Assignment - 2

Unit 3 and 4

Ques 1 Solve using substitution:

$$\int_{1/30}^{2} \frac{e^{2/\omega}}{\omega^2}\, d\omega.$$

Sol^n Let $e^{2/\omega} = t$

$\frac{2}{\omega} e^{2/\omega} = \frac{dt}{d\omega}$ $\qquad$ $\frac{1}{\omega} d\omega = \frac{dt}{2e^{2/\omega}}$

$\int \frac{1}{\omega} \times \frac{1}{2e^{2/\omega}}\, dt$ $\qquad$ $e^{2/\omega} = t$

$\frac{2}{\omega} = \log t$

$\boxed{\omega = \frac{2}{\log t}}$

$= \int \frac{\log t}{2} \times \frac{1}{2e^{\cancel{2}/\cancel{2}\log t}}\, dt$

$= \frac{1}{4}\int \frac{\log t}{\log t}\, dt$

Let $\log t = v$; $\frac{1}{t} dt = dv$

$= \frac{1}{4}\int v\,dv = \frac{1}{4} \times \frac{v^2}{2} + c$

$= \frac{1}{\cancel{2}4}\frac{(\log t)^2}{2} + c$

$= \frac{1}{\cancel{8}} \times \frac{4}{\omega^2} + c$ $\qquad$ $\boxed{\text{Ans} = \frac{1}{2\omega^2} + c}$

$$= \left[\frac{1}{2w^2}\right]_{1/30}$$

$$= \frac{1}{2}\left[\frac{1}{4} - \frac{1}{(1/30)^2}\right]$$

$$= \frac{1}{2}\left[\frac{1}{4} - \frac{1}{1/900}\right]$$

3600
3599

$$= \frac{1}{2}\left[\frac{1}{4} - 900\right]$$

$$= \frac{1}{2}\left[\frac{-3599}{4}\right] = -\frac{3599}{8}$$

Ques2 Solve using substitution:

$$\int \frac{2t^3+1}{(t^4+2t)^3}\,dt$$

Let $t^4+2t = x$

$4t^3+2 = \frac{dx}{dt}$; $2(2t^3+1) = \frac{dx}{dt}$

$(2t^3+1)\,dt = \frac{1}{2}\,dx$

$$= \frac{1}{2}\int \frac{1}{x^3}\,dx = \frac{1}{2}\int x^{-3}\,dx$$

$$= \frac{1}{2}\left(\frac{x^{-2}}{-2}\right) + c$$

$$\boxed{\text{Ans} = -\frac{1}{4x^2} + c}$$

$$\boxed{= \frac{-1}{4(t^4+2t)^2} + c}$$

Ques 3 If $f'(x) = x^4 + 3x - 9$, what was $f(x)$

Integrating both sides.

$$\int f'(x)\,dx = \int (x^4 + 3x - 9)\,dx.$$

$$f(x) = \int x^4\,dx + 3\int x\,dx - 9\int dx$$

$$= \frac{x^5}{5} + \frac{3x^2}{2} - 9x + C$$

$$\boxed{f(x) = \frac{x^5}{5} + \frac{3}{2}x^2 - 9x + C}$$

Ques 4 Given

$$f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \le 1 \end{cases}$$

Evaluate $\int_{-2}^{3} f(x)\,dx.$

Ans: $$\int_{-2}^{3} f(x)\,dx = \int_{-2}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx$$

$$= \int_{-2}^{1} 3x^2\,dx + \int_{1}^{3} 6\,dx$$

$$= \cancel{3}\left[\frac{x^3}{\cancel{3}}\right]_{-2}^{1} + 6[x]_1^3 \ \cancel{+C}$$

$$= 1 - (-8) + 6(3-1)$$

$$= 1 + 8 + 6(2)$$

$$= 9 + 12$$

$$\boxed{\text{Ans} = 21}$$

Ques 5 Solve.

$$\int 3(8y-1)\, e^{4y^2-y}\, dy$$

Let $4y^2 - y = t$

$(8y-1)\, dy = dt$

$$= 3\int e^t\, dt = 3e^t + C$$

$$\boxed{\text{Ans} = 3e^{4y^2-y} + C}$$

Ques 6 find $f(x)$ when.

$f''(x) = 15\sqrt{x} + 5x^3 + 6$

$f'(1) = -5/4$. $f(4) = 404$.

$$\int f''(x)\, dx = \int (15\sqrt{x} + 5x^3 + 6)\, dx$$

$$f'(x) = 15\int x^{1/2}\, dx + 5\int x^3\, dx + 6\int dx$$

$$= \frac{\cancel{15}^{5} \times 2x^{3/2}}{\cancel{3}} + \frac{5x^4}{4} + 6x$$

$$f'(x) = 10x^{3/2} + \frac{5}{4}x^4 + 6x + C$$

$$f(1) = 10(1)^{3/2} + \frac{5}{4}(1)^4 + 6(1) + C = -\frac{5}{4}$$

$$= 10 + \frac{5}{4} + 6 + C = -5/4$$

$\frac{32}{2}$

$$\Rightarrow 16 + \frac{5}{2} = -C$$

$$\boxed{C = -\frac{37}{2}}$$

$f'(x) = 10x^{3/2} + \frac{5}{4}x^4 + 6x - \frac{37}{2}$

$\int f'(x)\,dx = 10\int x^{3/2}dx + \frac{5}{4}\int x^4 du + 6\int x\,du - \frac{37}{2}\int du.$

$f(x) = \frac{\cancel{10}^{2} \times 2x^{5/2}}{\cancel{5}1} + \frac{\cancel{5}}{4} \times \frac{x^5}{\cancel{5}} + \frac{\cancel{6}^{3}x^2}{\cancel{2}} - \frac{37}{2}x + c$

$$\boxed{F(x) = 4x^{5/2} + \frac{1}{4}x^5 + 3x^2 - \frac{37}{2}x + C}$$

Ques 7 formulate the PDE from $f(x+iy) + g(x-iy)$

$\cancel{\phi(x+iy) = g(x-iy)}$

Soln let $z = f(x+ay) + \phi(x-ay)$.

diff w.r.t x and y.

$\frac{dz}{dx} = f'(x+ay) + \phi'(x-ay)$ —— ①

$\frac{dz}{dy} = f'(x+ay)\,a + \phi(x-ay)\,-a$. —— ②

diff ① w.r.t x.

$\frac{d^2z}{dx^2} = f''(x+ay) + \phi''(x-ay)$

diff ② w.r.t y

$\frac{d^2z}{dy^2} = f''(x+ay)a^2 + \phi''(x-ay)a^2$

$= a^2\left(f''(x+ay) + \phi''(x-ay)\right)$

$\frac{d^2\cancel{z}}{dy^2} = a^2 \frac{d^2\cancel{z}}{dx^2}$

$\therefore \left(\frac{dy}{dx}\right)^2 = \frac{1}{a^2} \Rightarrow \boxed{\frac{dy}{dx} = \frac{1}{a}}$

Ques 8 Solve the following.

$$\frac{dr}{d\theta} = \frac{r^2}{\theta} \; ; \; r(1) = 2$$

$$\int \frac{1}{r^2} dr = \int \frac{1}{\theta} d\theta$$

doubt

$$\frac{r^{-1}}{-1} = \log\theta + c$$

$$-\frac{1}{r} = \log\theta + C$$

$$r = -\frac{1}{\log\theta + c} + C$$

Ques 9 find the solution of the following differential equation.

$$\frac{dv}{dt} = 9.8 - 0.196v \; ; \; \frac{dv}{dt} + 0.196v = 9.8.$$

Soln

$$P = +0.196 \qquad Q = 9.8$$

$$I.F = e^{+\int P dt}$$

$$= e^{+\int 0.196 dt}$$

$$= e^{+0.196t}$$

$$v e^{0.196t} = 9.8 \int e^{0.196t} dt$$

$$v \cancel{e^{0.196t}} = 9.8 \frac{\cancel{e^{0.196t}}}{0.196} + c$$

$$\boxed{V = 50}$$

Ques 10 find the solution to the following initial Value Problem (IVP):-

$$ty' + 2y = t^2 - t + 1$$

Soln

$$t\left(\frac{dy}{dt}\right) + 2y = t^2 - t + 1.$$

$$\frac{dy}{dt} + \left(\frac{2}{t}\right)y = t - 1 + \frac{1}{t}.$$

$$\boxed{P = \frac{2}{t}} \quad \text{or} \quad \boxed{Q = t - 1 + 1/t}$$

$$If = e^{\int P dt} = e^{\int 2/t \, dt} = e^{2\log t}$$

$$y \, e^{2\log t} = \int \left(t - 1 + \frac{1}{t}\right) e^{2\log t} \, dt.$$

$$=$$

Ques 12 find the Taylor Series for $f(x) = x^3 - 10x^2 + 6$ for $x = 3$.

Taylor series. at $x = a + h$

$$f(a+h) = f(a) + hf'(a) + \frac{h^2 f''(a)}{2!} + \frac{h^3 f'''(a)}{3!} + \cdots$$

$f(a) = a^3 - 10a^2 + 6 \rightarrow f(3) = 3^3 - 90 + 6 = -57$

$f'(a) = 3a^2 - 20a \rightarrow f'(3) = 3(3)^2 - 20(3) = -33$

$f''(a) = 6a - 20 \quad f''(3) = 6(3) - 20 = -2$

$f'''(a) = 6. \quad f'''(3) = 6.$

$$f(3+h) = f(3) + (x-3)f'(3) + \frac{(x-3)^2 f''(3)}{2!} + \frac{(x-3)^3 f'''(3)}{3!}$$

$$= -57 + (x-3) - 33 + \frac{(x-3)^2 - 2}{2!} + \frac{(x-3)^3 6}{3!}$$

$$= -57 - 33(x-3) - (x-3)^2 + (x-3)^3.$$

Taylor series

Expansion $\rightarrow$

$$\boxed{f(3+h) = -57 - 33(x-3) - (x-3)^2 + (x-3)^3}$$

Qus13 find the maclurian series for $(1+x)^4$.

Sol Maclaurin series:

$$f(a+h) = f(0) + hf'(0) + \frac{h^2 f''(0)}{2!} + \frac{h^3 f'''(0)}{3!} \ldots$$

$f(a) = (1+a)^4$

$f'(a) = 4(1+a)^{4-1}$

$f''(a) = (4)(4-1)(1+a)^{4-2}$

$\cancel{f'''(a) = (4)(4-1)(4-2)(1+a)^{4-3}}$

$f(0) = (1)^4 = 1$

$f'(0) = 4(1)^{4-1} = 4$

$f''(0) = (4)(4-1)(1)^{4-2} = 6$

$(4)(4-1)$

$\cancel{f'''(0) = (4)(4-1)(4-2)}$

$$f((1+x)^4) = 1 + [x-(1+x)^4]1 + \frac{[x-(1+x)^4]^2\, 4}{2} + \frac{[x-(1+x)^4]^3 \ldots}{3!}$$

$$= 1 + [x-(1+x)^4]\left(1 + \frac{(x-(1+x)^4)4}{2} + \frac{(x-(1+x)^4)^2 (4)(4-1)}{6}\right.$$

$$\cancel{\text{Ans } 1 + [x-(1+x)^4]\left[1 + \frac{[x-(1+x)^4]4}{2} + \frac{[x-(1+x)^4]^2 (4)(4-1)}{6}\right]}$$

$$\boxed{\text{Ans } 1 + (1+x)^4\left[1 + \frac{(1+x)^4\, 4}{2} + \frac{(1+x)^4]^2\, 4(4-1)}{6}\right.}$$

Ques 14: determine the maclaurin series for $f(x) = \sqrt{1+x}$.

$f(a) = \sqrt{1+x}$ $\quad f(0) = \sqrt{1} = 1$

$f'(a) = \dfrac{1}{2\sqrt{1+x}}$ $\quad f'(0) = \dfrac{1}{2\sqrt{1+0}} = \dfrac{1}{2}$

$f''(a) = \dfrac{-(1)(\cancel{2}) \times \frac{1}{\cancel{2}\sqrt{1+x}}}{4(1+x)}$ $\quad f''(0) = -\dfrac{1}{4}.$

$= -\dfrac{1}{4(1+x)^{3/2}}$

Ans $1 + [\cancel{x} - \sqrt{1+x}]\dfrac{1}{2} + \left(-\dfrac{1}{4}\right)(\cancel{x} - \sqrt{1+x})^2$

$= 1 + [\cancel{x} - \sqrt{1+x}]\dfrac{1}{2} - \dfrac{1}{4}(\cancel{x}(\sqrt{1+x})^2$

$\boxed{\text{Ans} = 1 + \dfrac{(x - \sqrt{1+x})}{2}\left\{1 + \dfrac{1}{2}(x - \sqrt{1+x})\right\}}$

$\boxed{= 1 + \dfrac{\sqrt{1+4}}{2} - \dfrac{(x+1)}{4}}$

Qu15 Find the Taylor series for $f(x) = e^{-6x}$ about $x = -4$.

Sol

$f(a) = e^{-6a}$

$f'(a) = -6e^{-6a}$

$f''(a) = 36e^{-6a}$

$f'''(a) = -216e^{-6a}$

$f(-4) = e^{24}$

$f'(-4) = -6e^{24}$

$f''(-4) = 36e^{24}$

$f'''(-4) = -216e^{24}$

~~-4+h=x~~

~~h=x+4~~

$-4 + h = x$

$h = x + 4$

Taylor series

$$\Rightarrow e^{24} + \frac{(x+4) \times -6e^{24}}{1!} + \frac{(x+4)^2\, 36e^{24}}{2} + \frac{(x+4)^3\,(-216)e^{24}}{6}$$

$$\boxed{e^{24}\left[\frac{1}{6} - (x+4) + 3(x+4)^2 + 6(x+4)^3\right]}$$

Qu16: Find the taylor series for $f(x) = \ln(3+4x)$ about $x = 0$.

$f(a) = \ln(3+4x)$

$f'(a) = \frac{1}{3+4x} \times 4 = \frac{4}{3+4x}$

$f''(a) = \frac{-4(4)}{(3+4x)^2} = \frac{-16}{(3+4x)^2}$

$f(0) = \ln 3$

$f'(0) = \frac{4}{3}$

$f''(0) = \frac{-16}{9}$

$0 + h = 4$

$\boxed{h = 4}$

$\Rightarrow \ln 3 + \frac{\cancel{x} \times 4}{3} + \frac{x^2 \times -16^{8}}{\cancel{2}!\,9}$

$$\boxed{= \ln 3 + \frac{4}{3} x \left(1 - \frac{2x}{3} \right)}$$