

Lagrange's Interpolation Homework - 1

Q1.

x	7	8	9	10
$f(x)$	3	1	1	9

We need $f(9.5)$

$$f(9.5) = \frac{(9.5-8)(9.5-9)(9.5-10)}{(7-8)(7-9)(7-10)} \times 3$$

$$+ \frac{(9.5-7)(9.5-9)(9.5-10)}{(8-7)(8-9)(8-10)} \times 1$$

$$+ \frac{(9.5-7)(9.5-8)(9.5-10)}{(9-7)(9-8)(9-10)} \times 1$$

$$+ \frac{(9.5-7)(9.5-8)(9.5-9)}{(10-7)(10-8)(10-9)} \times 9$$

$$= \frac{1.5 \times 0.5 \times (-0.5) \times 3}{(-1) \times (-2) \times (-3)} + \frac{2.5 \times 0.5 \times (-0.5)}{2 \times 1 \times (-1)}$$

$$+ \frac{2.5 \times 1.5 \times (-0.5)}{2 \times 1 \times (-1)} + \frac{2.5 \times 1.5 \times 0.5 \times 9}{3 \times 2 \times 1}$$

$$f(9.5) = \frac{3}{16} - \frac{5}{16} + \frac{15}{16} + \frac{45}{16}$$

$$f(9.5) = 3.625$$

Q2.

x	8	3	7	9	10
$f(x)$	168	120	72	63	

We need $f(6)$

$$f(6) = \frac{(6-7)(6-9)(6-10)}{(8-7)(3-7)(3-9)(3-10)} \times 168$$

$$+ \frac{(6-3)(6-9)(6-10)}{(8-5)(7-3)(7-9)(7-10)} \times 120$$

$$+ \frac{(6-3)(6-7)(6-10)}{(8-4)(9-3)(9-7)(9-10)} \times 72$$

$$+ \frac{(6-3)(6-7)(6-9)}{(10-3)(10-7)(10-9)} \times 63$$

$$f(6) = \frac{(-1 \times -3 \times -4)}{(-4 \times -6 \times -7)} \times 168 + \frac{3 \times -3 \times -7}{4 \times -2 \times -3} \times 120$$

$$+ \frac{3 \times (-1) \times (-4)}{6 \times 2 \times (-1)} \times 72 + \frac{3 \times (-1) \times (-3)}{7 \times 3 \times 1} \times 63$$

$$f(6) = 12 + 180 - 72 + 27$$

$$f(6) = 147$$

Q3.

x	1	2	7	8
$f(x)$	4	5	5	4

We need $f(6)$.

$$f(6) = \frac{(6-2)(6-7)(6-8) \times 4}{(1-2)(1-7)(1-8)}$$

$$+ \frac{(6-1)(6-7)(6-8) \times 5}{(2-1)(2-7)(2-8)}$$

$$+ \frac{(6-1)(6-2)(6-8) \times 5}{(7-1)(7-2)(7-8)}$$

$$+ \frac{(6-1)(6-2)(6-7) \times 4}{(8-1)(8-2)(8-7)}$$

$$f(6) = \frac{4 \times -1 \times -2 \times 4}{-1 \times -6 \times -7} + \frac{5 \times -1 \times -2 \times 5}{1 \times -5 \times -6}$$

$$+ \frac{5 \times 4 \times -2 \times 5}{-7 \times 6 \times 1} + \frac{5 \times 4 \times -1 \times 4}{7 \times 6 \times 1}$$

$$= \frac{-16}{-42} + \frac{50}{30} - \frac{70}{42} + \frac{40}{42}$$

$$= \frac{-16}{42} + \frac{50}{42} - \frac{70}{42} + \frac{40}{42}$$

$$= \frac{-16 + 50 - 70 + 40}{42}$$

$$= \frac{4}{42}$$

$$= \frac{2}{21}$$

$$f(6) = 5.667$$

Q4.

x	1	2	3	4	7
$f(x)$	2	4	8	16	128

We need $f(5)$.

$$f(5) = -2(5+2)(5-3)(5-4)(5+7) \times 2$$

$$(1-1)(1+2)(1-3)(1-4)(1-7)$$

$$+ (5-1)(5-3)(5-4)(5-7) \times 4$$

$$(2-1)(2-3)(2-4)(2-7)$$

$$+ (5-1)(5-2)(5-4)(5-7) \times 8$$

$$(3-1)(3-2)(3-4)(3-7)$$

$$+ (5-1)(5-2)(5-3)(5-7) \times 16$$

$$(4-1)(4-2)(4-3)(4-7)$$

$$+ (5-1)(5-2)(5-3)(5-4) \times 128$$

$$(7-1)(7-2)(7-3)(7-4)$$

$$f(5) = \frac{3 \times 2 \times 1 \times -2 \times 2}{-1 \times -2 \times -3 \times -6} + \frac{4 \times 2 \times 1 \times -2 \times 4}{1 \times -1 \times -2 \times -5} + \frac{4 \times 3 \times 1 \times -2 \times 8}{2 \times 1 \times -1 \times -4}$$

$$+ \frac{4 \times 3 \times 2 \times -2 \times 16}{3 \times 2 \times 1 \times -3} + \frac{4 \times 3 \times 2 \times 1 \times 128}{6 \times 5 \times 4 \times 3}$$

$$f(5) = \frac{-2}{3} + \frac{32}{5} + \frac{(-24)}{3} + \frac{128}{3} + \frac{128}{15}$$

$$f(5) = 32.9334$$

Q5.

x	10	20	14	16
$f(x)$	0.25	0.2	0.15	0.10

We need $f(15)$.

$$f(15) = \frac{(15-20)(15-14)(15-16)}{(10-20)(10-14)(10-16)} \times 0.25$$

$$+ \frac{(15-10)(15-20)(15-16)}{(20-10)(20-14)(20-16)} \times 0.2$$

$$+ \frac{(15-10)(15-20)(15-14)}{(14-10)(14-20)(14-16)} \times 0.15$$

$$+ \frac{(15-10)(15-20)(15-14)}{(16-10)(16-20)(16-14)} \times 0.10$$

$$f(15) = \frac{-5 \times 1 \times -1 \times 0.25}{(-10 \times -4 \times -6)} + \frac{5 \times 1 \times -1 \times 0.2}{(10 \times 6 \times 4)}$$

$$+ \frac{5 \times -5 \times -1 \times 0.15}{(4 \times -2 \times 6)} + \frac{5 \times -5 \times 1 \times 0.10}{(8 \times -4 \times 2)}$$

$$= -1 - 1 + 5 + 5$$

$$= 29$$

$$= 29$$

$$f(15) = 0.120833$$

Homework - 2

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Regula Falsi method

Q1.

$$x^3 + x - 1 = 0$$

Function is: $y = x^3 + x - 1$

x	y
0.5	-0.375
0.6	-0.184
0.7	0.043

$$x_1 = 0.6 \quad x_2 = 0.7$$
$$y_1 = -0.184 \quad y_2 = 0.043$$

$$x_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$
$$= 0.681057$$

$$y_n = -0.003042$$

$$\text{Now, } x_1 = 0.681057 \quad x_2 = 0.7$$
$$y_1 = -0.003042 \quad y_2 = 0.043$$

$$x'_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$
$$= 0.682309$$

$$y'_n = -0.000045$$

Thus, root of $x^3 + x - 1 = 0$ is 0.682309

Q2! $x^3 - 2x + 0.5 = 0$ b/w 0 and 1

Function is, $y = x^3 - 2x + 0.5$

x	y
0	0.5
0.1	0.301
0.2	0.108
0.3	-0.073

$$x_1 = 0.2$$

$$x_2 = 0.3$$

$$y_1 = 0.108$$

$$y_2 = -0.073$$

$$x_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 0.259669$$

$$y_n = -0.001829$$

Now,

$$x_1 = 0.25, x_2 = 0.259669$$

$$y_1 = 0.108, y_2 = -0.001829$$

$$x'_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 0.258675$$

$$y'_n = -0.000041$$

Thus, root of $x^3 - 2x + 0.5$ is 0.258675

Q3. $x^3 + 2x - 20 \neq 0$ b/w 2 and 2.5

Function is: $y = x^3 + 2x - 20$

x	y
2.0	-8
2.1	-6.539
2.2	-4.952
2.3	-3.233
2.4	-1.376
2.5	0.625

$x_1 = 2.4$ $x_2 = 2.5$

$y_1 = -1.376$ $y_2 = 0.625$

$x_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$

$= 2.468766$

$y_n = -0.015819$

Now, $x_1 = 2.468766$ $x_2 = 2.5$
 $y_1 = -0.015819$ $y_2 = 0.625$

$x'_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$

$= 2.469537$

$y'_n = -0.000176$

Thus, root of $x^3 + 2x - 20$ is 2.469537

Q4. $3x - \cos x - 1 = 0$, b/w 0 and 1

Function is, $y = 3x - \cos x - 1$

x	y
0.0	-2
0.1	-1.699998
0.2	-1.399994
0.3	-1.099986
0.4	-0.799976
0.5	-0.499962
0.6	-0.199945
0.7	0.100075

$x_1 = 0.6$

$x_2 = 0.7$

$y_1 = -0.199945$

$y_2 = 0.100075$

$$x_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 0.666644$$

$y_n = -0.000000313$

Thus, root of $3x - \cos x - 1$ is 0.666644 .

Q5. $x^3 - 3x - 5$, b/w 2 and 2.5

Function is, $y = x^3 - 3x - 5$

x	y
2.0	-3
2.1	-2.039
2.2	-0.952
2.3	0.267

$$x_1 = 2.2 \quad x_2 = 2.3$$

$$y_1 = -0.952 \quad y_2 = 0.267$$

$$x_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 2.278097$$

$$y_n = -0.011592$$

$$\text{Now, } x_1 = 2.278097 \quad x_2 = 2.3$$

$$y_1 = -0.011592 \quad y_2 = 0.267$$

$$x'_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 2.279008$$

$$y'_n = -0.000136$$

Thus, the root of $x^3 - 3x - 5$ is 2.279008.

Q6. $x \log_{10} x - 1.2 = 0$, b/w 2 & 3.

Function is: $y = x \log_{10} x - 1.2$

x	y
2.0	-0.597940
2.1	-0.523339
2.2	-0.446670
2.3	-0.368026
2.4	-0.287493
2.5	-0.205149
2.6	-0.121069
2.7	-0.035318
2.8	0.052042

$x_1 = 2.7$, $x_2 = 2.8$
 $y_1 = -0.035318$, $y_2 = 0.052042$

$$x_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 2.740428$$

$$y_n = -0.0001902$$

Thus, root of $x \log_{10} x - 1.2$ is 2.740428.

Q7. $x^3 - 4x + 1 = 0$

Function is: $y = x^3 - 4x + 1$

x	y
0	1
0.1	0.601
0.2	0.208
0.3	-0.173

$$x_1 = 0.2, \quad x_2 = 0.3$$

$$y_1 = 0.208, \quad y_2 = -0.173$$

$$x_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 0.254593$$

$$y_n = -0.001869$$

Now, $x_1 = 0.2, \quad x_2 = 0.254593$

$$y_1 = 0.208, \quad y_2 = -0.001869$$

$$x'_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 0.254107$$

$$y'_n = -0.00002022$$

Thus, root of $x^3 - 4x + 1$ is 0.254107.

Q8. $x^3 + 2x^2 - 8 = 0$

Function is, $y = x^3 + 2x^2 - 8$

x	y
0	-8
1	-5
2	8

$x_1 = 1$, $x_2 = 2$

$y_1 = -5$, $y_2 = 8$

$$x_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 1.384615$$

$$y_n = -1.51156$$

Now, $x_1 = 1.384615$, $x_2 = 2$

$y_1 = -1.51156$, $y_2 = 8$

$$x'_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 1.482389$$

$$y'_n = -0.347529$$

Now, $x_1 = 1.482389$, $x_2 = 2$

$y_1 = -0.347529$, $y_2 = 8$

$$x''_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 1.503938$$

$$y''_n = -0.074689$$

$$\text{Now, } x_1 = 1.503938, \quad x_2 = 2$$

$$y_1 = -0.074689, \quad y_2 = 8$$

$$x'''_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 1.508526$$

$$y'''_n = -0.0158204$$

$$\text{Now, } x_1 = 1.508526, \quad x_2 = 2$$

$$y_1 = -0.0158204, \quad y_2 = 8$$

$$x''''_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 1.509496$$

$$y''''_n = -0.003339$$

$$\text{Now, } x_1 = 1.509496, \quad x_2 = 2$$

$$y_1 = -0.003339, \quad y_2 = 8$$

$$x_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 1.50970064$$

$$y_n = -0.000704$$

Thus, root of $x^3 + 2x^2 - 8$ is 1.50970064 .

Q9. $x^3 - 9x + 1 = 0$

Function is, $y = x^3 - 9x + 1$

x	y
0	1
0.1	0.101
0.2	-0.792

$$x_1 = 0.1, \quad x_2 = 0.2$$

$$y_1 = 0.101, \quad y_2 = -0.792$$

$$x_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 0.1113102$$

$$y_n = -0.000413$$

Thus, root of $x^3 - 9x + 1$ is 0.1113102

Q10. $x^3 - x - 4 = 0$

Function is: $y = x^3 - x - 4$

x	y
0	-4
1	-4
2	2

$$\begin{aligned} x_1 &= 1 & x_2 &= 2 \\ y_1 &= -4 & y_2 &= 2 \end{aligned}$$

$$x_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 1.333334$$

$$y_n = -2.96296$$

$$\begin{aligned} \text{Now, } x_1 &= 1.333334 & x_2 &= 2 \\ y_1 &= -2.96296 & y_2 &= 2 \end{aligned}$$

$$x'_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 1.7313434$$

$$y'_n = -0.541555$$

$$\begin{aligned} \text{Now, } x_1 &= 1.7313434 & x_2 &= 2 \\ y_1 &= -0.541555 & y_2 &= 2 \end{aligned}$$

$$x''_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 1.788589$$

$$y''_n = -0.066802$$

$$\text{Now, } x_1 = 1.788589 \quad x_2 = 2$$

$$y_1 = -0.066802 \quad y_2 = 2$$

$$x'''_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 1.795422$$

$$y'''_n = -0.007807$$

$$\text{Now, } x_1 = 1.795422 \quad x_2 = 2$$

$$y_1 = -0.007807 \quad y_2 = 2$$

$$x_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= 1.796217$$

$$y_n = -0.0009105$$

Thus, root of $x^3 - x - 4$ is 1.796217.