Application of Matrices in Robotics

Group: 90 - 99

Group Members-

Kartik Sundrani (90)	How Transformation Works In Robotics And its explanation.
Pranav Vaswani (93)	Robot Kinematics and its utility.
Bhavya Gupta (94)	Example problem 1 and its geometric and algebraic approach
Shagun Alag (97)	Forward Transformation for Rotation in Robotics.
Aditya Panchal (98)	Frame interpretation of Homogeneous Transformation in Robotics.
Aakanksha Kulkarni (99)	Computation of instantaneous velocity In matrix theory,application of jacobian matrices in robotics.

Kartik Sundrani (90)

How Transformation Works In Robotics

- Transformation Matrices can be used to describe that at what angle the servos need to be to reach the desired position in space or maybe an underwater autonomous vehicle needs to reach or align itself with several different obstacles inside the water.

- In other words, the transformation helps us to determine the movement of the parts of the objects or robots with respect to one another.

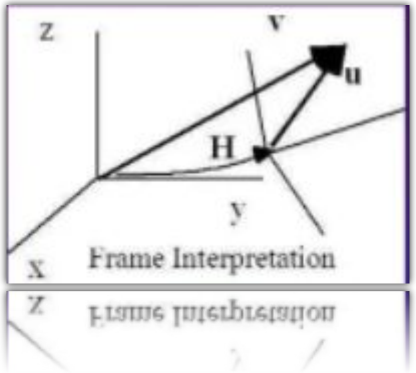
Kartik Sundrani (90)

Theoretical Explanation

- The Homogeneous Transformation effectively merges a frame orientation matrix, and frame translation vector into one frame.
- The order of the operation should be viewed as Rotation First, then translation.
- It can be viewed as position or orientation relationship of one frame relative to another frame called the reference frame.
- It can be interpreted as frame a described relative to the first or base frame while frame B described relative to frame A.

Aditya Panchal (98)

Frame Interpretation of Transformation



- Here we have been given with the vector u and its Transformation is represented by :

$$\boxed{V = Hu}$$

- Now this vector has components as

  Ux , Uy , Uz in a column and it has to

  Expanded to 4*1

Aditya Panchal (98)

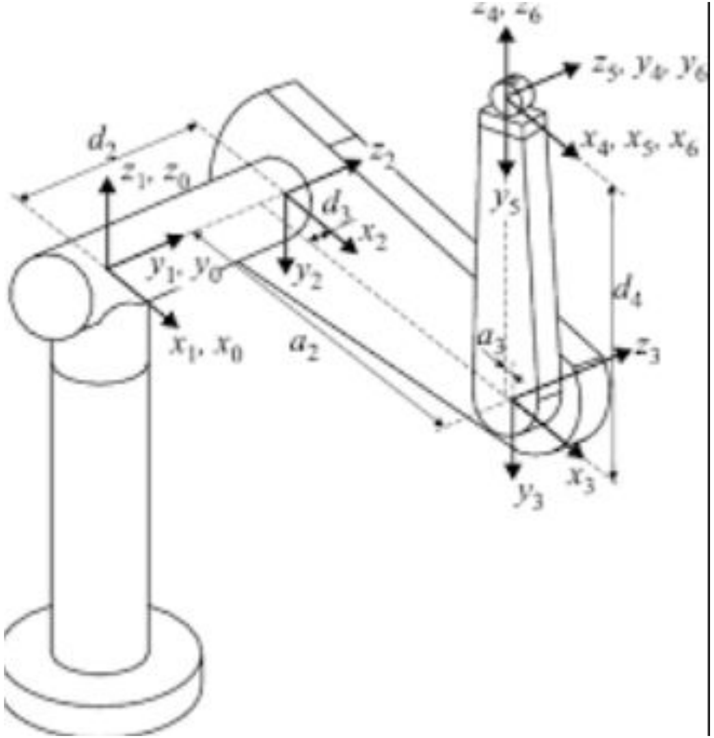
Homogeneous Transformation of matrices

$$F = \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- A transformation matrix must be in square form .
- It is much easier to calculate the inverse of a square matrix.
- In order to multiply the dimensions of matrices must match.

Aditya Panchal (98)

Graphical Representation of homogeneous transformation



$$^{A}P = {}^{A}_{B}R\,{}^{B}P + {}^{A}P_{BORG}$$

$$\begin{bmatrix} {}^{A}P_x \\ {}^{A}P_y \\ {}^{A}P_z \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} & {}^{A}_{B}R & & \begin{matrix} {}^{A}P_{BORG\_x} \\ {}^{A}P_{BORG\_y} \\ {}^{A}P_{BORG\_z} \end{matrix} \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{{}^{A}_{B}T} \begin{bmatrix} {}^{B}P_x \\ {}^{B}P_y \\ {}^{B}P_z \\ 1 \end{bmatrix}$$

$$^{A}P_{[4\times1]} = {}^{A}_{B}T_{4\times4}\,{}^{B}P_{[4\times1]}$$

Shagun Alag (97)

Forward Transformation in Robotics

- Forward transformation matrices capture the relationship between the reference frames of different links of the robot.
- A reference frame is basically the point of view of each of the robotic links, where if you were an arm joint yourself what you would consider ‘looking forward’.

Shagun Alag (97)

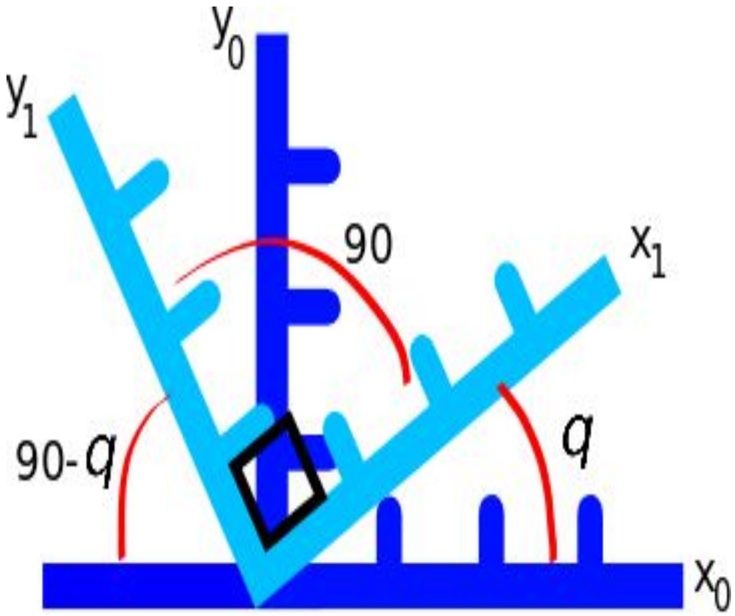
Forward Transformation for Rotation (1)

- The total contributions of a point defined in the *(x1,y1)* axes to the *x0* axis are,

$$p_{0_x} = cos(q)p_{x1} - sin(q)p_{y1}.$$

- Similarly for the *y0* axis contributions we have

$$p_{0_y} = sin(q)p_{x1} + cos(q)p_{y1}.$$



Shagun Alag (97)

Forward Transformation for Rotation (2)

- Rewriting the above equations in matrix form gives:

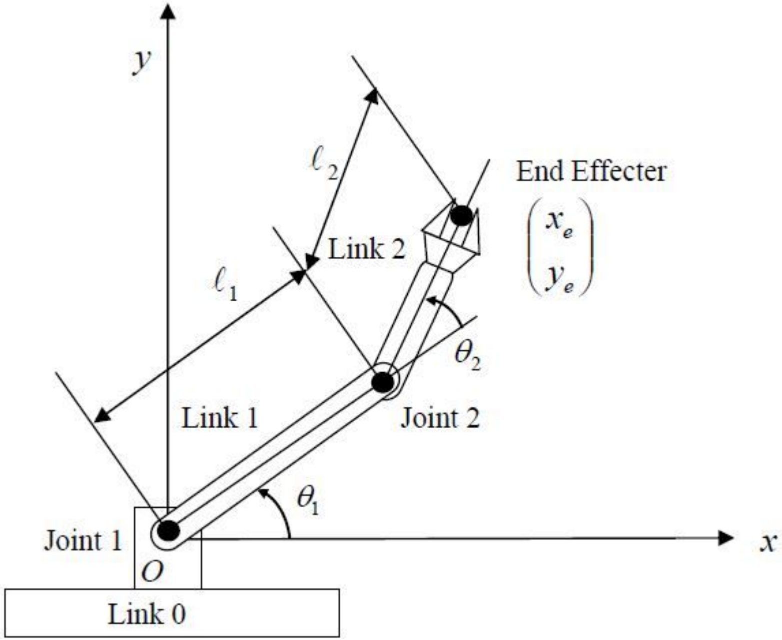
$$ {}^{1}_{0}\mathrm{R}\,\mathrm{p}_1 = \begin{bmatrix} cos(q_0) & -sin(q_0) \\ sin(q_0) & cos(q_0) \end{bmatrix} \begin{bmatrix} p_{x_1} \\ p_{y_1} \end{bmatrix}, $$

where ${}^{1}_{0}\mathbf{R}$ is called a rotation matrix.

Aakanksha Kulkarni (99)

Differential Kinematics

Robotic arm:



- Instantaneous velocity mappings can be obtained through time derivation of the direct kinematics function.

- It states the relationship between joint velocities & end-effector velocities of a robot manipulator.

Aakanksha Kulkarni (99)

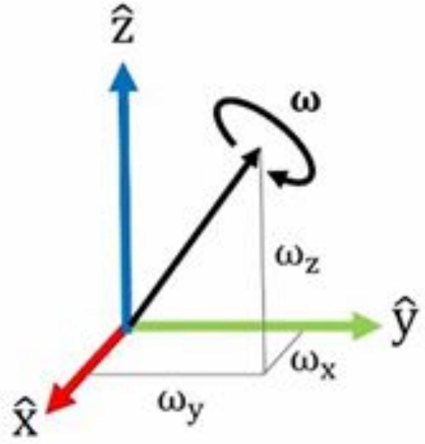
Returning to the time derivative.....

The time derivative of a rotation matrix is defined as a skew symmetric matrix as:

$$\dot{R}R^T = -R\dot{R}^T$$
$$\dot{R} = -\dot{R}^T$$

And

$$\boxed{\dot{R} \triangleq S(\omega)R}$$



Thus, we get the relationship:

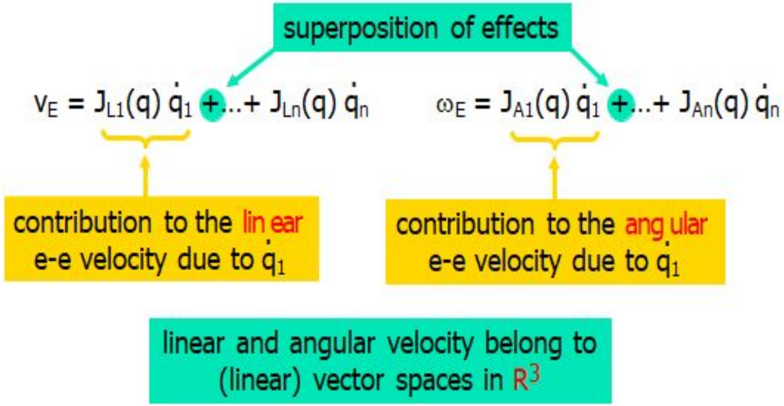
$$\boxed{\dot{R} = S(\omega)\,R} \Longleftrightarrow \boxed{S(\omega) = \dot{R}\,R^T}$$

Aakanksha Kulkarni (99)

Jacobian matrix theory applied to robotics:

- When jacobian matrices are applied to robotics and automation, then we get a 6 x n matrix of form:

$$\begin{pmatrix} v_E \\ \omega_E \end{pmatrix} = \begin{pmatrix} J_L(q) \\ J_A(q) \end{pmatrix} \dot{q} = \begin{pmatrix} J_{L1}(q) & \cdots & J_{Ln}(q) \\ J_{A1}(q) & \cdots & J_{An}(q) \end{pmatrix} \begin{pmatrix} \dot{q}_1 \\ \vdots \\ \dot{q}_n \end{pmatrix}$$

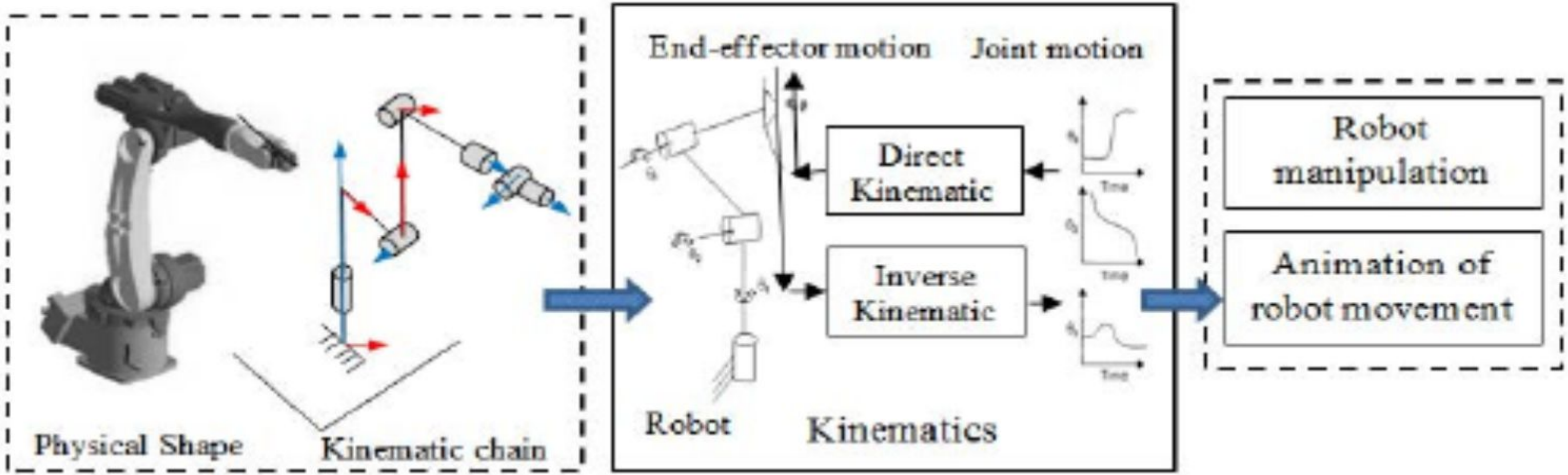


- We see that the derived 6 x n matrix calculates end effector instantaneous linear and angular velocity of a robotic arm.

Pranav Vaswani (93)

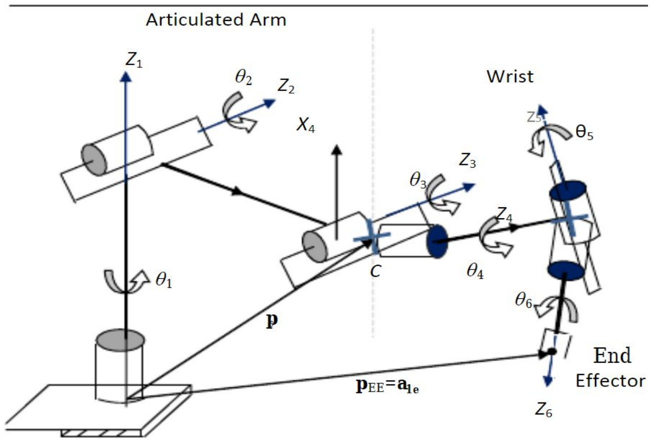
Robot kinematics and its utility.

The goal of a robot controller is to generate an acceptable motion of the end-effector by precisely actuating its joints for a specified task.



Pranav Vaswani (93)

Kinematic model of 6R robot with last three links intersecting

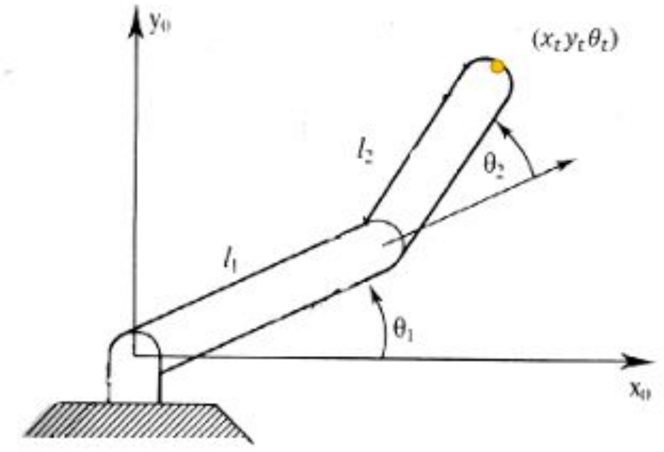


- The net transformation matrix to obtain the position and orientation of the End Effector can be given as:-

$$\mathbf{T} = \prod_{i=1}^{6} \mathbf{T}_i = \underbrace{(\mathbf{T}_1\mathbf{T}_2\mathbf{T}_3)}_{\text{Arm}} \overbrace{\mathbf{T}_4\mathbf{T}_5\mathbf{T}_6}^{\text{Wrist}}$$

Bhavya Gupta (94)

Example Problem 1



Set up:-

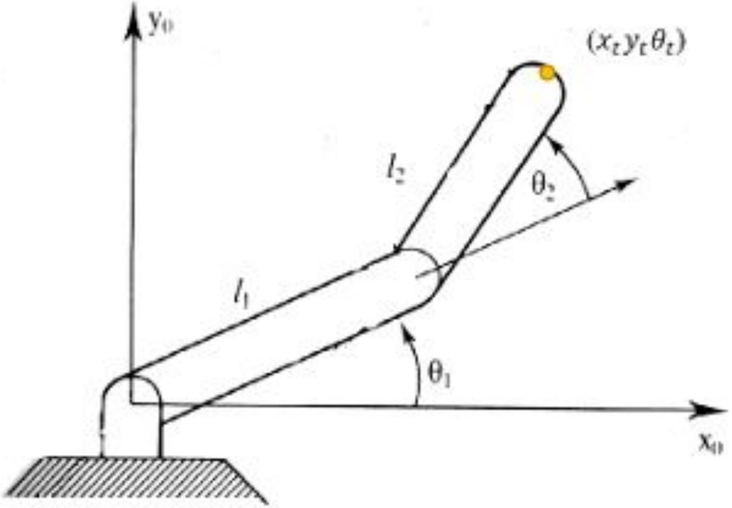
You have an RR robotic arm with base at the origin.- The first link moves th1 with respect to the x-axis. The second link moves th2 with respect to the first link.

Question-

What is the position and orientation of the end effector of the robotic arm?

Bhavya Gupta (94)

Geometric Approach-



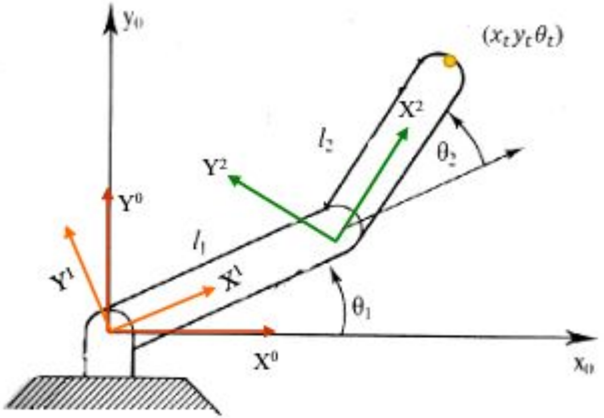
$$\theta t = \theta 1 + \theta 2$$

$$x_t = l1 * \cos(\theta 1) + l2 * \cos(\theta 1 + \theta 2)$$

$$y_t = l1 * \sin(\theta 1) + l2 * \sin(\theta 1 + \theta 2)$$

Bhavya Gupta (94)

Algebraic Approach-



- In the X0Y0 frame, the X1X1 frame is at orientation $\begin{bmatrix}\cos(\theta_1) & -\sin(\theta_1)\\ \sin(\theta_1) & \cos(\theta_1)\end{bmatrix}$.

$$\bar{V}^{X0Y0} = \begin{bmatrix}\cos(\theta_1) & -\sin(\theta_1)\\ \sin(\theta_1) & \cos(\theta_1)\end{bmatrix}\bar{V}^{X1Y1}$$

- In the X1Y1 frame, the X2X2 frame is at position $\begin{bmatrix}l_1\\ 0\end{bmatrix}$ and orientation $\begin{bmatrix}\cos(\theta_2) & -\sin(\theta_2)\\ \sin(\theta_2) & \cos(\theta_2)\end{bmatrix}$.

$$\bar{V}^{X0Y0} = \begin{bmatrix}\cos(\theta_2) & -\sin(\theta_2)\\ \sin(\theta_2) & \cos(\theta_2)\end{bmatrix}\left(\begin{bmatrix}l_1\\ 0\end{bmatrix} + \begin{bmatrix}\cos(\theta_2) & -\sin(\theta_2)\\ \sin(\theta_2) & \cos(\theta_2)\end{bmatrix}\bar{V}^{X2Y2}\right)$$

- In the X2Y2 frame, the end effector is at position $\begin{bmatrix}l_2\\ 0\end{bmatrix}$.

$$\bar{V}^{X0Y0} = \begin{bmatrix}\cos(\theta_2) & -\sin(\theta_2)\\ \sin(\theta_2) & \cos(\theta_2)\end{bmatrix}\left(\begin{bmatrix}l_1\\ 0\end{bmatrix} + \begin{bmatrix}\cos(\theta_2) & -\sin(\theta_2)\\ \sin(\theta_2) & \cos(\theta_2)\end{bmatrix}\left(\begin{bmatrix}l_2\\ 0\end{bmatrix} + \begin{bmatrix}x_t\\ y_t\end{bmatrix}\right)\right)$$

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