

Numerical Method & Algebra

Assignment - 2

1)

$$\begin{bmatrix} 1 & 2 \\ a & 0 \end{bmatrix} \begin{bmatrix} 3 & b \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 6 \\ 12 & 16 \end{bmatrix}$$

$$\begin{bmatrix} 3-8 & b+2 \\ 3a & ab \end{bmatrix} = \begin{bmatrix} -5 & 6 \\ 12 & 16 \end{bmatrix}$$

$$b+2=6$$

$$\underline{b=4}$$

$$3a=12$$

$$\boxed{a=4}$$

$$a=4, b=4$$

2) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$A^{20} = [A^2]^{10}$$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^2 = I$$

$$\therefore \text{Now, } (I)^{10} = I$$

$$A^{20} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\textcircled{3} \quad \begin{bmatrix} 1 & \lambda & 0 \\ 3 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix} = 1(2) - \lambda(3) = 2 - \lambda 3 = 0$$

$$2 = \lambda 3$$

$$\lambda = \frac{2}{3}$$

④ Total Revenue - Sales $\times$ each price.

Matrix for sales.

	W.B	C.F	F.M
Mumbai	20	10	8
Pune	15	12	4

Price matrix

Not dis. discounted

M.B	700	600
C.F	300	200
P.M	1200	1000

Revenue matrix :-

20	10	8
15	12	4

 $\times$

700	600
300	200
1200	1000

3

$$\begin{bmatrix} 14000 + 3000 + 9600 & 12000 + 2000 + 800 \\ 10500 + 3600 + 4800 & 9000 + 2400 + 4000 \end{bmatrix}$$

$$\begin{array}{r} 14 \\ 96 \\ \hline 1100 \end{array}$$

$$\begin{bmatrix} 26600 & 22000 \\ 18900 & 15400 \end{bmatrix}$$

Revenue matrix is

	undiscount	discount
Mumbai	26,600	22,000
Pune	18,900	15,400

Q.5 - To find first term & Common difference
nth term of AP: $a_n = a + (n-1)d$

Given conditions: -

A.P.:- a $a+d$ $a+2d$ $a+3d$

$$a + 3d = 3a \quad \text{--- ①}$$

$$a + 6d = 2(a + 2d) + 1 \quad \text{--- ②}$$

according to ①

$$3d = 2a$$

$$d = \frac{2}{3}a$$

put in ②.

$$a + 6 \times \frac{2}{3}a = 2\left(a + 2 \times \frac{2}{3}a\right) + 1$$

$$a + 4a = 2\left(a + \frac{4a}{3}\right) + 1$$

$$5a = 2\left(\frac{7a}{3}\right) + 1$$

$$5a = \frac{14a}{3} + 1$$

$$5a = \frac{14a + 3}{3}$$

$$15a = 14a + 3$$

$$\boxed{a = 3}$$

$$\boxed{d = 2}$$

Q6 p.p.

$$a + a+d \quad a+2d \quad a+3d$$

$$7(a+6d) = 11(a+10d)$$

$$\text{To show: } -(a+17d) = 0$$

So,

$$7a + 42d = 11a + 110d$$

$$7a - 11a = 110d - 42d$$

$$-4a = 68d$$

$$4a + 68d = 0$$

$$a + 17d = 0$$

Hence Proved.

Q7 Sum of Arithmetic series

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{20} = \frac{20}{2} [2a + 19d] = 10 [2a + 19d]$$

$$S_{22} = \frac{22}{2} [2a + 21d] = 11 [2a + 21d]$$

$$10 [2a + 19d] = 11 [2a + 21d]$$

$$\boxed{d = -2}$$

$$10[2a + 19x^{-2}] = 11[2a + 21x^{-2}]$$

$$10[2a - 38] = 11[2a - 42]$$

$$20a - 380 = 22a - 462$$

$$462 - 380 = 22a - 20a$$

$$\frac{82}{2a - 41} = 2a$$

Q8]

$$S_n = 2n^2 + 5n \quad \text{--- (1)}$$

Sum of A.P.

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$= na + \frac{(n-1)nd}{2}$$

$$= na + \frac{[n^2 - n] \times d}{2} \left[\frac{(n-1)d}{2} \right] \times n$$

$$= na + \frac{n^2d}{2} - \frac{nd}{2}$$

$$= n^2 \left(\frac{d}{2} \right) + n \left(a - \frac{d}{2} \right)$$

Comparing with eqⁿ 0

$$\frac{d}{2} = 2 \quad 1 \quad a - \frac{d}{2} = 5$$

$$\boxed{d = 4}$$

$$a - \frac{d}{2} = 5$$

$$a - \frac{4}{2} = 5$$

$$a = 5 + 2$$

$$\boxed{a = 7}$$

Q9

1 1 1 1 (91 times)

$$1 + 10 + 100 + 1000 + \dots + 10^{90}$$

$$a = 1, n = 91, r = 10$$

$$S_n = \frac{a_1 (r^n - 1)}{(r - 1)}$$

$$= \frac{1 \times (10^{91} - 1)}{10 - 1}$$

$$\boxed{a = 4}$$

$$a - \frac{a}{2} = 5$$

$$a - \frac{4}{2} = 5$$

$$a = 5 + 2$$

$$\boxed{a = 7}$$

99 1 1 1 1 (91 times)

$$1 + 10 + 100 + 1000 + 10^9$$

$$a = 1, n = 91, r = 10$$

$$S_n = \frac{a_1 (r^n - 1)}{(r - 1)}$$

$$= \frac{1 \times (10^{91} - 1)}{10 - 1}$$

$$(6) \text{ G.P. :- } a, ar, ar^2, ar^3.$$

$$a + ar + ar^2 = 31$$

$$a^2 + (ar)^2 + (ar^2)^2 = 651$$

$$a(1+r+r^2) = 31 \quad \text{--- (1)}$$

$$a^2(1+r^2+r^4) = 651 \quad \text{--- (2)}$$

Squaring eqⁿ (1)

$$[a(1+r+r^2)]^2 =$$

$$a^2(1+r^2+r^4 + 2r + 2r^2 + 2r^3) = 961$$

$$a^2(1+r^2+r^4) + \cancel{2a^2r} + \cancel{2a^2r^2} + \cancel{2a^2r^3} = 961$$

equation (1) & (2)

$$651 + \cancel{2a^2r} (31) = 961$$

$$2a^2r \cdot 31 = 310$$

$$62a^2r = 310$$

$$ar = \frac{5}{2}$$

$$a = \frac{5}{r}$$

Ans

equation in eqn ①

$$a(1+x+a^2) = 31$$

$$\frac{5}{x}(1+x+a^2) = 31$$

$$\frac{5}{x} + 5 + 5x = 31$$

$$\frac{5}{x} + 5x = 26$$

$$\frac{5 + 5x^2}{x} = 26$$

$$5 + 5x^2 = 26x$$

$$5x^2 - 26x + 5 = 0$$

$$5x^2 - 25x - x + 5 = 0$$

$$5x(x-5) - 1(x-5) = 0$$

$$(x-5)(5x-1) = 0$$

$$x = 5, \quad 5x = 1$$

$$x = \frac{1}{5}$$

$$x = 5, \quad a = \frac{5}{x} = \frac{5}{5} = 1$$

$$x = \frac{1}{5}, \quad a = \frac{5}{\frac{1}{5}} = 25$$

∴ the numbers are 1, 5, 25

