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NMA ASSIGNMENT

Observed Radius = 12.65 cm

Actual Radius = 12.5 cm

Observed area = πr^2 .

$$A_0 = \pi \times (12.65)^2$$

$$\therefore A_0 = 502.75 \text{ cm}^2$$

Actual area = πr^2

$$A_1 = \pi \times (12.5)^2$$

$$\therefore A_1 = 490.874 \text{ cm}^2$$

i) Absolute error = Observed value - actual value

$$= 502.725 - 490.874$$

$$= 11.851 \text{ cm}^2$$

ii) Relative error = $\frac{\text{Absolute error}}{\text{Actual value}}$

$$= \frac{11.851}{490.874}$$

$$= 0.02414$$

iii) Percentage error = Relative error $\times 100$

$$= 2.414\%$$

2. (i)

$x_1 = 4$

$x = 5$

$x_2 = 6$

$y_1 =$

$y_2 =$

$y = ?$

$y_1 = 0.60206$

$y_2 = 0.7781513$

$$\therefore y = \frac{y_2 (x - x_1) + y_1 (x_2 - x)}{(x_2 - x_1)}$$

$$= \frac{0.7781513(1) + 0.60206(1)}{2}$$

$$\therefore y = 0.6901$$

ii)

$x_1 = 4.5$

$x = 5$

$x_2 = 5.5$

$y_1 = 0.6532125$

$y = ?$

$y_2 = 0.7403627$

$$\therefore y = \frac{y_2 (x - x_1) + y_1 (x_2 - x)}{x_2 - x_1}$$

$$= \frac{0.7403627(0.5) + 0.6532125(0.5)}{1}$$

$$\therefore y = 0.6901$$

$$x^4 - x - 10 = 0$$

3.

$$a = 1.5$$

$$b = 2$$

$$\therefore f(a) = f(1.5) = (1.5)^4 - 1.5 - 10 = -6.4375$$

$$f(b) = f(2) = 2^4 - 2 - 10 = 4$$

$\therefore$ 1st iteration:

$$x_1 = \frac{2 \cdot 1.5}{2} = 1.75$$

$$f(x_1) = f(1.75) = 1.75^4 - 1.75 - 10 = -23.11$$

2nd iteration.

$$x_2 = \frac{1.75 + 2}{2} = 1.875$$

$$f(x_2) = 1.875^4 - 1.875 - 10 = \underline{\underline{0.4846}}$$

3rd iteration.

$$x_3 = \frac{1.75 + 1.875}{2} = 1.8125$$

$$f(x_3) = f(1.8125) = 1.8125^4 - 1.8125 - 10 = \underline{\underline{-1.0202}}$$

4th iteration

$$x_4 = \frac{1.875 + 1.8125}{2} = 1.84375$$

$$f(x_4) = 1.84375^4 - 1.84375 - 10 = \underline{\underline{-0.2877}}$$

5th iteration

$$x_5 = \frac{1.875 + 1.84375}{2} = 1.859375$$

$$f(x_5) = 1.859375^4 - 1.859375 - 10 = \underline{\underline{0.093}}$$

$\therefore$ root of $x^4 - x - 10 = 0$ is
 1.859375

which is approximately 1.86

c.

$$f(x) = x^3 - x - 1$$
$$f'(x) = 3x^2 - 1$$

x	0	1	2
y	-1	-1	7

$$\therefore x_0 = \frac{1+2}{2} = 1.5$$

$$f(x_0) = 0.875 \quad f'(x_0) = 5.75$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 1.5 - \frac{0.875}{5.75}$$

$$= \underline{\underline{1.34783}}$$

$$f(x_1) = 0.10068 \quad f'(x_1) = 4.44991$$

$$x_2 = 1.34783 - \frac{0.10068}{4.44991}$$

$$= 1.3252$$

$$\therefore f(x_2) = 0.002502 \quad f'(x_2) = 4.2684$$

$$x_3 = 1.3252 - \frac{0.002502}{4.2684}$$

$$\therefore x_3 = 1.3247$$

$$f(x_3) = 0.0000007545$$

$$f'(x_3) = 4.2645$$

$$\therefore x_4 = 1.3247 - 0$$

$$\therefore x_4 = 1.3247$$

$$\therefore \text{Root of } x^3 - x - 6 = 0 \text{ is } 1.3247$$

5. $A^2 = A$

$$7A - (1-A)^3 = 90 \text{ find}$$

$$\therefore 7A - (1-A)^3 = 7A - (1^3 + A^3 + 3A^2 \cdot 1 + 3A \cdot 1^2)$$

$$= 7A - 1 - A^3 - 3A^2 - 3A$$

$$= 7A - 1 - A^3 - 3A^2 - 3A$$

$$= 7A - 1 - A - 3A$$

$$= 7A - 7A - 1$$

$$\therefore 7A (1+A)^3 = -1$$

6.) let $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$$\therefore |A| = 1 \begin{vmatrix} 0 & 6 \\ 1 & -1 \end{vmatrix} + 1 \begin{vmatrix} 4 & 6 \\ 0 & -1 \end{vmatrix} + 2 \begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix}$$

$$= 1(-6) + 1(-4) + 2(4) = -2$$

$|A| \neq 0$ inverse exists.

Co-factor matrix = $\begin{bmatrix} -6 & 4 & 4 \\ 1 & -1 & 1 \\ -6 & 2 & 4 \end{bmatrix}$

$\therefore$ Adjoint matrix = $\begin{bmatrix} -6 & 4 & 4 \\ 1 & -1 & 1 \\ -6 & 2 & 4 \end{bmatrix}$

$$A^{-1} = \frac{1}{|A|} \times \text{Adjoint}$$

$$= \frac{-1}{2} \begin{bmatrix} -6 & 4 & 4 \\ 1 & -1 & 1 \\ -6 & 2 & 4 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -1/2 & -2 \\ -1/2 & 1/2 & -1 \\ -3 & 1 & 2 \end{bmatrix}$$

9) The smallest 3 digit no. that leaves remainder of 2 is 101 and the largest is 998.

$$\therefore a = 101, \quad l = 998.$$

$$\therefore l_n = a + (n-1)d,$$

$$998 = 101 + (n-1)3.$$

$$\frac{998 - 101}{3} = n - 1$$

$$\underline{n = 300.}$$

$$S_n = \frac{n}{2} (a + l) = \frac{300}{2} (101 + 998)$$

$$= 164850$$

$\therefore$ Sum of all numbers that leave a remainder of 2 when divided by 2

$$= 164850$$

10.) $46 = 32.$

$48 = 128$

$$\therefore 46 = ar^n$$

$$32 = ar^5 \quad \text{--- (1)}$$

$$48 = ar^{2+}$$

$$128 = ar^{2+} \quad \text{--- (2)}$$

Dividing (2) and (1)

$$\frac{ar^2}{ar^5} = \frac{128}{32}$$

$$\therefore x^2 = 4$$

$$\Rightarrow \boxed{x = 2}$$

11.) Expand $(4+3x)^5 = (3x+4)^5$

$$\therefore (a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 +$$

$$\frac{n(n-1)(n-2)}{3!}$$

$$= 243x^5 + 1620x^4 + 4320x^3 + 5760x^2 + 3840x + 1024$$

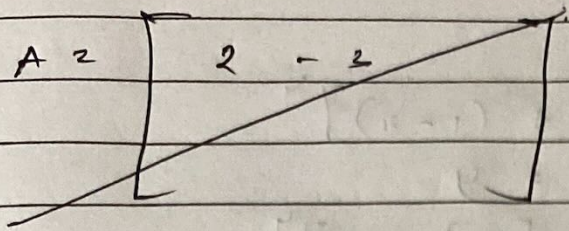
12. $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$$\lambda^3 - (0)\lambda^2 + (-13)\lambda + (30 + 18 + 0) = 0$$

$$\therefore \lambda^3 - 13\lambda + 48 = 0$$

$$\therefore \lambda = 1 \text{ satisfies}$$

$$\therefore \begin{array}{cccc|c} 1 & 1 & 0 & -13 & 48 \\ \text{---} & \text{---} & \text{---} & \text{---} & \text{---} \\ 1 & 1 & 0 & -13 & 48 \\ \text{---} & \text{---} & \text{---} & \text{---} & \text{---} \\ 0 & 0 & 0 & 0 & 0 \end{array}$$



$$\therefore A^2 + A - 12 = 0$$

$$A^2 + 4A - 3A - 12 = 0$$

$$(A + 4)(A - 3) = 0$$

$$A = -4 \text{ or } A = 3$$

13

$$f(x) = x^3 - 7x^2 + 8x - 3$$

$$\therefore f(x) = x^3 - 7x^2 + 8x - 3$$

$$f(5) = -13$$

$$f'(x) = 3x^2 - 14x + 8$$

$$f'(5) = 13$$

$$\therefore x_1 = 5 - \left(\frac{-13}{13} \right)$$

$$= 6$$

$$f(6) = 9$$

$$f'(6) = 32$$

$$x_2 = 6 - \left(\frac{9}{32} \right)$$

$$\therefore x_2 = 5.71875$$

14.) $(1-x+x^2)^4$
 $\therefore (x^2 + (1-x))^4$
 let $1-x = y$

$\therefore [x^2 + y]^4$
 Using binomial theorem

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{6}a^{n-3}b^3 + \dots$$

$$\therefore (x^2 + y)^4 = (x^2)^4 + 4(x^2)^3y + \frac{4 \times 3}{2}(x^2)^2y^2 + \frac{4 \times 3 \times 2}{6}(x^2)y^3 + y^4$$

$$\therefore [x^2 + (1-x)]^4 = x^8 + 4x^6(1-x) + 6x^4(1-x)^2 + 4x^2(1-x)^3 + (1-x)^4$$

15) $e^{-x} = 3 \log(x)$
 $\therefore f(x) = 3 \log x - e^{-x}$
 $a = 1$ $b = 1$

$$\therefore f(a) = 0.367876$$

$$f(b) = 1.944106$$

$$\therefore \frac{a+b}{2} = \frac{1+1}{2} = 1$$

$$f(x_1) = 0.993265$$

$$x_2 = 1 + \frac{1.5}{2} = 1.25$$

$$f(x_2) = 3 \log(1.25) - e^{-1.25} \\ = 0.372925$$

$$x_3 = \frac{1 + 1.25}{2} = 1.125$$

$$f(x_3) = 3 \log(1.125) - e^{-1.125} \\ = \underline{0.02569664}$$

$$\therefore x_4 = \frac{1 + 1.125}{2} = 1.0625$$

$$f(x_4) = 3 \log(1.0625) - e^{-1.0625} \\ = \underline{\underline{-0.16372}}$$