Probability & statistics - II

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Q1.] $X \sim N(800, 100^2)$
$Y \sim N(1200, 300^2)$

$P(Y_1 + Y_2 + Y_3 > X_1 + X_2 + X_3 + X_4)$

$P(Y_1 + Y_2 + Y_3 - (X_1 + X_2 + X_3 + X_4) > 0)$

$(Y_1 + Y_2 + Y_3) - (X_1 + X_2 + X_3 + X_4) \sim N(3 \times 1200 -$
$\sim N(400, 3100$

$P\left(Z > \frac{800 - 400}{\sqrt{310000}}\right)$

$P(Z > 0.71842) = 1 - P(Z < 0.\cancel{77}842)$
$= 1 - P(Z < 0.72)$
$= 1 - 0.76424$
$= 0.23576$

Q2.] $N \sim$ Neg-binomial(n, p)

$P(N = n) = {}^{n+2}C_n (0.9)^3 (0.1)^n$

$= {}^{(n+3)-1}C_{n-1} (0.9)^3 (0.1)^n$

$n = 3, p = 0.9$

$X \sim$ Gamma$(6, 2)$

$M_Y(t) = E(e^{tY}/N = n)$

$E(e^{tY}/N = n) = E(e^{t(X_1 + X_2 \cdots X_n)})$
$= E(e^{tX_1} + e^{tX_2} + e^{tX_3} \cdots e^{tX_n}$
$= E\left(1 - \frac{t}{\lambda}\right)^{-\alpha}$

$= E\left(1 - \frac{t}{\lambda}\right)^{-6}$

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$$E(E(e^{tY}/N=n)) = E\left(1-\frac{t}{2}\right)^{-6}$$
$$= E\left(e^{N\log\left(1-\frac{t}{2}\right)^{-6}}\right)$$
$$= M_N\left(\log\left(1-\frac{t}{2}\right)^{-6}\right)$$

$$\cancel{M_N(t) = \left(\frac{pt}{1-qt}\right)^{r} = 0.9}$$

$$M_N(t) = \left(\frac{pe^t}{1-qe^t}\right)^{r} = \left(\frac{pe^{\log\left(1-\frac{t}{2}\right)^{-6}}}{1-qe^{\log\left(1-\frac{t}{2}\right)^{-6}}}\right)$$
$$= \left[\frac{0.9\left(1-\frac{t}{2}\right)^{-6}}{1-0.1\left(1-\frac{t}{2}\right)^{-6}}\right]^{3}$$

ii.] $$V(Y) = E(N)\,V(X) + [E(X)]^2\,V(N)$$
$$= \left[\frac{3}{0.9}\times\frac{6}{4}\right] + \left[\left(\frac{6}{2}\right)^2 \times \frac{3(0.1)}{0.9^2}\right]$$
$$= \frac{25}{3}$$

$SD = 2.887$

Q3.] $$f(x) = \lambda e^{-\lambda(x-\alpha)}, \quad x \geq \alpha \;\&\; \lambda, \alpha > 0$$

$$MGF = E(e^{tX}) = \int_{\alpha}^{\infty} e^{tx}\,\lambda e^{-\lambda(x-\alpha)}\,dx$$

$$M_X t = \lambda\int_{\alpha}^{\infty} e^{tx} e^{-\lambda x} e^{\lambda\alpha}\,dx$$
$$= e^{\lambda\alpha}\lambda\int_{\alpha}^{\infty} e^{(t-\lambda)x}\,dx$$
$$= e^{\lambda\alpha}\cdot\lambda\left[\frac{e^{(t-\lambda)x}}{t-\lambda}\right]_{\alpha}^{\infty} = \frac{e^{\lambda\alpha}\lambda}{t-\lambda}\left[\frac{e^{(t-\lambda)}}{t-\lambda}\right]_{\alpha}^{\alpha}$$

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$$\frac{e^{\lambda\alpha}\lambda}{t-\lambda}\left[-e^{-(\lambda-t)\alpha}\right]$$

$$\frac{e^{\lambda\alpha}\lambda}{\lambda-t}\, e^{-\lambda\alpha}\, e^{t\alpha} = e^{t\alpha}\frac{\lambda}{\lambda-t}$$

ii) $M_x(t) = \dfrac{e^{t\alpha}\lambda}{\lambda-t} = \lambda e^{t\alpha}(\lambda-t)^{-1}$

$M'_x(t) = \lambda\left[e^{t\alpha}(\lambda-t)^{-1}\right]$

$= \lambda\left[(\lambda-t)^{-1}e^{t\alpha}(\alpha) + e^{t\alpha}(\lambda-\right.$

$= \lambda\left[e^{t\alpha}(\lambda-t)^{-2}\left[\alpha(\lambda-t)+1\right]\right.$

$= \dfrac{1}{\lambda}\left[\lambda\alpha+1\right]$

$M''_x(t) = \dfrac{1}{\lambda}\left[\lambda(1)+0\right]$

$= \dfrac{\lambda}{\lambda} = 1$

$V(X) = 1 - \dfrac{1}{\lambda}(\lambda\alpha+1)$

$= \dfrac{\lambda-\lambda\alpha-1}{\lambda}$

iii)

~~Q4~~ 07.

		X			
		1	2	3	4
Y	1	0.2	0	0.05	0.15
	2	0	0.3	0.1	0.2

$V(X/Y=2) = E(X^2/Y=2) - E(X/Y=$

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$E(X^2/Y=2) = \Sigma x^2 \dfrac{P(X=x \cap Y=2)}{P(Y=2)}$

$\dfrac{1^2 \, P(X=1, Y=2)}{P(Y=2)} + \dfrac{2^2 \, P(X=2, Y=2)}{P(Y=2)} +$

$\dfrac{3^2 \, P(X=3, Y=2)}{P(Y=2)} + \dfrac{4^2 \, P(X=4, Y=2)}{P(Y=2)}$

$\dfrac{(1)(0)}{0.6} + \dfrac{4(0.3)}{0.6} + \dfrac{9(0.1)}{0.6} + \dfrac{16(0.2)}{0.6}$

8.8333

$E(X/Y=2) = \Sigma \dfrac{x(P X=x, Y=2)}{P(Y=2)}$

$\dfrac{1(0)}{0.6} + \dfrac{2(0.3)}{0.6} + \dfrac{3(0.1)}{0.6} + \dfrac{4(0.2)}{0.6}$

0.807

iii) $f(u,v) = \dfrac{48}{67}(2uv - u^2)$

$E(U/V=v)$

$\dfrac{f(u,v)}{f(v)} = f\,U/V=v$

$f(v) = \dfrac{48}{67}\int_0^1 (2uv - u^2)\,du \Rightarrow \dfrac{48}{67}\left[2\dfrac{u^2}{2}v - \dfrac{u^3}{3}\right]$

$= \dfrac{48}{67}\left[v - \dfrac{1}{3}\right] \Rightarrow \dfrac{16}{67}(3v-1)$

$f(v) = \dfrac{16}{67}(3v-1)$

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$$\frac{f(u,v)}{f(v)} = \frac{\overset{3}{\cancel{48}}}{\cancel{67}} \frac{(2uv^2 - u^2)}{\cancel{16}(3v-1)} \times \cancel{67}$$

$$= \frac{3}{3(v-1} (2uv^2 - u^2) \cdot 2u^2v^2$$

$$E(U|V=v) = \frac{3}{3v-1} \int_0^1 u\,(2uv^2 - u^2)\,du$$

Q10.)

$$= \frac{3}{3v-1}\left[\frac{2u^3v^2}{3} - \frac{u^4}{4}\right]_0^1$$

$$= \frac{3}{3v-1}\left[\frac{8v^2-3}{12}\right]$$

$$= \frac{8v^2-3}{4(3v-1)}$$

Q7.) $X \sim \text{Gamma}(3,2)$

$E(Y|X=x) = 3x+1$

$V(Y|X=x) = 2x^2+5$

$$E(Y) = E(E(Y|X=x) \Rightarrow E(3x+1)$$

$$= 3E(X)+1$$

$$= 3\left(\frac{3}{2}\right)+1 \Rightarrow \frac{9}{2} + \frac{1}{1} = \frac{9+2}{2}$$

$$= \frac{11}{2}$$

$$V(Y) = E(V(X|Y)) + V(E(X|Y))$$

$$= E(2x^2+5) + V(3x+1)$$

$$= E(2x^2+5) + V(3x+1)$$

$$= 2E(X^2) + 5 + 9V(x)$$

$$= E(X)^2 - [E(X)]^2$$

$$= \frac{3}{4} + \frac{9}{4} = E(X)^2$$

$$E(X)^2 = \frac{12}{4} \qquad E(X^2) = 3$$

$$V(Y) = 2(3) + 5 + 9\left(\frac{3}{4}\right)$$
$$= \frac{71}{4}$$

Q10.) $f(x,y) = k x^{-\alpha} e^{-y/\beta}$

$$k \int_1^\infty \int_1^\infty x^{-\alpha} e^{-y/\beta} \, dx \, dy = 1$$

$$k \int e^{-y/\beta} \left[\frac{x^{-(\alpha-1)}}{-\alpha+1} \right]_1^\infty = 1$$

$$\frac{k}{1-\alpha} \int e^{-y/\beta} [-1]$$

$$\frac{k}{\alpha-1} \int e^{-y/\beta} \, dy = 1$$

$$\frac{k}{\alpha-1} \left[\frac{e^{-y/\beta}}{-\frac{1}{\beta}} \right]_1^\infty$$

$$\frac{-k}{\alpha-1} (\beta) \left(e^{-1/\beta} \right) = 1$$

$$k = \frac{\alpha-1}{\beta\left(e^{-1/\beta}\right)}$$