Date........./........./.........

Page...................

Roll No. 51

Calculus Assignment

1. $$\int_{\frac{1}{50}}^{2} \frac{e^{2/w}}{w^2}\, dw$$

Let $\frac{2}{w} = u$, $du = \frac{-2}{w^2} . dw$, $\frac{du}{-2} = \frac{dw}{w^2}$

$w = 2, u = 1$ $\quad$ $w = 1/50$, $u = 100$

$$\frac{1}{-2} \int_{1}^{100} e^u \, du$$

$$\frac{-1}{2} [e^u]_1^{100}$$

$$\frac{-1}{2} [e^{100} - e^1]$$

$$\Rightarrow \boxed{\frac{-e^{99}}{2}}$$

3. $$f'(x) = x^4 + 3x - 9$$

$$f(x) = \int f'(x)\, dx$$

$$= \int x^4 + 3x - 9 \quad dx$$

$$f(x) \Rightarrow \frac{x^5}{5} + \frac{3x^2}{2} - 9x + C$$

4. $$\int_{-2}^{3} f(x)\,dx$$

$$f(x) = \begin{cases} 6 & (x > 1) \\ 3x^2 & (x \leq 1) \end{cases}$$

$$\int_{-2}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx$$

$$= \int_{-2}^{1} 3x^2 \, dx + \int_{1}^{3} 6 \, dx = \cancel{3}\left[\frac{x^3}{\cancel{3}}\right]_{-2}^{1} + [6x]_1^3$$

$$= \left[(1)^3 - (-2)^3\right] + \left[(6 \times 3) - (6 \times 1)\right] = (1+8) + (12) = 21$$

$$= \boxed{21}$$

Date......../......../........

Page..........................

5. $\int 3(8y-1)\, e^{4y^2-y}\, dy$

Let $u = 4y^2-y$

$du = 8y-1\ dy$

Hence, the equation will be,

$3\int e^{u}\, du$

$= 3e^{u}$

$\Rightarrow 3e^{4y^2-y}$

6. $f''(x) = 15\sqrt{x}+5x^3+6$

$f(1) = \frac{-5}{4}$, $f(4) = 404$

$f'(x) = \int f''(x)dx = \int 15\sqrt{x}+5x^3+6\ dx$

$f'(x) = \left(\frac{2\times 15\, x^{3/2}}{3}\right) + \frac{5x^4}{4} + 6x$

$f'(x) = 10x^{3/2} + \frac{5}{4}x^4 + 6x + C_1$

$f(x) = \int f'(x)\,dx = \int 10x^{3/2}+\frac{5}{4}x^4+6x+C_1\,dx$

$= \left(\frac{10\times 2\, x^{5/2}}{5}\right) + \left(\frac{5}{4}\times\frac{x^5}{5}\right) + \left(\frac{6x^2}{2}\right) + C_1x + C_2$

$f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 + C_1x + C_2$

$f(1) = 4+\frac{1}{4}+3+C_1+C_2 = \frac{-5}{4}$, $C_2+C_1 = \frac{-5}{4} - \frac{7}{1} - \frac{1}{4} = \frac{-34}{4} \Rightarrow$

$f(4) = 432+4C_1+C_2 = 404$, $4C_1+C_2 \Rightarrow -28$

Date......../......../........

Page........................

$\times 4\ [\ C_1+C_2 = -17/2\ ,\ 4C_1+C_2 = -28$

$$4C_1 + 4C_2 = -34$$
$$\underline{\cancel{4C_1} + C_2 = -28}$$

$$3C_2 = -6$$
$$\boxed{C_2 = -2}$$

$$C_1 = \frac{-17}{2} + 2$$
$$\boxed{C_1 = \frac{-13}{2}}$$

$$\boxed{f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 \cancel{+} - \frac{13}{2}x - 2}$$

7. $\underset{\text{Let}}{Z} = f(x+iy) + g(x-iy)$ —— (1)

Differentiating (1) partially w.r.t. x & y, we get,

$$p = f'(x+iy) + g'(x-iy) \quad \text{—— (2)}$$
$$q = f'(x+iy)i - g'(x-iy)(-i) \quad \text{—— (3)}$$

Differentiating (2) & (3) again partially w.r.t. x & y, we get

$$r = f''(x+iy) + g''(x-iy)$$

$$t = f''(x+iy).i^2 + g''(x-iy)(-i)^2$$

i.e. $t = i^2 \{ f''(x+iy) + g''(x-iy) \}$

or $t = a^2 r$

Date......../......../........

Page..........................

8. $\frac{dr}{d\theta} = \frac{r^2}{\theta}$, $r(1) = 2$

$r'(\theta) = \frac{r^2}{\theta}$

$r(\theta) = \int r'(\theta)\, dr$

$r(\theta) = \int \frac{r^2}{\theta}\, dr = \frac{1}{\theta}\int r^2\, dr = \frac{1}{\theta}\frac{r^3}{3} + C$

$r(\theta) = \frac{r^3}{3\theta} + C$

$r(1) = 2 = \frac{1}{3\theta} + C = \cancel{1}$

$C = 2 - \frac{1}{3\theta} = \frac{6\theta - 1}{3\theta}$

$r(\theta) = \frac{r^3}{3\theta} + \frac{6\theta - 1}{3\theta} \Rightarrow \boxed{\frac{r^3 + 6\theta - 1}{3\theta}}$

9. $\frac{dv}{dt} = 9.8 - 0.196v$

$\frac{dv}{9.8 - 0.196v} = dt$

$\cancel{I.F = e^{\int -0.196\, dt}}$

$\frac{dv}{dt} + 0.196v = 9.8$

$p(t) = 0.196$, $u(t)$ the integration factor is given by

$u(t) = e^{\int 0.196\, dt}$

Multiply each term by $u(t)$ and rearrange,

$(e^{\int 0.196 dt})\left(\frac{dv}{dt}\right) + (0.196)(e^{\int 0.196 dt})(v) = 9.8(e^{\int 0.196 dt})$

Let $e^{\int 0.196 dt} = D$.

$(f(x) * f(y))' = f'(x) f'(y) + f(x) f'(y)$

then,

$\underline{(D * v)' = 9.8D}$ or $(e^{\int 0.196 dt} * v)' = 9.8 e^{\int 0.196 dt}$

Date....../....../......

Page..............

12. $f(x) = x^3 - 10x^2 + 6$ at $x = 3$

Taylor Series Expansion,

$$\sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n$$, f^n as n^{th} derivative a as center

$f(x) = x^3 - 10x^2 + 6$ $\quad f(3) = -57$

$f'(x) = 3x^2 - 20x$ $\quad f'(3) = -33$

$f''(x) = 6x - 20$ $\quad f''(3) = -2$

$f'''(x) = 6$ $\quad f'''(3) = 6$

$f''''(x) = 0$, we stop here

$$f(x) = x^3 - 10x^2 + 6 = \frac{-57}{0!}(x-3)^0 + \frac{-33}{1!}(x-3) + \frac{-2}{2!}(x-3)^2 + \frac{6}{3!}(x-3)^3$$

Taylor Series for $f(x) = x^3 - 10x^2 + 6$

Ans. $f(x) = -57 - 33(x-3) - (x-3)^2 + (x-3)^3$

Date........./........./.........

Page...............................

13. Maclaurin series for $(1+x)^n$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \ldots\ldots$$

$f(x) = (1+x)^n$ $\qquad$ $f(0) = 1$

$f'(x) = n(1+x)^{n-1}$ $\qquad$ $f'(0) = n$

$f''(x) = n(n-1)(1+x)^{n-2}$ $\qquad$ $f''(0) = n(n-1)$

$f'''(x) = n(n-1)(n-2)(1+x)^{n-3}$ $\qquad$ $f'''(0) = n(n-1)(n-2)$

Ans. $\Rightarrow$ $f(x) = (1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots\ldots$

14. $f(x) = \sqrt{1+x}$ $\qquad$ $f(0) = 1$

$f'(x) = \frac{1}{2}(1+x)^{-1/2}$ $\qquad$ $f'(0) = \frac{1}{2}$

$f''(x) = \frac{-1}{4}(1+x)^{-3/2}$ $\qquad$ $f''(0) = \frac{-1}{4}$

$f'''(x) = \frac{3}{8}(1+x)^{-5/2}$ $\qquad$ $f'''(0) = \frac{3}{8}$

Using Maclaurin's series expansion function, we have

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \ldots$$

Ans. $f(x) = \sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} + \ldots\ldots$

Date......./......./.......

Page...................

15. $f(x) = e^{-6x}$ at $x = -4$] Taylor Series

Taylor Series Expansion,

$$\sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n, \quad f^n \text{ as } n^{th} \text{ derivative } a \text{ as center}$$

$f(x) = e^{-6x}$ $\qquad f(-4) = e^{24}$

$f'(x) = -6e^{-6x}$ $\qquad f'(-4) = -6e^{24}$

$f''(x) = 36e^{-6x}$ $\qquad f''(-4) = 36e^{24}$

$f'''(x) = -216e^{-6x}$ $\qquad f'''(-4) = -216e^{24}$

we stop here,

$$f(x) = e^{-6x} = \frac{e^{24}}{0!}(x+4)^0 + \frac{-6e^{24}}{1!}(x+4) + \frac{36e^{24}}{2!}(x+4)^2 + \frac{-216e^{24}}{3!}(x+4)^3$$

Taylor series for $f(x) = e^{-6x}$

Ans. $f(x) = e^{24} - 6e^{24}(x+4) + 18e^{24}(x+4)^2 - 36e^{24}(x+4)^3$

Date......./......../......
Page..........

16. $f(x) = \ln(3+4x)$ about $x=0$] Taylor series

$f(x) = \ln(3+4x)$ $\quad f(0) = \ln(3)$

$f'(x) = \dfrac{1}{3+4x}$ $\quad f'(0) = \dfrac{1}{3}$

$f''(x) = \dfrac{-4}{(3+4x)^2}$ $\quad f''(0) = \dfrac{-4}{9}$

$f'''(x) = \dfrac{9}{(3+4x)^3}$ $\quad f'''(0) = \dfrac{8}{27}$

$$f(x) = \frac{\ln(3)}{0!} \cdot x^0 + \left(\frac{1x}{3 \times 1!}\right) + \left(\frac{-4x^2}{9 \times 2!}\right) + \left(\frac{8}{27 \times 3!}x^3\right)$$

Ans. $f(x) = \ln(3) + \dfrac{x}{3} - \dfrac{2}{9}x^2 + \dfrac{4}{81}x^3 + \ldots\ldots$