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Calculus HW

① $\lim\limits_{h\to 0} \dfrac{2(-3+h)^2-18}{h}$

$\longrightarrow \cancel{\lim\limits_{h\to 0} \dfrac{2(h^2+9-6h-18)}{h}}$

$= \cancel{\lim \dfrac{2(h^2-6h-9)}{h}}$

$= \lim\limits_{h\to 0} \dfrac{2(-3+h)^2-2(3)^2}{h}$

$= \lim\limits_{h\to 0} 2\left[\dfrac{(-3+h-3)(-\cancel{3}+h+\cancel{3})}{h}\right]$

$= \lim\limits_{h\to 0} 2\left[\dfrac{(h-6)\cancel{(h)}}{\cancel{h}}\right]$

$= \lim\limits_{h\to 0} 2[(h-6)]$

$= 2\times -6 = -12$

② $g(x) = \begin{cases} 2x & x<6 \\ x+1 & x\geq 6 \end{cases}$

$\longrightarrow$ ⓐ at $x=4$

$g(4) = 2\times 4 = 8$

$\lim\limits_{x\to 4^-} g(x) = 8$

$\lim\limits_{x\to 4^+} g(x) = 8$

$\therefore$ $g(x)$ is continuous at $x=4$

(b) $x = 6$

$\longrightarrow$ $g(6) = x - 1 = 6 - 1 = 5$

$= \lim_{x \to 6^-} g(x) = 2(6) = 12 \longrightarrow LHL$

$\lim_{x \to 6^+} g(x) = 6 - 1 = 5 \longrightarrow RHL$

$= \therefore$ Since $LHL \neq RHL$

Therefore, discontinuous at $x = 6$.

(3) $\lim_{x \to 9} \frac{x - 9}{3 - \sqrt{x}}$

$\longrightarrow \lim_{x \to 9} - \frac{\cancel{(\sqrt{x} - 3)}(\sqrt{x} + 3)}{\cancel{3 - \sqrt{x}}}$

$= \lim_{x \to 9} - (\sqrt{x} + 3)$

$= - (\sqrt{9} + 3) = -6$

(4) $f(x) = \frac{4x + 5}{9 - 3x}$

$\longrightarrow$ a) $x = -1$

$= f(-1) = \frac{4(-1) + 5}{9 - 3(-1)} = \frac{1}{12}$

b) $x = 0$

$f(0) = \frac{5}{9}$

c) $x = 3$

$f(3) = \frac{4(3) + 5}{9 - 3(3)}$

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(5) $$\lim_{x\to\infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$$

$$\to \lim_{x\to\infty} \frac{\dfrac{6e^{4x} - e^{-2x}}{e^{4x}}}{\dfrac{8e^{4x} - e^{2x} + 3e^{-x}}{e^{4x}}}$$

$$= \lim_{x\to\infty} \frac{6 - e^{-6x}}{8 - e^{-2x} + 3e^{-5x}}$$

$$= \frac{6-0}{8-0+0}$$

$$= \frac{6}{8} = \frac{3}{4}.$$

(6) $$g(y) = \begin{cases} y^2+5 & \text{if } y < -2 \\ 1-3y & \text{if } y \geq -2 \end{cases}$$

a) $\lim_{y\to 6} g(y)$

$$= \lim_{y\to 6} (1-3y) = 1-3(6) = -17$$

b) $$\lim_{y\to -2} g(y) = \lim_{y\to -2} (1-3y) = 1-3(-2)$$
$$= 1+6$$
$$= 7.$$

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(7) $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

By product Rule,

$\rightarrow f'(t) = (4t^2 - t)\frac{d}{dt}(t^3 - 8t^2 + 12) +$

$(t^3 - 8t^2 + 12)\frac{d}{dt}(4t^2 - t)$

$= (4t^2 - t)(3t^2 - 16t + 0)$

$+ (t^3 - 8t^2 + 12)(8t - 1)$

$= 12t^4 - 64t^3 - 3t^3 + 16t^2$

$+ 8t^4 - t^3 - 64t^3 + 8t^2$

$+ 96t - 12$

$= 20t^4 - 132t^3 + 24t^2 + 96t - 12$

(8) $R(w) = \dfrac{3w + w^4}{2w^2 + 1}$

$\rightarrow R'(w) = \dfrac{(2w^2+1)\frac{d}{dw}(3w + w^4) - (3w + w^4) \times \frac{d}{dw}(2w^2+1)}{(2w^2+1)^2}$

$= \dfrac{(2w^2+1)(3 + 4w^3) - (3w + w^4)(4w)}{(2w^2+1)^2}$

$= \dfrac{6w^2 + 8w^5 + 3 + 4w^3 - 12w^2 - 4w^5}{(2w^2+1)^2}$

$= \dfrac{4w^5 - 6w^2 + 4w^3 + 3}{(2w^2+1)^2}$

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(9) $f(x) = 2e^x - 8^x$

$\longrightarrow f'(x) = 2\frac{d}{dx}(e^x) - \frac{d}{dx}(8^x)$

$= 2e^x - 8^x \ln 8$

(10) $y = z^5 - e^z \ln(z)$

$\longrightarrow \frac{dy}{dz} = 5z^4 - \left[e^z \frac{d}{dz}(\ln z) + \ln z \frac{d}{dz}(e^z)\right]$

$= 5z^4 - \left[e^z \frac{1}{z} + \ln z(e^z)\right]$

$= 5z^4 - e^z\left[\frac{1}{z} + \ln z\right]$

(11) a) $f(x) = (6x^2 + 7x)^4$

$\longrightarrow f'(x) = 4(6x^2 + 7x)^3 \frac{d}{dx}(6x^2 + 7x)$

$= 4(6x^2 + 7x)^3 (12x + 7)$

$= (6x^2 + 7x)^3 (48x + 28)$

b) $f(t) = 5 + e^{4t} + t^7$

$f'(t) = 0 + e^{4t+t^7} \frac{d}{dt}(4t + t^7)$

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$= e^{4t+t^7} \cdot (4+7t^6)$

(12) a) $z = \ln(7-x^3)$

$\rightarrow \frac{dz}{dx} = \frac{1}{7-x^3} \cdot \frac{d}{dx}(7-x^3)$

$= \frac{1}{7-x^3} \cdot (0-3x^2)$

$= \frac{dz}{dx} = \frac{-3x^2}{7-x^3}$

$= \frac{d^2z}{dx} = \frac{d}{dx}\left[\frac{-3x^2}{7-x^3}\right]$

$= \frac{(7-x^3)[-6x] - (-3x^2)(-3x^2)}{(7-x^3)^2}$

$= \frac{-42x + 6x^4 + -9x^4}{(7-x^3)^2}$

$= \frac{-42x - 3x^4}{(7-x^3)^2}$

$= \cancel{3(14} \; \frac{-3x(x^3+14)}{(7-x^3)^2}$

b) $Q(v) = \frac{2}{(6+2v-v^2)^4}$

$\rightarrow Q'(v) = \frac{(6+2v-v^2)^4(0) - 2(4)(6+2v-v^2)^3 \times (2-2v)}{(6+2v-v^2)^8}$

$= \frac{-8(2-2v)\cancel{(6+2v-v^2)^3}}{(6+2v-v^2)^{\cancel{8}\,5}} = \frac{16v-16}{(6+2v-v^2)^5}$

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$$= \quad Q''(v) = 16 \frac{d}{dv} \frac{(v-1)}{(6+2v-v^2)^5}$$

$$= 16 \left[\frac{(6+2v-v^2)^5 (1) - (v-1) \times (5)(6+2v-v^2)^4 (2-2v)}{(6+2v-v^2)^{5 \times 2}} \right]$$

$$= \frac{16 \left((6+2v-v^2)^5 + 4(1-v)^2 (6+2v-v^2)^4 \right)}{(6+2v-v^2)^{10}}$$

$$= \frac{16}{(6+2v-v^2)^5} + \frac{4(1-v)^2}{(6+2v-v^2)^6}.$$

c) $f(t) = \ln(1+t^2)$

$\longrightarrow \quad f'(t) = \frac{1}{1+t^2} \cdot 2t$

$= \quad f''(t) =$ By u/v rule,

$$= \frac{(1+t^2)(2) - (2t)(2t)}{(1+t^2)^2}$$

$$= \frac{2+2t^2 - 4t^2}{(1+t^2)^2}$$

$$= 2 - 2t^2$$

(13) $f(x) = x^3 - 3x^2 - 9x + 12$

$\rightarrow$ $f'(x) = 3x^2 - 6x - 9$

Put $f'(x) = 0$,

$\therefore 3(x^2 - 2x - 3) = 0$

$(x-3)(x+1) = 0$

$\underline{x = 3,\ x = -1}$

$f''(x) = 6x - 6$

$f''(3) = 18 - 6 = 12 > 0$

$f''(-1) = -6 - 6 = -12 < 0$

$\therefore 3 \longrightarrow$ pt of Minima

Min value $= (3)^3 - 3(3)^2 - 9(3) + 12$
$= -15$

$-1 \longrightarrow$ pt of Maxima

Max value $= (-1)^3 - 3(-1)^2 - 9(-1) + 12$
$= 21 - 3 - 1$
$= 21 - 4$
$= \underline{\underline{17}}$.

(14) $f(x) = 4x^3 - 18x^2 + 24x - 7$

$\rightarrow$ $f'(x) = 12x^2 - 36x + 24$
$= 12(x^2 - 3x + 2)$
$= 12(x-2)(x-1)$

$f'(0) = \cancel{12}$ at $\underline{x = 2, 1}$.

$= \quad f''(x) = 24x - 36$

$= \quad \cancel{f''(x)}\ f''(1) = 24 - 36 = -12 \rightarrow$ Pt of Max

$f''(2) = 48 - 36 = 12 \rightarrow$ Pt of min

$= \therefore \quad f(1) = 4(1)^3 - 18(1)^2 + 24(1) - 7$

$= 4 - 18 + 24 - 7$

$= 3 \longrightarrow$ Max Value

$f(2) = 4(2)^3 - 18(2)^2 + 24(2) - 7$

$= 32 - 72 + 48 - 7$

$= \cancel{8} - 7 = 1 \longrightarrow$ Minimum value

(15) $f(x) = x^2(x+3)$

$\rightarrow \quad f'(x) = x^2 \frac{d}{dx}(x+3) + (x+3)\frac{d}{dx}(x^2)$

$= x^2(1) + (x+3)(2x)$

$= 3x^2 + 6x$

$=$ Put $f'(x) = 0$

$3x(x+2) = 0$

$x = 0,$

$x = -2$

$= \quad f''(x) = 6x + 6$

$= \quad f''(0) = 6 \longrightarrow$ Pt of Minima

$f''(-2) = -12 + 6 = -6 \longrightarrow$ Pt of Maxim

$=$ Minimum Value $= 0$

Max $= 4(3-2) = 4$

(1b) $f(x) = 3x^2 - 6x$

→ $f'(x) = 6x - 6$

$f'(x) = 0$

at $\underline{\underline{x = 1}}$

∴ $y = 3(x^2) - 6x$

$= 3 - 6$

$\underline{y = -3}$

∴ $\underline{\underline{1, -3}}$ is the lowest point in the graph

Now

at $x = 0$,

$y = 0$

∴ It passes throu' $(0,0)$.

at $y = 0$, $x = 2$ $(2,0)$