

Assignment-2

Saathi

$$1. \int_{1/50}^2 \frac{e^{2/w}}{w^2} dw$$

$$\text{Let } t = e^{2/w}$$

Diff w.r.t w

$$\frac{dt}{dw} = -2e^{2/w} \cdot \frac{1}{w^2}$$

$$= \int_{1/50}^2 \frac{t w^2}{-2 w^2 e^{2/w}} dt$$

$$= -\frac{1}{2} \int_{1/50}^2 dt$$

$$= -\frac{1}{2} [t]_{1/50}^2$$

$$= -\frac{1}{2} (2 - 0.02)$$

$$= \boxed{-0.99}$$

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2. $\int \frac{2t^3+1}{(t^4+2t)^3} dt$

Let $x = (t^4+2t)^3$

Diff w.r.t. t .

$$\frac{dx}{dt} = 3(t^4+2t)^2 (4t^3+2)$$

$$= \int \frac{2t^3+1}{x(3)(t^4+t)^2 (4t^3+2)} dx$$

= $\otimes$ and $\odot$ by 2

$$= \int \frac{4t^3/t^2}{(x)(6)(4t^3/2)(x)^{2/3}} \cdot dx$$

$$= \frac{1}{6} \int x^{-5/3}$$

$$= \frac{1}{6} \left(\frac{x^{-5/3+1}}{-\frac{5}{3}+1} \right)$$

$$= \frac{1}{6(-2)} x^{-2/3} + C$$

$$= -\frac{1}{4} (t^4+2t)^{-2/3} + C$$

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Q3. $f'(x) = x^4 + 3x - 9$

$$f(x) = \int f'(x) dx$$

$$f(x) = \int (x^4 + 3x - 9) dx$$

$$= \frac{x^5}{5} + \frac{3x^2}{2} - 9x + C$$

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5.4. $\int 3(8y-1)e^{4y^2-y} dy$

$$x = 4y^2 - y$$

Diff w.r.t. x

$$\frac{dx}{dy} = (e^{4y^2-y}) (8y-1)$$

$$= \int \frac{3(8y-1)e^{4y^2-y}}{e^{4y^2-y}(8y-1)} dx$$

$$= 3x + C$$

$$= \boxed{3(e^{4y^2-y}) + C}$$

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4. $f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \leq 1 \end{cases}$

$$= \int_{-2}^3 f(x) dx$$

$$= \int_1^{\infty} 6 dx + \int_{-\infty}^1 3x^2 dx$$

$$= [6x]_1^{\infty} + \left[\frac{3x^3}{3} \right]_{-\infty}^1$$

$$= -6 + 1$$

$$= -5 //$$

$$9. \frac{dv}{dt} = 9.8 - 0.196v.$$

$$= \int \frac{dv}{9.8 - 0.196v} = \int dt$$

$$= \log(9.8 - 0.196v)$$

$$= -0.196v + C //$$

$$12. f(x) = x^3 - 10x^2 + 6 \quad x=3$$

$$f(x) = x^3 - 10x^2 + 6 \quad f(3) = 3^3 - 10(3)^2 + 6$$

$$= -57$$

$$f'(x) = 3x^2 - 20x \quad f'(3) = -33$$

$$f''(x) = 6x - 20 \quad f''(3) = -2$$

$$f'''(x) = 6 \quad f'''(3) = 6$$

$$f^{IV}(x) = 0 \quad f^{IV}(3) = 0$$

$$\therefore x^3 - 10x^2 + 6 = -57 - 33(x-3) + \frac{2}{2!}(x-3)^2 + \frac{6}{3!}(x-3)^3 + 0$$

$$= -57 - 33(x-3) - (x-3)^2 + (x-3)$$

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$$11. \frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}} \quad y(0) = -1$$

$$\therefore \int \frac{1}{y^3} dy = \int \frac{x}{\sqrt{1+x^2}} dx$$

$$\begin{aligned} 1+x^2 &= t \\ 2x dx &= dt \\ dx &= \frac{1}{2} dt \end{aligned}$$

$$\therefore \frac{-1}{2y^2} = \frac{1}{2} \int \frac{1}{\sqrt{t}} \cdot dt$$

$$\therefore -\frac{1}{2} \cdot \frac{1}{y^2} = 2 \times \frac{1}{2} \sqrt{1+x^2} + C$$

$$-\frac{1}{2} \cdot \frac{1}{(-1)^2} = \sqrt{1+(0)^2} + C$$

$$-\frac{1}{2} = 1 + C$$

$$-\frac{3}{2} = C$$

$$\frac{-1}{2y^2} = \sqrt{1+x^2} = \frac{3}{2} //$$