Assignment 2

Q1) Sol^n:-

$$\int_{1/50}^{2} \frac{e^{2/\omega}}{\omega^2} \, d\omega$$

for, integrand $e^{2/\omega}$, substitute $u = \frac{2}{\omega}$ and $du = -\frac{2}{\omega^2} d\omega$

This gives new lower ~~bound~~ bound $u = \frac{2}{\frac{1}{50}} = 100$

and upper bound $u = \frac{2}{2} = 1$

$$= -\frac{1}{2} \int_{100}^{1} e^{u} \, du$$

$$= \frac{-1}{2} \left(-e^{100} + e \right)$$

$$= \frac{1}{2} \left(e^{100} + e \right)$$

Q2) Sol^n is

$$\int \frac{2t^3+1}{(t^4+2t)^3}\,dt$$

For the integrand $\frac{2t^3+1}{(t^4+2t)^3}$, substitute $u = t^4+2t$

and $du = (4t^3+2)\,dt$.

$$= \frac{1}{2}\int \frac{1}{u^3}\,du$$

$$= \frac{1}{2}\left(-\frac{1}{2u^2}+c\right)$$

$$= -\frac{1}{4(t^4+2t)^2} + c$$

$$= -\frac{1}{4(t^4+2t)^2} + c$$

Q3) $f'(x) = x^4 + 3x - 9$

then

$$f(x) = \frac{x^5}{5} + \frac{3x^2}{2} - 9x$$

Q4) $f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \leq 1 \end{cases}$

$$\int_{-2}^{3} f(x)\,dx = \int_{-2}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx$$

$$\int_{-2}^{1} 3x^2\,dx + \int_{1}^{3} 6\,dx$$

$$\left[x^3\right]_{-2}^{1} + \left[6x\right]_{1}^{3}$$

$$1+8 + 18-6 = 21$$

Q5) Sol^n :-

$$\int 3(8y-1)\, e^{4y^2-y}\, dy$$

applying u-substitution

$$= 3 \cdot \int e^u \, du$$

$$= 3e^u$$

$$= 3e^{4y^2-y}$$

$$= 3e^{4y^2-y} + c$$

Q6) $f''(x) = 15\sqrt{x} + 5x^3 + 6$

$f'(x) = \int 15\sqrt{x} + 5x^3 + 6\,dx$

$= \frac{15x^{3/2}}{3/2} + \frac{5x^4}{4} + 6x + k$

$f(x) = \int \frac{15x^{3/2}}{3/2} + \frac{5x^4}{4} + 6x + k\,dx$

$f(x) = 4x^{5/2} + \frac{x^4}{4} + 3x^2 + kx + c$

$f(1) = \frac{-5}{4} = 4 + \frac{1}{4} + 3 + k + c$

$k + c = -8.5$

$f(4) = 404 = 4(4)^{5/2} + \frac{(4)^4}{4} + 3(4)^2 + 4k + c$

$4k + c = 164$

$4k + c = 164$

$-\ k + c = -8.5$

$3k = 172.5$

$k = 57.5$

$c = -66$

$\therefore f(x) = 4x^{5/2} + \frac{x^4}{4} + 3x^2 + 57.5x - 66$

Q2) $\frac{dv}{dt} = 9.8 - 0.196v$

- $\int \frac{dv}{(0.196v - 9.8)} = \int dx$

- $\left(\frac{1}{0.196}\right) \ln(0.196v - 9.8) = x + c$

- $\ln(0.196v - 9.8) = 0.196x + C_1$ where $C_1 = 0.196c$

- $0.196v - 9.8 = e^{(0.196x + C_1)}$

$= e^{(0.196x)} \times e^{(C_1)}$

$= C_2 \times e^{(0.196x)}$

Q10) $t\dfrac{dy}{dt} +$ [redacted] $2y = t^2 - t + 1$

$$\frac{dy}{dt} + \frac{2y}{t} = t - 1 + \frac{1}{t}$$

$$\mu\frac{dy}{dt} + 2\frac{\mu}{t} = \mu\left(t - 1 + \frac{1}{t}\right)$$

let $\dfrac{d\mu}{dt} = \dfrac{2\mu}{t}$

$$\mu y = \int \mu\left(t - 1 + \frac{1}{t}\right) dt$$

$$\int \frac{d\mu}{\mu} = \int \frac{2\,dt}{t}$$

$$\ln|\mu| = 2\ln|t| + c$$

$$= \ln t^2 c$$

$$\mu = At^2$$

$$y = \left(\frac{1}{At^2}\right)\int At^2\left(t - 1 + \frac{1}{t}\right) dt$$

$$y = \frac{t^2}{4} - \frac{t}{3} + \frac{1}{2} + \frac{c}{t^2}$$

Q12) $f(x) = x^3 - 10x^2 + 6$ at $x = 3$

$$\sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n, \quad f^n$$

$f(x) = x^3 - 10x^2 + 6$ $\qquad f(3) = -57$

$f'(x) = 3x^2 - 20x$ $\qquad f'(3) = -33$

$f''(x) = 6x - 20$ $\qquad f''(3) = -2$

$f'''(x) = 6$ $\qquad f'''(3) = 6$

$f''''(x) = 0$, we stop here

$$\therefore f(x) = x^3 - 10x^2 + 6 = \frac{-57}{0!}(x-3)^0 + \frac{-33}{1!}(x-3)^1 + \frac{-2}{2!}(x-3)^2 + \frac{6}{3!}(x-3)^3$$

$$\therefore f(x) = -57 - 33(x-3) - (x-3)^2 + (x-3)^3$$

14) Let

$f(x) = \sqrt{1+x}$

The maclaurin series is given by

$$f(x) = f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)\,x^2}{2!} + \frac{f'''(0)\,x^3}{3!} + \cdots \frac{f^{(n)}(0)\,x^n}{n!} + \cdots$$

For the given function we have

$f(x) = (1+x)^{1/2}$

$$= 1 + (1/2)x + \frac{1/2(-1/2)}{2!}x^2 + \frac{1/2(-1/2)(-3/2)}{3!}x^3 + \cdots$$

$$= 1 + \frac{1}{2}x - \frac{1/4}{2}x^2 + \frac{3/8}{6}x^3 - \frac{15/16}{24}x^4 + \cdots$$

$$= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + \cdots$$

To estimate $f(1/2)$ we put $x = 1/2$

$$f(1/2) \approx 1 + 1/2\,(1/2) - 1/8\,(1/2)^2 + 1/16\,(1/2)^3$$
$$- 5/128\,(1/2)^4 + \cdots$$

$$\approx 1 + \frac{1}{4} - \frac{1}{32} + \frac{1}{128} - \frac{5}{2048} + \cdots$$

$$\approx \frac{9983}{8224}$$

$$= 1.2132886$$

Q 15) $f(x) = e^{-6x}$ $\qquad x = -4$

Sol^n:-

$$= e^{24} + \frac{d/dx(e^{-6x})(-4)}{1!}(4+x) + \frac{d^2/dx^2(e^{-6x})(-4)}{2!}(x+4)^2$$

$$+ \frac{d^3/dx^3(e^{-6x})(-4)}{3!}(x+4)^3 + \cdots$$

$$= e^{24} + \frac{-6e^{24}}{1!}(x+4)) + \frac{36e^{24}}{2!}(x+4)^2 + \frac{-216e^{24}}{3!}(x+4)^3$$

$$= e^{24} - 6e^{24}(x+4) + 18e^{24}(x+4)^2 + (-36e^{24}(x+4)^3) + \cdots$$

Q16) $f(x) = \ln(3+4x) \quad x=0$

Soln :-

$$= \ln(3) + \frac{d/dx(\ln(3+4x))}{1!} + \frac{d^2/dx^2(\ln(3+4x))}{2!}$$

$$+ \frac{d^3/dx^3(\ln(3x+4x))}{3!} + \cdots$$

$$= \ln(3) + \frac{4/3}{1!}x + \frac{-16/9}{2!}(x)^2 + \frac{128/27}{3!}(x)^3 + \cdots$$

$$= \ln(3) + \frac{4}{3}(x) + \left[-\frac{8}{9}(x)^2\right) + \frac{64}{81}(x)^3 + \cdots$$