series

$$f(x) = 1 + x\cdot\frac{1}{2} + \frac{x^2}{2}\left(-\frac{1}{4}\right) + \frac{x^3}{\cancel{3}\times 2}\left(\frac{\cancel{3}}{8}\right)$$

$$= 1 + x/2 - x^2/8 + x^3/16$$

Q15] $f(x) = e^{-6x}$ $\quad f(-4) = e^{24}$

$f'(x) = -6e^{-6x}$ $\quad f(-4) = -6e^{24}$

$f''(x) = 36e^{-6x}$ $\quad f(-4) = 36e^{24}$

$f'''(x) = -216e^{-6x}$ $\quad f(-4) = -216e^{24}$

$\cancel{f(x) = e^{24} - 6e^{24} + \frac{36e^{24}}{2}}$

$$f(x) = e^{24} + (x+4)(-6e^{24}) + \frac{(x+4)^2}{2!}(36e^{24}) + \frac{(x+4)^3}{6}(-216e^{24}) \dots$$

Q1b] $f(x) = \ln(3+4x)$ $\qquad f(0) = \ln(3)$

~~Tay~~ $f'(x) = \frac{1}{3+4x}(4)$ $\qquad f'(0) = \frac{4}{3}$

$f''(x) = \frac{-4}{(3+4x)^2}$ $\qquad f''(0) = -\frac{4}{9}$

$f'''(x) = \frac{8}{(3+4x)^3}$ $\qquad f'''(0) = \frac{8}{27}$

∴ Series

$$f(x) = \ln(3) + x\cdot\frac{4}{3} - x^2\cdot\frac{2}{9} + x^3\cdot\frac{4}{81} + \dots$$