

10/02/22

Home work - 3.

Lagrange's Interpolation.

classmate

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Roll NO. 77. S.C.B.

①

$$y = f(x)$$

x	7	8	9	10
$f(x)$	3	1	1	9

find value of $y = f(x)$ at $x = 9.5$.

→

$$\begin{aligned}
 f(x) &= \frac{(9.5-8)(9.5-9)(9.5-10)}{(7-8)(7-9)(7-10)} \times (3) \\
 &\quad + \frac{(9.5-7)(9.5-9)(9.5-10)}{(8-7)(8-9)(8-10)} \times (1) \\
 &\quad + \frac{(9.5-7)(9.5-8)(9.5-10)}{(9-7)(9-8)(9-10)} \times (1) \\
 &\quad + \frac{(9.5-7)(9.5-8)(9.5-9)}{(10-7)(10-8)(10-9)} \times (9)
 \end{aligned}$$

$$\therefore f(x) = f(9.5) = 1.5$$

$$\begin{aligned}
 &= \frac{(1.5)(0.5)(-0.5)}{(-1)(-2)(-3)} \cdot (3) + \frac{(-2.5)(0.5)(-0.5)}{(1)(-1)(-2)} \cdot (1) \\
 &\quad + \frac{(2.5)(1.5)(-0.5)}{(2)(1)(-1)} \cdot (1) + \frac{(2.5)(1.5)(0.5)}{(3)(2)(1)} \cdot (9)
 \end{aligned}$$

$$\therefore f(9.5) = \frac{-0.375(3)}{6} + \frac{(-0.625)}{2} + \frac{(-1.875)}{-2} + \frac{1.875}{6}$$

②

x	3	7	9	10
y	168	120	72	63

find y when $x=26$.

$$f(6) = \frac{(6-7)(6-9)(6-10)}{(3-7)(3-9)(3-10)} (168) +$$

+

$$\frac{(6-3)(6-9)(6-10)}{(7-3)(7-9)(7-10)} (120) +$$

$$\frac{(6-3)(6-7)(6-10)}{(9-3)(9-7)(9-10)} (72) +$$

$$\frac{(6-3)(6-7)(6-9)}{(10-3)(10-7)(10-9)} (63)$$

$$f(6) = \frac{(-1)(-3)(-4)(168)}{(-4)(-6)(-7)} + \frac{(3)(-3)(-4)(120)}{(4)(-2)(-3)}$$

+

$$\frac{(3)(-1)(-4)(72)}{(8)(2)(-1)} + \frac{(3)(-1)(-3)(63)}{(7)(3)(1)}$$

$$\therefore f(6) = 12 + 180 - 72 + 27 = \underline{147}$$

3	x	1	2	7	8
	f(x)	4	5	5	4

find $f(6) = ?$

→

$$f(6) = \frac{(6-2)(6-7)(6-8)(4)}{(1-2)(1-7)(1-8)} + \frac{(6-1)(6-7)(6-8)(5)}{(2-1)(2-7)(2-8)} \\ + \frac{(6-1)(6-2)(6-8)(5)}{(7-1)(7-2)(7-8)} + \frac{(6-1)(6-2)(6-7)(4)}{(8-1)(8-2)(8-7)}$$

$$\therefore f(6) = \frac{(4)(-1)(-2)(4)}{(-1)(-6)(-7)} + \frac{(5)(-1)(-2)(5)}{(1)(-5)(-6)} +$$

+

$$\frac{(5)(4)(-2)(5)}{(6)(5)(-1)} + \frac{(5)(4)(-1)(4)}{(7)(6)(1)}$$

$$\therefore f(6) = -16/21 + 5/3 + 20/3 - 40/21 = \underline{\underline{5.6667}}$$

4	x	1	2	3	4	7
	f(x)	2	4	8	16	128

find $f(5)$

→

$$f(5) = \frac{(5-2)(5-3)(5-4)(5-7)(2)}{(1-2)(1-3)(1-4)(1-7)} +$$

+

$$\frac{(5-1)(5-3)(5-4)(5-7)(4)}{(2-1)(2-3)(2-4)(2-7)} +$$

+

$$\frac{(5-1)(5-2)(5-4)(5-7)(8)}{(3-1)(3-2)(3-4)(3-7)}$$

$$+ \frac{(s-1)(s-2)(s-3)(s-7)}{(4-1)(4-2)(4-3)(4-7)} (16)$$

+

$$\frac{(s-1)(s-2)(s-3)(s-4)}{(7-1)(7-2)(7-3)(7-4)} (128)$$

$$\begin{aligned} \therefore f(s) &= \frac{(3)(2)(1)(-2)(2)}{(-1)(-2)(-3)(-6)} + \frac{(4)(2)(1)(-2)(4)}{(1)(-1)(-2)(-5)} \\ &+ \frac{(4)(3)(1)(-2)(8)}{(2)(1)(-1)(-4)} + \frac{(4)(3)(2)(-2)(16)}{(3)(2)(1)(-3)} \\ &+ \frac{(4)(3)(2)(1)(128)}{(6)(5)(4)(3)} \end{aligned}$$

$$\therefore f(s) = -\frac{2}{3} + 32/s - 24 + 128/3 + 128/s.$$

$$\therefore f(s) = 32.934$$

(5)	x	10	20	14	16
	$f(x)$	0.25	0.2	0.15	0.10

find $f(15)$.

$$\rightarrow A(15) = \frac{(15-20)(15-14)(15-16)(0.25)}{(10-20)(10-14)(10-16)} + \frac{(15-10)(15-14)(15-16)(0.2)}{(20-10)(20-14)(20-16)}$$

$$+ \frac{(15-10)(15-20)(15-16)(0.15)}{(14-10)(14-20)(14-16)} + \frac{(15-10)(15-20)(15-14)(0.10)}{(16-10)(16-20)(16-14)}$$

$$\begin{aligned} f(15) &= \frac{(-5)(1)(-1)(0.25)}{(-10)(-4)(-6)} + \frac{(5)(1)(-2)(0.2)}{(10)(6)(4)} + \frac{(5)(-5)(-1)(0.15)}{(4)(-6)(-2)} \\ &+ \frac{(5)(-5)(1)(0.1)}{(16)(-4)(2)} \Rightarrow f(15) = 0.10521 \end{aligned}$$

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Homework - 2

Regula Falsi method

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Yasgi Tendulkar

Roll No. 71 Sec. B.

Q Find Approx. value of root of eqn

(1) $x^3 + x - 1 = 0$ lying b/w 0.5 and 1

x	$y = x^3 + x - 1$
0.5	-0.375
1	1

$$\bar{x} = \frac{0.5(1) - 1(-0.375)}{1 - (-0.375)} = \frac{0.5 + 0.375}{1.375} = \frac{0.875}{1.375} = 0.6364$$

$$\bar{y} = (0.6364)^3 + 0.6364 - 1 = -0.1058549 \Rightarrow \bar{y} = -0.1059$$

x	$y = x^3 + x - 1$
0.6364	-0.1059
1	1

$$\bar{x} = \frac{(0.6364)(1) - 1(-0.1059)}{1 - (-0.1059)} = \frac{0.6364 + 0.1059}{1.1059} = \frac{0.7423}{1.1059} = 0.6712$$

$$\bar{y} = (0.6712)^3 - (0.6712) - 1 = -0.0264$$

x	$y = x^3 + x - 1$
0.6712	-0.0264
1	1

$$\bar{x} = \frac{0.6712(1) - 1(-0.0264)}{1 - (-0.0264)} = \frac{0.6712 + 0.0264}{1.0264} = \frac{0.6976}{1.0264} = 0.6797$$

$$\bar{y} = (0.6797)^3 + (0.6797) - 1 \Rightarrow \bar{y} = 0.0063$$

x	y
0.6797	-0.0063
1	1

$$\bar{x} = \frac{0.6797(1) - 1(-0.0063)}{1 - (-0.0063)} = \frac{0.6797 + 0.0063}{1.0063}$$

$$\bar{y} = \frac{(0.6817)^3 + (0.6817) - 1}{-0.0015} = \underline{\underline{-0.0015}}$$

x	y
0.6817	-0.0015
1	1

$$\bar{x} = \frac{0.6817(1) - 1(-0.0015)}{1 - (-0.0015)} = \frac{0.6817 + 0.0015}{1.0015}$$

$$\bar{y} = \frac{(0.6822)^3 + (0.6822) - 1}{-0.0003}$$

$\therefore$ Root of $x^3 + x - 1 = 0$ is approx 0.6822

② $x^3 - 2x + 0.5 = 0$, lying b/w in 0 and 1

x	$y = x^3 - 2x + 0.5$
0	0.5
1	-0.5

$$\bar{x} = \frac{0(-0.5) - 1(0.5)}{-0.5 - 0.5} = \underline{0.5}$$

$$\bar{y} = (0.5)^3 - 2(0.5) + 0.5 = 0$$

$$= \underline{-0.375}$$

x	$y = x^3 - 2x + 0.5$
0.5	-0.375

x	$y = x^3 - 2x + 0.5$
0.5	0.5
0.5	-0.375

$$\bar{x} = \frac{0(-0.375) - 0.5(0.5)}{-0.375 - 0.5} = \underline{0.2857}$$

$$\bar{y} = (0.2857)^3 - 2(0.2857) + 0.5$$

$$= \underline{-0.04808}$$

x	$y = x^3 - 2x + 0.5$
0	0.5
0.2857	-0.04808

$$\bar{x} = \frac{0(0.04808) - 0.2857(0.5)}{-0.04808 - 0.5} = \underline{0.2606}$$

$$\bar{y} = (0.2606)^3 - 2(0.2606) + 0.5$$

$$= \underline{+0.0035}$$

∴ Root of $x^3 - 2x + 0.5$ is approx 0.2606

(3)

x	$y = x^3 + 2x - 20$
2	0
2.5	0.625

The root of $x^3 + 2x - 20 = 0$ is 2.

(4)

x	$y = 3x - \cos x - 1$
0	-2
1	1.0002

$$\bar{x} = \frac{0(1.0002) - 1(-2)}{1.0002 - (-2)} = \underline{\underline{0.6667}}$$

$$\bar{y} = 3(0.6666) - \cos(0.6666) - 1 = \underline{\underline{-0.00013}}$$

The root of $3x - \cos x - 1$ is approx 0.6666

(5)

x	$y = x^3 - 3x - 5$
2	-3
2.5	3.125

$$\bar{x} = \frac{2(3.125) - 2.5(-3)}{3.125 - (-3)} = \underline{\underline{2.2449}}$$

$$\bar{y} = (2.2449)^3 - 3(2.2449) - 5 = \underline{\underline{-0.4214}}$$

x	$y = x^3 - 3x - 5$
2.2449	-0.4214
2.5	3.125

$$\bar{x} = \frac{2.2449(3.125) - 2.5(-0.4214)}{(3.125) - (-0.4214)} = \underline{\underline{2.2752}}$$

$$\bar{y} = \frac{(2.2752)^3 - 3(2.2752) - 5}{4} = \frac{-0.0053}{-0.0479}$$

(c)

x	y
2.2752	-0.0479
2.5	3.125

$$\bar{x} = \frac{2.2752(3.125) - 2.5(-0.0479)}{3.125 - (-0.0479)} = 2.2786$$

$$\bar{y} = (2.2786)^3 - 3(2.2786) - 5 = -0.0053$$

(d)

x	y
2.2786	-0.0053
2.5	3.125

$$\bar{x} = \frac{2.2786(3.125) - 2.5(-0.0053)}{3.125 - (-0.0053)} = 2.279$$

$$\bar{y} = (2.279)^3 - 3(2.279) - 5 = -0.0002$$

The root of $x^3 - 3x - 5 = 0$ is approx 2.279

(E)

(a)

x	$y = x \log_{10} x - 1.2$
2	-0.5979
3	0.2314

$$\bar{x} = \frac{2(0.2314) - 3(-0.5979)}{0.2314 - (-0.5979)} = 2.72097$$

$$\bar{y} = 2.72097 \log_{10}(2.72097) - 1.2 = -0.01713$$

(b)

x	y
2.72097	-0.01713
3	0.2314

$$\bar{x} = \frac{2.72097(0.2314) - 3(-0.01713)}{0.2314 - (-0.01713)} = \underline{2.7402}$$

$$\bar{y} = 27402 \log_{10}(2.7402) - 1.2 = \underline{0.0004}$$

The root of $x \log_{10} x - 1.2$ is approx 2.7402.

(7)

(a)	x	$y = x^3 - 4x + 1$
	0	1
	1	-2

$$\bar{x} = \frac{0(-2) - 1(1)}{-2 - 1} = \underline{0.3333}$$

$$\bar{y} = (0.333)^3 - 4(0.333) + 1 = \underline{-0.2962}$$

(b)	x	$y = x^3 - 4x + 1$
	0	1
	0.3333	-0.2962

$$\bar{x} = \frac{0(-0.2962) - 0.3333(1)}{-0.2962 - 1} = \underline{0.2572}$$

$$\bar{y} = (0.2572)^3 - 4(0.2572) + 1 = \underline{-0.0118}$$

(c)	x	y
	0	1
	0.2572	-0.0118

$$\bar{x} = \frac{0(-0.0118) - 0.2572(1)}{-0.0118 - 1} = \underline{0.2542}$$

$$\bar{y} = (0.2542)^3 - 4(0.2542) + 1 = \underline{-0.0004}$$

The root of $x^3 - 4x + 1 = 0$ is approx 0.2542