ProbnStat | Assignment-2

Q1] $X \sim Poi(\theta)$.

$\frac{X-\theta}{\sqrt{\theta}} \overset{a}{\sim} N(0,1)$.

$X = 200$,

Z will be used (Normal Approximation) to X

By Principle,

$P[x_1 < X < x_2] = 0.95$

Using Continuity

~~$P\left[\frac{x_1-\theta}{\sqrt{\theta}} < Z < \frac{x_2-\theta}{\sqrt{\theta}}\right]$~~

$P[x_1 - 0.5 < X < x_2 + 0.5] = 0.95$

$P\left[\frac{x_1-0.5-200}{\sqrt{200}} < Z < \frac{x_2+0.5-200}{\sqrt{200}}\right] = 0.95$

Now, $y_1 = x_1 - 0.5$

$y_2 = x_2 + 0.5$

$P\left[\frac{y_1-200}{\sqrt{200}} < Z < \frac{y_2-200}{\sqrt{200}}\right] = 0.95$

$y_1/y_2 = 200 \pm 1.96\sqrt{200}$

$= 200 \pm 27.7186$

$= (172.2814, 227.7186)$.

$x_1 = y_1 + 0.5 = 172.7814$

$x_2 = y_2 - 0.5 = 227.2186$

$\Rightarrow (x_1, x_2) = (172.7814, 227.2186)$

$v(\bar{x}) = E[\bar{x}^2] - [E(\sum x_i)]^2$

$E(\sum x_i)^2 - n [E(\sum x_i)]^2$

PAGE NO.:
DATE: / /

Q2) $\bar{X} = \frac{\sum X_i}{n}$, $S^2 = \frac{1}{n-1} \sum_n (X_i - \bar{X})^2$.

i) $E(\bar{X}) = E\left(\frac{\sum X_i}{n}\right) = \frac{1}{n} E(\sum X_i)$

$= \frac{1}{n} \sum E(X_i)$

$= \frac{1}{n}(n\mu) = \boxed{\mu}$

$V(\bar{x}) = V\left(\frac{\sum X_i}{n}\right)$

$= \frac{1}{n^2} V(\sum X_i)$

$= \frac{1}{n^2} \sum V(X_i)$ [Since they are iid].

$= \frac{1}{n^2}(n\sigma^2) = \boxed{\frac{\sigma^2}{n}}$

ii) $E(S^2) = \sigma^2$

$E(S^2) = E\left[\frac{1}{n-1} \sum (X_i - \bar{X})^2\right]$

$V(x) = E(x^2) - (E(x))^2$

$V(\bar{x}) = E(\bar{x}^2) - (E(\bar{x}))^2$

$-\mu^2$

$= \frac{1}{n-1} E\left[\sum (X_i - \bar{X})^2\right]$

$= \frac{1}{n-1} E\left[\sum X_i^2 - n\bar{X}^2\right]$

$= \frac{1}{n-1} E\left[\sum X_i^2\right] - E\left[n\bar{X}^2\right]$

$= \frac{1}{n-1} \left\{ E(\sum X_i^2) - n E(\bar{X}^2) \right\}$

~~$= \frac{1}{n-1} \left\{ \frac{\sigma^2}{n} + n\bar{X}^2 - n\left(\frac{\sigma^2}{n} + n\mu^2\right) \right\}$~~

~~$= \frac{1}{n-1} \left\{ E(n^2 \bar{X}^2) \right.$~~

~~$= \frac{1}{n-1} \left\{ E(\sum X_i^2) - n E(\sum X_i \right.$~~

DATE: / /

$$= \frac{1}{n-1}\left[n\sigma^2 + n^2\mu^2 - n\left(\sigma^2/n + n\mu^2\right)\right]$$

$$= \frac{1}{n-1}(n-1)\sigma^2 = \boxed{\sigma^2}$$

iii) $\because \frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$

$$Var\left[\frac{(n-1)s^2}{\sigma^2}\right] = Var(\chi^2_{n-1})$$

$$\frac{(n-1)^2}{\sigma^4} Var(s^2) = 2(n-1)$$

$$\boxed{Var(s^2) = \frac{2\sigma^4}{(n-1)}}$$

iv) $E(s^2) = \sigma^2 = 100$

50% of $E(s^2) \Rightarrow 0.5 \times 100 = \underline{50}$

$$P[-50 < s^2 < 50] = P[-50.5 < s^2 < 50.5]$$

$$= P\left[\frac{-50.5(9)}{100} < \chi^2_9 < \frac{50.5(9)}{100}\right]$$

$$= P[-4.545 < \chi^2_9 < 4.545]$$

$=$

Q3) i) MLE $= \prod^{n} f_X(X = x_i)$

$$(L(\theta)) = [P(X=0)]^{86} (P(X=1))^{75} (P(X=2))^{16} \cdot (P(X=3))^2 (P(X=4))$$

Now, $\boxed{n=10,\ p} \Rightarrow \underline{Bin}$

$P[X=0] = {}^nC_0 p^0 q^n = (1-p)^n$.

$P[X=1] = {}^nC_1 p^1 q^{n-1} = np(1-p)^{n-1}$

$P[X=2] = {}^nC_2 p^2 q^{n-2} = {}^nC_2 p^2 (1-p)^{n-2}$

$P[X=3] = {}^nC_3 p^3 q^{n-3} = {}^nC_3 p^3 (1-p)^{n-3}$.

$P[X=4] = {}^nC_4 p^4 q^{n-4}$

$$= \boxed{{}^nC_4 p^4 (1-p)^{n-4}}$$

$$L(p) = (1-p)^{86n} \times \left(np(1-p)^{n-1}\right)^{75} \times \left({}^nC_2 p^2 (1-p)^{n-2}\right)^{16} \times \left({}^nC_3 p^3 (1-p)^{n-3}\right)^{2} \left({}^nC_4 p^4 (1-p)^{n-4}\right).$$

$$\cancel{= C\{(1-p)^{86n}}$$

$$= C\{(1-p)^{86n+75(n-1)+16(n-2)+2(n-3)+n-4}.$$
$$p^{75+32+6+4}.\}$$

$$L(p) = C\{(1-p)^{180n-117} p^{117}\}$$

Differentiating both the sides, we get that :-

(Before that, Taking log on both sides :)

$$\cancel{\frac{d\,L(p)}{dp} = C}$$

$$\log(L(p)) = 180n-117 \log(1-p) + 117\log p.$$

Differentiating both the sides,

$$\frac{L'(p)}{L(p)} = \frac{180n-117}{1-p}(-1) + \frac{117}{p}.$$

$$\frac{d\log(p)}{dp} = \frac{117-180n}{1-p} + \frac{117}{p}$$

If $\frac{d\ln(p)}{dp} = 0 \Rightarrow$ then $\Rightarrow$

PAGE NO.:

DATE: / /

$$\frac{-117}{p} = \frac{117-180n}{1-p}$$

$$117(1-p) = \cancel{117}\ \cancel{180np}\ p(180n-117)$$

$$117 = p\left[180n-\cancel{117}+\cancel{117}\right]$$

$$\boxed{p = \frac{117}{180n}} \Rightarrow \boxed{p = \frac{117}{1800}} = \underline{0.065}$$

Q4] $f_X(x) = \dfrac{C\beta^3}{(x+\beta)^4}$, $\boxed{x>0,\ \beta>0}$

i) $\because$ By Fundamental Property:-

$$\int_0^1 \frac{C\beta^3}{(x+\beta)^4}\,dx = 1$$

$$C\beta^3\int_0^1 \frac{dx}{(x+\beta)^4} = 1$$

$$\left[\left(\frac{(x+\beta)^{-4+1}}{-4+1}\right)\beta\right]_0^1 = \frac{1}{C\beta^3}$$

$$\left[(x+\beta)^{-3}\right]_0^1 = \frac{-1}{3C\beta^4}$$

Prob n Stat Assignment

Page No.
Date

Q3] i) X: No of claims ~~on a day~~ on a particular type of policy

$X \sim Bin(4,p)$.

$\boxed{n=180}$.

i) MLE of P:-

Let L be the likelihood fn:-

$\frac{180}{5} = 36$

$$L = P(X=0)\, P(X=1)\, P(X=2)\, P(X=3).$$
$$= {}^nC_0 p^0(1-p)^3 \; {}^nC_1 p^1(1-p)^2 \; {}^nC_2 p^2(1-p)^1 \; {}^nC_3 p^3(1-p)^0$$
$$L = (\text{Const})\, p^{0+1+2+3}\,(1-p)^{3+2+1+0}$$
$$L = p^6(1-p)^6$$

Taking log on both the sides,

$$\log L = 6\log p + 6\log(1-p).$$

Differentiating,

$$\frac{dL}{dp} = \frac{6}{p} + \frac{6(-1)}{1-p} = 6\left[\frac{1}{p} - \frac{1}{1-p}\right]$$

Put $\frac{dL}{dp} = 0$, we get that.

$$\frac{1}{1-p} = \frac{1}{p} \Rightarrow p = 1-p. \Rightarrow \boxed{\hat{p} = \frac{1}{2}}$$

$$\Rightarrow \boxed{\hat{p} = \frac{1}{2}}$$

ii)

	O_i	E_i	$(O_i-E_i)^2/E_i$
1	86	36	$(50)^2/36 =$
2	75	36	$(39)^2/36 =$
3	16	36	$(-20)^2/36 =$
4	2	36	$(-34)^2/36 =$
5	1	36	$(-35)^2/36 =$

Page No.
Date

i.e $f(x) = \frac{\alpha \lambda^{\alpha}}{(\lambda + x)^{\alpha+1}}$, $x > 0$,

Here $\boxed{\alpha = 3}$, $\boxed{\lambda = \beta}$.

$\Rightarrow f(x) = \frac{3\beta^3}{(x+\beta)^4}$, where from,

$\boxed{C = 3}$

$\Rightarrow 3 = \frac{\beta(1+\beta)^5}{\beta^5 - (1+\beta)^5}$.

$3\left[\frac{\beta^5}{(1+\beta)^5} - 1\right] = \beta$.

$\left(\frac{\beta}{1+\beta}\right)^5 = \frac{\beta}{3} + 1 = \frac{\beta + 3}{3}$.

Let $1+\beta = x$,

$\left(\frac{x-1}{x}\right)^5 = \frac{x-1+1}{3}$

$\left(1 - \frac{1}{x}\right)^5 = \frac{x+2}{5}$.

$\Rightarrow \boxed{x = \quad}$

$\Rightarrow \boxed{\beta = \quad}$

By table,

$\Rightarrow$ Mean $= \frac{\lambda}{\alpha - 1} = \frac{\beta}{3}$

$\Rightarrow Var(X) = \frac{\alpha\lambda^2}{(\alpha-1)^2} = \frac{3\beta^2}{(2)^2} = \frac{3\beta^2}{4}$

$=$

iii) By MOM:-

$\overline{X_n} = \frac{\lambda}{\alpha - 1} = \frac{\hat{\beta}}{2}$.

$\Rightarrow \boxed{\hat{\beta} = 2\overline{X_n}}$

Page No

Date

$$\text{Bias} \Rightarrow E[g(X)] - \frac{\beta}{2}$$

$$\Rightarrow E\left[2\bar{X}_n\right] - \frac{\beta}{2}$$

$$\Rightarrow 2\, E(\bar{X}_n) - \frac{\beta}{2}$$

$$\Rightarrow 2\left(\frac{\beta}{2}\right) - \frac{\beta}{2} \Rightarrow \boxed{\frac{\beta}{2}}$$

Thus, the nature of the estimator is unbiased.

iii) Here, $\hat{\beta} = b\bar{X}_n$.

Q5] To show, $\hat{a} = \bar{x} - \sqrt{3}s$.

By MOM, Estimator of the parameter a is:-

$$\bar{x} = \frac{a+b}{2} \Rightarrow \cancel{\hat{a} = \frac{1-b\bar{x}}{\bar{x}} = \frac{1}{\bar{x}} - b}$$

$$\cancel{Var(g) = Var\left(\frac{1}{\bar{x}} - b\right) = Var\left(\frac{1}{\bar{x}}\right) + 0}$$

$$\cancel{\hat{a} = 2\bar{x} - b \Rightarrow g(X)}$$

$$\cancel{\Rightarrow Var(g) = Var(2\bar{x} - b) = 4\,Var(\bar{x}) = 4(}$$

Q5] To show, $\hat{a} = \bar{x} - \sqrt{3}s$

By MOM, Estimator of the parameter is:-

$$\bar{x} = \frac{a+b}{2} \Rightarrow \hat{a} = 2\bar{x} - b.$$

$$\Rightarrow 2\bar{x} = b + \hat{a} \quad \text{--- (1)}$$

$$\bar{s}^2 = \frac{1}{12}(b-a)^2.$$

$$\bar{s} = \frac{1}{\sqrt{12}}(b-a).$$

$$\sqrt{12}\,\bar{s} = b - \hat{a} \quad \text{--- (2)}$$

(1) - (2), we get that

$$2\bar{x} - \sqrt{12}\,\bar{s} = 2\hat{a}.$$

$$\boxed{\bar{x} - \sqrt{3}\,\bar{s} = \hat{a}}$$

ii) ① + ②, we get that:-

$$2\bar{x} + \sqrt{12}\bar{s} = 2\hat{b}$$

$$\Rightarrow \boxed{\hat{b} = \bar{x} + \sqrt{3}\bar{s}}$$

iii) Take any Random sample i.e
let $a = 5$, $b = 20$.
Sample of size 6:-

6, 9, 15, 18, 7, 10

$\bar{x} =$

$\bar{s} =$

Now, according to MOM Estimation:-

$$\hat{b} = \bar{x} + \sqrt{3}\bar{s}$$

$$=$$

$$\boxed{\hat{b} = \quad}.$$

and

$$\hat{a} = \bar{x} - \sqrt{3}\bar{s}$$

$$=$$

$$\boxed{\hat{a} = \quad}$$

Thus,

iv) Likelihood fn $= L = \prod_{i=1}^{n} f(x_i)$

$$= \underbrace{\left(\frac{1}{b-a}\right) \cdots \cdots \left(\frac{1}{b-a}\right)}_{n\ times}$$

$$L = \frac{1}{(b)^n}$$

Taking log on both the sides,

$\log L = -n \log (b)$

Taking Derivative

$$\frac{d \log L}{db} = -\frac{n}{b}$$

$$0 = +\frac{n}{b}.$$

$H_0 : p = 0.1,$ $X \sim Bin(n, p)$.

$H_1 : p > 0.1$.

Type-1 error :- Ho is Rejected but it is true.

If $X \Rightarrow$ proportion of the hits of missiles, then

In Case 1, Ho is rejected if $\underline{X} > 1/6$.

In Case 2, Ho is rejected, if $\underline{X} > \frac{3}{24} = \frac{1}{8}$.

i) $\alpha = P\left(\text{Ho is Rejected} / \text{It is true}\right)$

$$= P\left(\underline{X} > 1/6 / p = 0.1\right) + \frac{P\left(\underline{X} \leq 1/6 \cap X > 1/8\right)_{p=0.1}}{P\left(X > 1/8\right)_{p=0.1}}$$

$$= 1 - P\left(X \leq 1/6 / p = 0.1\right) + \frac{P\left(0.125 \leq X \leq 0.1667\right)_{p=0.1}}{P\left(X > 0.1/8\right)_{p=0.1}}$$

By table :- $P\left(\frac{X \le 1/6 = 0.1667}{p=0.1}\right) \Rightarrow$ By Interpolation,
The value calculated here is 0.34518 :
Again, By Interpolation, we get that :

$$P\left(\frac{0.125 < X < 0.1667}{p=0.1}\right) = 0.014105.$$

(i) $\alpha = 1 - 0.34518 + \frac{0.014105}{1 - 0.331075}.$

~~= 0.65482 +~~

$$\boxed{\alpha = 0.71626934}$$

ii) We calculate the power of the test in terms of p :-

~~$N = P\left[\frac{X > 1/6}{p=0.3}\right] + P\left[\frac{0.125 < X < 0.1667}{p=0.3}\right]$~~

~~iii) $\beta = P$ (H0 is not rejected / should be rejected) $P\left[\frac{X > 1/6}{p=0.3}\right]$~~

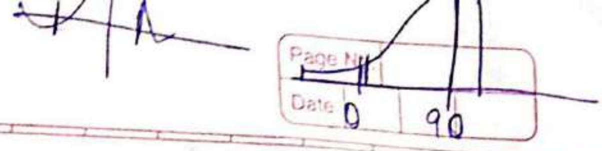
$$N = P\left[\frac{X > 1/6}{p=0.3}\right] + P\left(\frac{X \le 1/6}{p=0.3}\right) P\left(\frac{Y > 1/6}{p=0.3}\right)$$

$$P\left(\frac{X \le 1/6}{p=0.3}\right)$$

$$= P\left[\frac{X > 1/6}{p=0.3}\right] + P\left[\frac{Y > 1/6}{p=0.3}\right]$$

$$= 0.02567 + 0.02567$$

$$\boxed{N = 0.05134}$$



iv) $\boxed{n=10}$.

$$\boxed{p = \frac{40}{120} = \frac{1}{3}}$$

$$\hat{p} \sim N\left(p, \frac{p(1-p)}{n}\right)$$

$$\Rightarrow \hat{p} = p \pm Z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}$$

$$\hat{p} = p \pm Z_{10\%/2}\sqrt{\frac{\frac{1}{3}\left(\frac{2}{3}\right)}{\cancel{10}5}}$$

$$\hat{p} = \frac{1}{3} \pm Z_{5\%}\sqrt{\frac{1}{45}}$$

$$= \frac{1}{3} \pm \frac{(1.6449)}{\sqrt{45}} = \frac{1}{3} \pm 0.24520721$$

$$= 0.09126123.$$

Thus, the lower bound is 0.0916123.

v) $H_0: p = 0.278$.

$H_1: p > 0.278$.

Test statistic: $\frac{\hat{p} - p}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}}$.

$$\Rightarrow \frac{0.3 - 0.278}{\sqrt{\frac{0.3(2/3)}{10}}}$$

$$\Rightarrow \frac{0.022}{\sqrt{0.02}} \Rightarrow 0.3760603 \Rightarrow Z_{cal}$$

As $Z_{tab} = 1.6449$, Thus since.

$\frac{1}{5}\sqrt{95}$

1.649

~~$Z_{cal} < Z_{tab}$~~

$$= \frac{(Z+1/2)/n - \hat{p}}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}}$$

$$= \frac{(2.87+1/2)/10 - 0.278}{\sqrt{\frac{(0.278)(1-0.278)}{10}}}$$

$$= \frac{0.05}{0.27639} = 0.132841 < Z_{tab}$$

Hence, we donot reject the Hypothesis.

vi) From iv and v point, we see 2 ways of obtaining hypothesis conclusions, as they are:
→ Since the estimated probability (0.287) lies between the confidence interval of part iv, hence do not reject
→ And the TS is less than Z_{tab}, hence do not reject

vii) Let our H_0 be:-
H_0: Let $\delta = \mu_1 - \mu_2 = 0$
H_1: $\delta = \mu_1 - \mu_2 \neq 0$.
Let μ_1 = Institute 2 and μ_2 = Institute 1,
Since, no specification is given, we consider it with 5% los, from a normal distribution.
Test Statistic :- $\frac{(\hat{\mu}_1 - \hat{\mu}_2) - \delta}{\sqrt{\frac{\hat{\sigma}_1^2}{n_1} + \frac{\hat{\sigma}_2^2}{n_2}}}$.

$$\Rightarrow Z_{cal} = \frac{(85-70) - 0}{\sqrt{\frac{(54)^2}{12} + \frac{(70)^2}{15}}} = \frac{5}{\sqrt{\frac{(54)^2}{12} + \frac{(70)^2}{15}}}$$

[If we assume that $\boxed{\mu_1 = \mu_2}$]

$$= \frac{5}{\sqrt{243 + 326.667}} = \frac{5}{23.867696} = 0.020955$$

$< Z_{tab}$

Hence, we do not reject the hypothesis, i.e.
Thus, at some point of the dist^n, the differe--ence becomes zero.

Q7) i) Data 1 :-
[24, 45, 29, 33, 20, 40, 26.5, 26.5, 25, 27.5
$\rightarrow \Sigma = 296.5$

Thus SM $= \frac{296.5}{10} = 29.65$.

Data 2 :- Mean $= 31.5$.

Data 1 :- $SV_1 = \frac{1}{n-1}\left\{\Sigma x_i^2 - n(\bar{x}_1)^2\right\}$

$$= \frac{1}{9}\left[8614.5 - 296.5\right]$$

$$SV_1 = \frac{8318}{9} = \boxed{924.22}$$

$$SV_2 = \frac{1}{9}\left[\Sigma x_i^2 - n_2(\bar{x}_2)^2\right] .$$

$$= 1253.80556$$

ii) The assumption while testing for equal mean b/t^n Data 1 & 2 is that they have equal variance, which is not true in this case.

$\boxed{\sum x_i^* - \sum x_i}$

iii) ~~Hypothesis~~ LOS :- 5%.

$H_0 : \sum x_i = \sum x_i^* \Rightarrow \boxed{R = 0}$

$H_1 : \sum x_i < \sum x_i^*$ where

$x_i \rightarrow$ Individual claims in Public

$x_i^* \rightarrow$ Individual claims in Private

Test statistic:-

$$\frac{(\sum x_i^* - \sum x_i) - 0}{\sqrt{n[V(x_i) + V(x_i^*)]}}$$

$$\Rightarrow \left(\frac{315 - 296.5 - 0}{\sqrt{n[924.22 + 1253.80556]}} \right)$$

$$= \frac{18.5}{\sqrt{10(2178.02556)}} = \frac{18.5}{\sqrt{\cdot}}$$

$$= \frac{18.6}{147.5814} = \underline{0.125354} < (Z_{tab})_{5\%}$$

Thus, we do not reject the Hypothesis.

Q8] i) $\hat{\theta}$ is an unbiased estimator iff :-

$\boxed{E(\hat{\theta}) = \theta}$, where

$\hat{\theta}$ = estimator of parameter θ.

θ = True Parameter Value (θ).

ii) Bias is defined as the difference between the expectation of the estimator and the true ~~param~~ value of the parameter. i.e

If $g(X) \Rightarrow$ Estimator

$x \Rightarrow$ True parameter value

Page No.
Date

Then, Bias of the estimator is defined as:-

$Bias = E(g(\underline{x})) - x$.

~~Let Bias($\hat{\theta}$) = 0 and Estimator of~~

Let $E_1 = \tilde{\theta}$, $E_2 = \hat{\theta}$.

$Bias(\tilde{\theta}) = 0$

$Var(\tilde{\theta}) = (MSE)_1$, while $(MSE)_2 = [E(\hat{\theta}) - \theta]^2 + Var(\hat{\theta})$.

Since $(MSE)_1 < (MSE)_2$, thus.

efficiency of $\tilde{\theta}$ is more, as $n \to \infty$, $MSE \to 0$, which leads to greater efficiency.

v) Since $Bias(\tilde{\theta}) < Bias(\hat{\theta}) \Rightarrow \tilde{\theta}$ is more consistent than $\hat{\theta}$.

vi) 2 Methods of Estimating θ :-
[Assuming it is discrete] :-

→ Method of Moments i.e By equating Population and Sample Moments [Raw].

→ Maximizing Likelihood function i.e by calculating the maximum value of Likelihood function of a particular sample.

Q9) i) t_K variable is defined as:-

$\frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t_K$, where $\bar{X} \Rightarrow$ Sample Mean
$S \Rightarrow$ Sample Standard Deviation
$n \Rightarrow$ Size of the sample.
$\mu \Rightarrow$ pop^n mean.

Also, $t_K \sim \frac{N(0,1)}{\sqrt{\frac{\chi^2_K}{K}}}$, where $K = dog$ (degrees of freedom)

ii) $\text{Mean}(t_k) = 0$, and $\text{Var}(t_k) = \frac{k}{k-2}$, for $\boxed{k>2}$

iii) a) $n = 10$, $\bar{X} = 50$, $S^2 = 48.667$.
$\alpha = 1$

$$\mu = \bar{X} \pm t_{n-1, \frac{1}{2}\%} \sqrt{\frac{S^2}{n}}$$

$$= 50 \pm t_{9, 0.5\%} \sqrt{\frac{48.667}{10}}$$

$$= 50 \pm (3.250) \sqrt{4.8667}$$

$$= (42.83031, 57.1696945)$$

b) Also, $t_k \sim \dfrac{N(0,1)}{\sqrt{\frac{\chi^2_k}{k}}} \rightarrow \dfrac{\bar{X} - 0}{1/\sqrt{n}}$

$$P(x_1 < \bar{X} < x_2) = 0.99$$

$$\cancel{P\left(\frac{x_1 - \mu}{s/\sqrt{n}} < t_k < \frac{x_2 - \mu}{s/\sqrt{n}}\right)}$$

$$P\left(\frac{x_1 - 0}{1/\sqrt{n}} < N(0,1) < \frac{x_2 - 0}{1/\sqrt{n}}\right) = 0.99$$

$$P\left(x_1\sqrt{n} < N(0,1) < x_2\sqrt{n}\right) = 0.99$$

$$P\left(\frac{x_1\sqrt{n}}{\sqrt{\frac{\chi^2_9}{9}}} < t_{n-1} < \frac{x_2\sqrt{n}}{\sqrt{\frac{\chi^2_9}{9}}}\right) = 0.99$$

Test statistic.

$$\frac{\bar{X} - 0}{\frac{\sqrt{\frac{\chi^2_k}{k}}}{\sqrt{n}}} \Rightarrow \frac{\bar{X} - 0}{\sqrt{\frac{\chi^2_k}{nk}}}$$

$$\Rightarrow x_1/x_2 = \bar{X} \pm t_{9, 0.5\%} \sqrt{\frac{\chi^2 \cancel{\sigma} n-1}{n \cancel{\sigma} n-1}}$$

$$= 50 \pm (3.250) \sqrt{\frac{23.59}{9(10)}}$$

$$\doteq \underline{(48.336, 51.6639)}.$$

Q10] $X \Rightarrow$ No. of Claims ~~in a year~~ of an insurance policy.

$X \sim Poi(\lambda)$, where $\boxed{\lambda = 3}$.

$n\cancel{X} \Rightarrow$ No. of claims in a year.

$n\cancel{X} \sim Poi(n\mu)$.

Thus, our Hypothesis would be stated as:-

H_0: ~~nX = 3100~~ $\lambda = 3$

H_1: ~~nX < 3100~~ $\lambda \neq 3$.

i) Type 1 Error:- $P\left(\text{Ho is Reject} / \text{it is true}\right)$

$$= P\left(\frac{\cancel{X} 1000X < 3100}{\lambda = 3}\right)$$

$$= P\left(\frac{X > 3.1}{\lambda = 3}\right).$$

$$= \cancel{0.62484} = 1 - 0.62484$$

$$= \boxed{}$$

ii) Type 2 Error:- $P\left(\text{Ho is Not Rejected} / \text{it is not true}\right)$.

$$\Rightarrow P\left(\frac{X < 3.1}{\lambda = 3}\right)$$

$$\Rightarrow 0.62484$$

$\frac{\lambda^x e^{-\lambda}}{x!}$

Page No.

Date

iii) The power of a test is defined as Probability of Rejecting Ho when it is false, i.e

$(1-\beta)$, where β = Type II error.

$$\Rightarrow P\left(\frac{X > 3.1}{\lambda = \mu}\right) \Rightarrow 1 - F_X(3.1)$$

$$\Rightarrow \boxed{\frac{\mu^{3.1} e^{-\mu}}{(3.1)!}}$$