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Numerical Method

Roll no. 19//

Assignment - 2

1. $$\begin{bmatrix} 1 & 2 \\ a & 0 \end{bmatrix} \begin{bmatrix} 3 & b \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 3-8 & b+2 \\ 3a & ab \end{bmatrix}$$

$$\begin{bmatrix} 3-8 & b+2 \\ 3a & ab \end{bmatrix} = \begin{bmatrix} -5 & 6 \\ 12 & 12 \end{bmatrix}$$

$$b+2 = 6$$
$$3a = 12$$
$$ab = 12$$

$$a = 4 \qquad b = 4$$

2. $$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \underline{I}$$

$$A^{20} = (A^2)^{10} = I^{10} = I$$

3. The matrix is not invertible if the determinant is 0.

$$\begin{bmatrix} 1 & \lambda & 0 \\ 3 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix} = 1(2) - \lambda(3-1) = 0$$

$$2 - 2\lambda = 0$$

$$\lambda = 1$$

4. Let us find the sales matrix for the parts be S.

	W.B	C.F	F.M
Mumbai	20	10	8
Pune	15	12	4

Let the price matrix for the parts be P.

	Undiscounted	discounted
W.B	700	600
C.F	300	200
F.M	1200	1000

Revenue Matrix = S × P

$$\begin{bmatrix} 20 & 10 & 8 \\ 15 & 12 & 4 \end{bmatrix} \times \begin{bmatrix} 700 & 600 \\ 300 & 200 \\ 1200 & 1000 \end{bmatrix}$$

$$= \begin{bmatrix} 26600 & 22000 \\ 18900 & 14400 \end{bmatrix}$$

The revenue matrix is

	Undiscounted	Discounted
Mumbai	26000	22000
Pune	18900	14400

5. Let the first term be a and the common difference be d.

The n^{th} term $= a + (n-1)d$

According to the given conditions,

$$a + 3d = 3a$$
$$a + 6d = 2(a + 2d) + 1$$

$$2a = 3d$$

$$a = 2d - 1$$
$$4d - 2 = 3d \quad \text{or} \quad d = 2$$
$$a = 3$$

6. Let the first term be a and the common difference be d.

$$7(a + 6d) = 11(a + 10d)$$
$$4a + 68d = 0$$
$$a + 17d = 0$$

$a + 17d$ is nothing but the 18^{th} term

7. The sum to n terms is given be $\frac{n}{2}(2a + (n-1)d)$

$\therefore \quad 10(2a + 19d) = 11(2a + 21d)$

$20a + 190d = 22a + 231d$

$-2a = 41d$

$a = 41$ //

8. The sum to n terms is given to be

$\frac{n}{2}(2a + (n-1)d)$

$= na + \frac{n^2 d}{2} - \frac{nd}{2}$

$= n^2\left(\frac{d}{2}\right) + n\left(a - \frac{d}{2}\right)$

Comparing coefficients of n & n^2,

$\frac{d}{2} = 2$ and $a - \frac{d}{2} = 5$

$d = 4 \qquad a = 7$

9. 111 ... 1 (91 times) = $1 + 10 + 100 + \dots + 10^{90}$

This is the sum of a G.P with $a = 1$, $n = 91$, $r = 10$

$$\text{Sum} = \frac{1((10^{91} - 1)}{10 - 1}$$

$$= \frac{(10^7)^{13} - 1}{10^7 - 1} \times \frac{10^7 - 1}{10 - 1}$$

$$= (1 + 10^7 + 10^{14} + \dots + 10^{91}) \times (1 + 10 + 100 + \dots 10^9)$$

Since both the terms are natural numbers, the given number is a multiple of 2 numbers.

Therefore it a composite no.

10. Let the number be a, ar & ar^2

$$a + ar + ar^2 = 31$$
$$a(1 + r + r^2) = 31$$

$$a^2 + (ar)^2 + (ar^2)^2 = 651$$
$$a^2(1 + r^2 + r^4) = 651$$

$a^2(1 + r^2 + r^4 + 2r + 2r^2 + 2r^3 = 961$

$a^2(1 + r^2 + r^4) + (2ar)(a)(1 + r + r^2) = 961$

$651 + (2ar)(31) = 961$

$2ar(31) = 310$

$ar = 5$

$a = \frac{5}{r}$

$\frac{5}{r}(1 + r + r^2) = 31$

$\frac{5}{r} + 5 + 5r = 31$

$5r + \frac{5}{r} = 26$

$5r^2 - 26r + 5 = 0$

$5r^2 - 25r - r + 5 = 0$

$5r(r - 5) - (r - 5) = 0$

$(5r - 1)(r - 5) = 0$

$r = \frac{1}{5}$ or $r = 5$

$a = 25$ or $a = 1$

$\therefore$ 1, 5, 25