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Financial Mathematics. Assignment - 1

Q.1] The nominal rate of discount per annum convertible quaterly is 8%.

i.e $d^{(4)} = 8\%$ per quarter

i) Calculate equivalent force of interest.

i.e $\delta = ?$

$$\therefore e^{\delta} = \frac{1}{\left(1 - \frac{d^{(4)}}{4}\right)^{4}}$$

∴ Taking ln on both sides

$$\delta = \ln\left(1 - \frac{d^{(4)}}{4}\right)^{-4}$$

$$= \ln\left(1 - \frac{8\%}{4}\right)^{-4}$$

$$\therefore \delta = 8.08108\%$$

ii) Calculate the equivalent effective rate of interest per annum.

$\therefore i = ?$

$$(1+i) = \left(1 - \frac{d^{(4)}}{4}\right)^{-4}$$

$$\therefore i = \left(1 - \frac{d^{(4)}}{4}\right)^{-4} - 1$$

$$\therefore i = \left(1 - \frac{8\%}{4}\right)^{-4} - 1$$

$$= 1.0841657 - 1$$

$$= 0.841657$$

$$\therefore i = 8.41657\%$$

iii) Calculate the equivalent nominal rate of ~~interest~~ discount per annum convertible monthly i.e $d^{(12)} = ?$

$$\therefore \left(1 - \frac{d^{(12)}}{12}\right)^{12} = \left(1 - \frac{d^{(4)}}{4}\right)^{4}$$

$$\therefore \left(1 - \frac{d^{(12)}}{12}\right) = \left(1 - \frac{d^{(4)}}{4}\right)^{4/12}$$

$$\therefore \left(1 - \frac{d^{(12)}}{12}\right) = \left(1 - \frac{d^{(4)}}{4}\right)^{1/3}$$

$$\therefore d^{(12)} = 12\left[1 - \left(1 - \frac{d^{(4)}}{4}\right)^{1/3}\right]$$

$$\boxed{d^{(12)} = 8.0539339\%}$$

Q.2] $\delta(t) = 0.004t + 0.0002t^2$ for all t.

i) Find present value of sum of rupees 1000 payable at the end of 10 years from now. i.e $PV_0 = ?$

Amount due $= C = 1000$ $\quad n = 10$

$$PV_0 = C \cdot v(0,10) - \int_0^{10} 0.004t + 0.0002t^2 \cdot dt$$

$$= 1000 \cdot e^{-\left[\frac{0.004t^2}{2} + \frac{0.0002t^3}{3}\right]_0^{10}}$$

$$= 1000 \cdot e^{-\left[\left(0.002 \times (10)^2 + \frac{0.0002 \times 1000}{3}\right) - 0\right]}$$

$$= 1000 \cdot e^{\left[-0.2 - \frac{0.2}{3}\right]}$$

$$= 1000 \cdot e^{(-0.8/3)} = 1000 \cdot e$$

$$= ₹\, 765.928$$

$$\boxed{\therefore PV_0 = ₹\, 765.928}$$

ii) Find constant annual effective rate of interest over a 10 year period.

$$\therefore (1+i)^{10} = e^{\int_0^{10} \delta(t)\cdot dt}$$

$$(1+i)^{10} = e^{\int_0^{10} 0.004t + 0.0002t^2\, dt}$$

$$= e^{\left[\frac{0.004t^2}{2} + \frac{0.0002t^3}{3}\right]_0^{10}}$$

$$= e^{\left[\left(0.002\times 100 + \frac{0.0002\times 1000}{3}\right) - 0\right]}$$

$$= e^{\left[0.2 + \frac{0.2}{3}\right]}$$

$$(1+i)^{10} = e^{(0.8/3)}$$

$$(1+i)^{10} = 1.305605172$$

$$\therefore (1+i) = (1.305605172)^{1/10}$$

$$(1+i) = 1.027025404$$

$$\therefore i = 1.027025404 - 1$$

$$\boxed{\therefore i = 2.7025404\%}$$

Q.3] i) $d = iv$

By equivalent rates of interest,

$$d = \frac{i}{1+i}$$

Now $1/(1+i) = v$ $\therefore$ The given equation can be written as $d = iv$
where d is the annual effective rate of discount and i is the annual effective rate of interest
Moreover d is the interest paid at the start of the year and i is the interest paid at the end of the year. Therefore if we discount 'i' for 1 year with the help of discounting factor 'v' we get 'd'.

ii) Calculate nominal rate of discount per annum convertible half yearly

i.e $d^{(2)} = ?$

equivalent to

a) An effective rate of discount of 2.25% per quarter. i.e $\frac{d^{(4)}}{4} = 2.25\%$

$$\therefore \left(1 - \frac{d^{(4)}}{4}\right)^4 = \left(1 - \frac{d^{(2)}}{2}\right)^2$$

$$\therefore (1 - 2.25\%)^4 = \left(1 - \frac{d^{(2)}}{2}\right)^2$$

$$\therefore (1 - 2.25\%)^{4/2} = \left(1 - \frac{d^{(2)}}{2}\right)$$

$$\therefore d^{(2)} = 2\left[1 - (1 - 2.25\%)^2\right]$$

$$\therefore d^{(2)} = \cancel{9.875\%}\ 8.89875\%$$

b) A nominal rate of discount of 5% convertible every 2 years

$d^{(1/2)} = 5\%$

$$\left(1 - \frac{d^{(1/2)}}{1/2}\right)^{1/2} = \left(1 - \frac{d^{(2)}}{2}\right)^2$$

$$\therefore \left(1 - \frac{d^{(1/2)}}{1/2}\right)^{1/4} = \left(1 - \frac{d^{(2)}}{2}\right)$$ — Taking sq. root on both sides

$$\therefore d^{(2)} = 2\left[1 - \left(1 - \frac{d^{(1/2)}}{1/2}\right)^{1/4}\right]$$

$\therefore$

$$\therefore d^{(2)} = 2\left[1 - \left(1 - \frac{5\%}{1/2}\right)^{1/4}\right]$$

$$= \cancel{5}\ 0.0519925$$

$$\therefore d^{(2)} = 5.19925\%.$$

iii) A 91-day government bill provides the investor with an annual effective rate of return of 5%. Calculate simple discount rate at which the bill is discounted.

i.e $d(si)$ - ?

Let the 91-day goverment bill be of ₹ 100 & the discounted amount be ₹ x.

As the annual effective rate of return is 5%,

$$i \quad \therefore \frac{100 - x}{100} = 5\%.$$

$$\therefore \frac{100-x}{100} = 0.05$$

$$\therefore 100 - x = 5$$

$$\therefore x = 100 - 5 = ₹95$$

$$\therefore x = ₹95 //$$

$$\therefore PV = C \cdot (1 - nd)$$

$$\therefore 95 = 100\left(1 - \frac{91}{365}d\right)$$

$$\therefore \frac{95}{100} = \left(1 - \frac{91}{365}d\right)$$

$$\therefore \frac{91}{365}d = 1 - \frac{95}{100}$$

$$d = \left(\frac{100-95}{100}\right) \times \frac{365}{91}$$

$$= 0.200549451$$

$$\boxed{\therefore d = 20.0549451\%}$$

Q.4] i) $d = iv$

By equivalent rates of interest,

$$d = \frac{i}{1+i}$$

$$= i \times \frac{1}{1+i}$$

$$= i \times v$$

$$\therefore d = iv$$

d is the annual effective rate of discount which is interest paid at the start of the year and i is the annual effective rate of interest paid at the end of the year.
∴ If 'i' is discounted with the help of v i.e discounting factor we get d i.e interest paid at the start of the year.

ii) $i = 0.08$

Calculate

a) $d^{(12)}$

$$\therefore \left(1 - \frac{d^{(12)}}{12}\right)^{12} = (1+i)^{-1}$$

$$\therefore \left(1 - \frac{d^{(12)}}{12}\right) = (1+i)^{-1/12}$$

$$\therefore d^{(12)} = 12\left[1 - (1+i)^{-1/12}\right] = 12\left[1 - (1+0.08)^{-1/12}\right]$$

$$d^{(12)} = 0.07671477$$

b) $\left(1+\frac{i^{(365)}}{365}\right)^{365} = (1+i)$

$\therefore \left[1+\frac{i^{(365)}}{365}\right] = (1+i)^{1/365}$

$\therefore i^{(365)} = 365\left[(1+i)^{1/365} - 1\right]$

$= 365\left[(1+0.08)^{1/365} - 1\right]$

$i^{(365)} = 0.76969155$

c) $\delta = ?$

$\delta = \ln(1+i)$

$= \ln(1+0.08)$

$\delta = 0.076961041$

d) $i^{(0.5)} = ?$

$\therefore \left[1+\frac{i^{(0.5)}}{0.5}\right]^{0.5} = (1+i)$

$\therefore \left[1+\frac{i^{(0.5)}}{0.5}\right] = (1+i)^{1/0.5}$

$\therefore i^{(0.5)} = 0.5\left[(1+i)^{1/0.5} - 1\right]$

$= 0.5\left[(1+0.08)^{1/0.5} - 1\right]$

$i^{(0.5)} = 0.0832$

Q.5] i) The rate of discount per annum convertible quaterly is 6%.

$\therefore d^{(4)} = 6\%$.

a) $i^{(2)} = ?$

$$\left[1+\frac{i^{(2)}}{2}\right]^{2} = \left[1-\frac{d^{(4)}}{4}\right]^{-4}$$

$$\therefore \left[1+\frac{i^{(2)}}{2}\right] = \left[1-\frac{d^{(4)}}{4}\right]^{-4/2}$$

$$\therefore \quad i^{(2)} = 2\times\left\{\left[1-\frac{d^{(4)}}{4}\right]^{-2} - 1\right\}$$

$$= 2\left[\left(1-\frac{0.06}{4}\right)^{-2} - 1\right]$$

$$= 0.061377516$$

$$\therefore i^{(2)} = 6.1377516\%$$

b) $d^{(12)} = ?$

$$\left(1-\frac{d^{(12)}}{12}\right)^{12} = \left(1-\frac{d^{(4)}}{4}\right)^{4}$$

$$\therefore \left(1-\frac{d^{(12)}}{12}\right) = \left(1-\frac{d^{(4)}}{4}\right)^{4/12}$$

$$\therefore \quad d^{(12)} = 12\left[1-\left(1-\frac{d^{(4)}}{4}\right)^{1/3}\right]$$

$$= 12\left[1-\left(1-\frac{0.06}{4}\right)^{1/3}\right]$$

$$= 0.060302525$$

$$\therefore d^{(12)} = 6.0302525\%$$

ii) Amount borrowed on 15th April, 2005 = ₹ 2,00,000.

Amount payable after one year i.e on 15th April, 2006 = ₹ 2,20,000.

To find:-

Amount payed for early repayment of loan on 17^{th} July, 2005 i.e after 3 months = ?

$$2,20,000 = 2,00,000 \times A(0,1)$$

$$\therefore \quad A(0,1) = \frac{220,000}{2,00,000} = 1.1$$

$\therefore$ Amount payed on 17^{th} July, 2005

$$\text{i.e } AV(3/12) = C \cdot A(3/12)$$
$$= 2,00,000\,(1+0.1)^{3/12}$$
$$= 2,00,000\,(1.1)^{3/12}$$
$$= ₹\ 204,822.73$$

$\therefore$ Amit paid ₹ 204,822.73 on 17^{th} July, 2005 to terminate the contract.

Q.6] i) Let us assume the manufacturer makes a sale of ₹ 100.

then a) Cash payment - 30% below the retail price.

$\therefore$ ~~PV = ₹ 100 (1-d)~~   ~~Sale~~ Selling price $= 100 - (100 \times 30\%)$

~~= 100 (1 - 0.3)~~ $= 100 - 30$

~~= ₹70~~ $= ₹70$

b) Six month's credit - 25% below retail price

$\therefore$ ~~PV = 100 (1-d)^{1/2}~~   Selling price $= 100 - (100 \times 25\%)$

$= 100(1-0.25)^{1/2}$ $= 100 - 25$

$= ₹75$

~~The retailer gets the same effective rate of discount from both terms.~~

~~$\therefore 70 = 75$~~

If ₹75 is discounted for six month it give the PV at time 0

$$\therefore\ 70 = 75(1-d)^{1/2}$$

$$\therefore\ \frac{70}{75} = (1-d)^{1/2}$$

$$\therefore\ \left(\frac{70}{75}\right)^2 = (1-d)$$

$$\therefore\ d = \left[1-\left(\frac{70}{75}\right)^2\right]$$

$$\therefore\ d = 0.128889$$

$$\therefore\ d = 12.8889\%.$$

ii) a) $d^{(12)} = ?$

$$(1-d) = \left(1-\frac{d^{(12)}}{12}\right)^{12}$$

$$\therefore\ (1-d)^{1/12} = \left[1-\frac{d^{(12)}}{12}\right]$$

$$\therefore\ d^{(12)} = 12\left[1-(1-d)^{1/12}\right]$$

$$= 12\left[1-(1-\cancel{d}\,0.128889)^{1/12}\right]$$

$$= 0.137195565$$

$$\therefore\ d^{(12)} = 13.7195565\%.$$

ii) b) $i^{(2)} =$

$$\left[1+\frac{i^{(2)}}{2}\right]^2 = (1-d)^{-1}$$

$$\therefore\ \left[1+\frac{i^{(2)}}{2}\right] = (1-d)^{-1/2}$$

$$\therefore\ i^{(2)} = 2\left[(1-d)^{-1/2}-1\right].$$

$$= 2\left[(1-0.128889)^{-1/2}-1\right]$$

$\therefore i^{(2)} = 0.1428572$

$\therefore i^{(12)} = 14.28572\%$

c) $\delta = ?$

$\delta = \ln (1-d)^{-1}$

$= \ln (1 - 0.128889)^{-1}$

$= 0.1379858$

$\therefore \delta = 13.79858\%$

Q.7] An investment fund offers to grow from ₹20,000 to ₹26,000 in 17 months.

$\therefore \quad AV = C \cdot A(t)$

$\therefore \quad 26{,}000 = 20{,}000 \; A(0,17)$

$\therefore \quad \dfrac{26000}{20{,}000} = A(0,17)$

$\therefore A(0,17) = 1.3$

$\therefore \left[1 + \dfrac{i^{(17)}}{17} \right]^{17} = 1.3$

$\therefore \quad (1+i) = \left(1 + \dfrac{i^{(17)}}{17} \right)^{17} = 1.3$

$\therefore (1+i) = 1.3$

$\therefore i = 1.3 - 1$

$= 0.3 = 30\%$

$A(0,17)$ – Accumulating factor for 17 months.

$\therefore \quad (1+i)^{17/12} = A(0,17) = 1.3$

$\therefore \quad (1+i) = (1.3)^{12/17}$

$\therefore i = (1.3)^{12/17} - 1$

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$\therefore i = 0.20345706$

$\therefore i = 20.345706\%$

i) nominal rate of interest per annum convertible quarterly.

$i^{(4)} = ?$

$\therefore \left(1 + \frac{i^{(4)}}{4}\right)^4 = (1+i)$

$\therefore \left(1 + \frac{i^{(4)}}{4}\right) = (1+i)^{1/4}$

$\therefore i^{(4)} = 4\left[(1+i)^{1/4} - 1\right] = 4\left[(1+0.20345706)^{1/4} - 1\right]$

$= 0.1895525$

$\boxed{\therefore i^{(4)} = 18.95525\%}$

ii) The nominal rate of discount per annum convertible half yearly.

$\therefore d^{(2)} = ?$

$\therefore \left(1 - \frac{d^{(2)}}{2}\right)^2 = (1+i)^{-1}$

$\therefore \left[1 - \frac{d^{(2)}}{2}\right] = (1+i)^{-1/2}$

$\therefore d^{(2)} = 2\left[1 - (1+i)^{-1/2}\right]$

$= 2\left[1 - (1+0.20345706)^{-1/2}\right]$

$= 0.176882348$

$\boxed{\therefore d^{(2)} = 17.6882348\%}$

iii) Simple rate of interest per annum.

$26000 = 20000\left(1 + \frac{17}{12} \times i\right)$

$\therefore \frac{26000}{20,000} = \left(1 + \frac{17}{12} i\right)$

$$\therefore \left(\frac{26000}{20,000} - 1\right) = \frac{17}{12} i$$

$$\therefore i = \frac{12}{17}\left(\frac{26000 - 20,000}{20,000}\right)$$

$$= 0.211764706$$

$$\therefore i = 21.1764706\%$$

iv) Comment on the results.

We can see that the effective annual interest per annum is 20.345706%. and the simple interest per annum is 21.1764706%.

From this we can infer that in simple interest a higher interest rate is required to accumulate the same amount as effective annual rate of interest (compound interest) for the same period.

Moreover,

$$i > i^{(4)} > d^{(2)}.$$

~~This~~

Q.8] Explain with the help of graph, for an effective interest rate of 8% p.a, the relationship between equivalent nominal rates of interest convertible p^{thly} ($i^{(p)}$) and p.

$$\therefore i = 8\% \text{ p.a} = 0.08$$

$$\therefore (1+i) = \left(1+\frac{i^{(p)}}{p}\right)^p$$

$$\therefore (1+i)^{1/p} = \left(1+\frac{i^{(p)}}{p}\right)$$

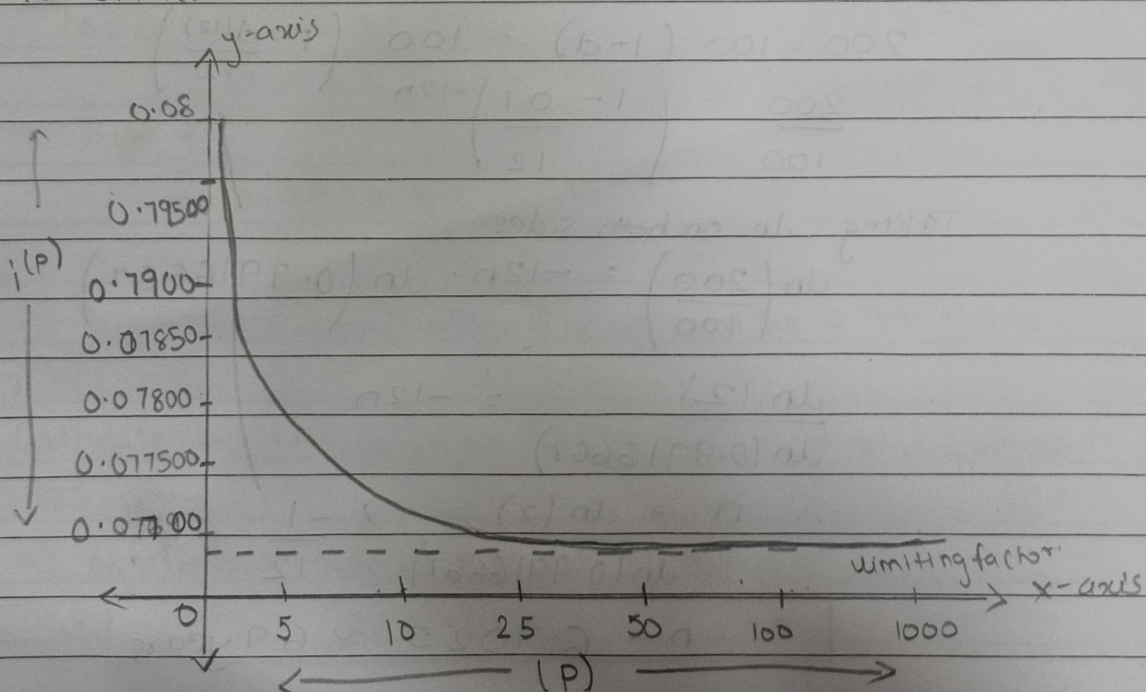
$$\therefore i^{(p)} = p\left[(1+i)^{1/p} - 1\right]$$

By substituting different p values the following $i^{(p)}$ values are obtained.

$\therefore$

p	$i^{(p)} = p\left[(1+i)^{1/p} - 1\right]$
1	0.08
2	0.078461
3	~~0.077906~~ 0.077957
4	0.077706
5	0.077556
10	0.077258
25	0.077079
50	0.0770203
75	0.0769907
100	0.0769907
365	0.0769692
1000	0.0769640
10000	0.0769613

Plotting the above points on the ~~gra~~ x-axis and y-axis the following graph is obtained.



The following inferences can be made from the above graph.

1) We can see that as the value of p increase the nominal interest rate i.e $i^{(p)}$ which is paid p^{th}ly decreases.

2) But after a certain value of p i.e when the interest is paid more frequently and the compounding gets more frequent, the decrease in $i^{(p)}$ becomes less significant.

3) As the value of p gets bigger and approaches infinity, the interest is compounded continuously. This is known as force of interest denoted by 'δ'.

Q.9] i) Define force of interest.

- Force of interest is the nominal rate of interest compounded continuously. It is represented by the greek letter δ. Force of interest measures the continuous change in the fund value.

ii) If $i^{(2)} = 0.0775$

Calculate

a) $i = ?$

$$\therefore (1+i) = \left(1 + \frac{i^{(2)}}{2}\right)^2$$

$$\therefore i = \left[1 + \frac{i^{(2)}}{2}\right]^2 - 1$$

$$= \left[1 + \frac{0.0775}{2}\right]^2 - 1$$

$$= 0.079001563$$

$$\therefore i = 7.9001563\%$$

b) $\delta = ?$

$$\delta = \ln(1+i)$$

$$= \ln(1 + 0.079001563)$$

$$= \ln(1.079001563)$$

$$= 0.076036135$$

$$\therefore \delta = 7.6036135\%$$

c) $d^{(12)} = ?$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = (1+i)^{-1}$$

$$\left[1-\frac{d^{(12)}}{12}\right] = (1+i)^{-1/12}$$

$$\therefore d^{(12)} = 12\left[1-(1+i)^{-1/12}\right]$$

$$= 12\left[1-(1+0.079001563)^{-1/12}\right]$$

$$= 0.075795747$$

$$= 7.5795747\%$$

d) $i^{(1/2)} = ?$

$$\therefore \left(1+\frac{i^{(1/2)}}{1/2}\right)^{1/2} = (1+i)$$

$$\therefore \left[1+\frac{i^{(1/2)}}{1/2}\right] = (1+i)^2$$

$$\therefore i^{(1/2)} = \frac{1}{2}\left[(1+i)^2-1\right]$$

$$= \frac{1}{2}\left[(1+0.079001563)^2-1\right]$$

$$= 0.0821221$$

$$= 8.21221\%$$

Q.10] $\delta(t)$ is the force of interest per annum at time t years is defined as:-

$$\delta(t) = \begin{cases} 0.08 & \text{for } 0 \leq t \leq 5 \\ 0.06 & \text{for } 5 \leq t \leq 10 \\ 0.04 & \text{for } t \geq 10 \end{cases}$$

Derive the expression for $v(t)$, the present value of 1 due at time t.

$PV_0 = C \times v(t)$

$\therefore PV_0 = 1 \cdot v(t) = v(t)$

$\therefore \cancel{v(t)} = e^{-\int}$

$\therefore v(t) = e^{-\int_0^t \delta(s)\,ds}.$

$\cancel{\therefore \text{for } 0 \leq t \leq 5}$

$\therefore v(t) = \cancel{e^{-\int_0^t 0.08\,dt} = e^{[-0.08t]}}$

$= e^{-\int_0^5 0.08\,dt} \cdot e^{-\int_5^{10} 0.06\,dt} \cdot e^{-\int_{10}^t 0.04\,dt}$

$= e^{-[0.08t]_0^5} \cdot e^{-[0.06t]_5^{10}} \cdot e^{-[0.04]_{10}^t}$

$= e^{[(-0.08 \times 5) - (-0.08 \times 0)]} \cdot e^{[(-0.06 \times 10) - (-0.06 \times 5)]} \cdot e^{[(-0.04t) - (-0.04 \times 10)]}$

$= e^{(-0.4 - 0)} \cdot e^{(-0.6 + 0.3)} \cdot e^{(-0.04t + 0.4)}$

$= e^{(-0.4 - 0.3 - 0.04t + 0.4)}$

$= e^{-0.04t - 0.3}.$

$= e^{-0.04t - 0.3}$

$\boxed{\therefore V(t) = e^{-0.04t - 0.3}}$

Q.11] a) The period of loan = 9 months $\therefore n = 9/12$

Repayable by a single payment of ₹50,000. $\therefore$ Amount due = C = ₹50,000.

rate of discount = d = 18% p.a.

To find:- $PV_0 = ?$

$PV_0 = C \times v(t) = C \times (1-d)^n$

$= 50{,}000 \cdot (1-d)^{9/12}$

$= 50{,}000 \cdot (1-0.18)^{9/12}$

$= 43\,085.42$ //

∴ ₹43,085.42 was the amount initially lent to the borrower.

b) i. If $\delta = 6\%$. find $d^{(12)}$

$$\left[1 - \frac{d^{(12)}}{12}\right]^{12} = e^{-\delta}$$

$$\therefore \left[1 - \frac{d^{(12)}}{12}\right] = e^{-\delta/12}$$

$$\therefore d^{(12)} = 12\left[1 - e^{-\delta/12}\right]$$

$$= 12\left[1 - e^{-0.06/12}\right]$$

$$= 0.05985025$$

$$= 5.985025\%$$

ii) If $d^{(4)} = 6\%$ find $i^{(12)}$

$$\therefore \left[1 + \frac{i^{(12)}}{12}\right]^{12} = \left[1 - \frac{d^{(4)}}{4}\right]^{-4}$$

$$\therefore \left[1 + \frac{i^{(12)}}{12}\right] = \left[1 - \frac{d^{(4)}}{4}\right]^{-4/12}$$

$$\therefore \left[1 + \frac{i^{(12)}}{12}\right] = \left[1 - \frac{d^{(4)}}{4}\right]^{-1/3}$$

$$\therefore i^{(12)} = 12\left[\left[1-\frac{d^{(4)}}{4}\right]^{-1/3} - 1\right]$$

$$= 12\left[\left[1-\frac{0.06}{4}\right]^{-1/3} - 1\right]$$

$$= 0.060607089$$

$$= 6.0607089\%$$

c) Arrange the following quantities in ascending order of numerical value,

$i, d, i^{(4)}, \delta$

$\therefore$ By ascending order

$d, \delta, i^{(4)}, i$

i.e $d < \delta < i^{(4)} < i$.

The ~~values~~ abole rates have the same effective interest rate i.e the interest earned/paid is the same. However when they are payable ~~is~~ i.e the time when the interest is paid earned is different.

i - annual effective interest rate is payable once per year at the end.

$i^{(4)}$ - is nominal interest rate payable quaterly is payable each quarter

δ is the nominal interest rate compounded continuously

d - annual effective discount rate is paid at the start of the year once and only.

As they give the same amount of total interest but payable at different periods they are adjusted according to the time value of money.

d is ~~be~~ the lowest because it is payable once at the start of the year and i is the highest because it is payable once at the end of the year.

$\therefore$

d) Explain in words why $d = iv$ holds true.

'd' is the effective annual rate of discount which is the interest (or discount) paid at the start of the measurement year i.e generally one year. 'i' is the effective annual rate of discount which is the interest earned at the end of a year. 'v' is the discounting factor.

$\therefore$ If 'i' is discounted for 1 year by multiplying it with 'v' we will get the present value of interest earned at the start of the year i.e interest at the start of the year - 'd'.

$$PV = C \cdot v(t)$$

Accordingly $\therefore d = iv$ //

Q.12] A deposit of 7049 is accumulated.

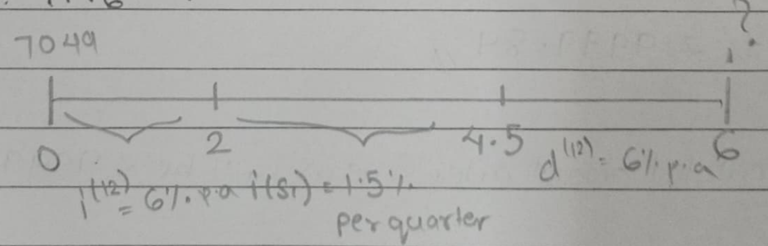
$\therefore$ Amount invested $= C =$ ₹ 7049

$i^{(12)} = 6\%$ p.a – for first 2 years.

Simple interest $= i_{(si)} = 1.5\%$ per quarter for next 2 and half years.

$d^{(12)} = 6\%$ p.a for the next 1 and half years.

To find :- $AV_6 = ?$



$\therefore AV_6 = C \cdot A(0,6)$

$= ₹\, 7049 \cdot A(0,2) \cdot A(2,4.5) \cdot A(4.5,6)$

$$\therefore A(0,2) = \left(1+\frac{i^{(12)}}{12}\right)^{12\times 2}$$

$$= \left(1+\frac{0.06}{12}\right)^{24}$$

$$= 1.127159776.$$

$$A(2,4.5) = (1+ \cancel{2}n \times i \times 4)$$

$$= (1+2.5\times 1.5\% \times 4)$$

$$= (1+2.5\times 0.015\times 4)$$

$$= \cancel{1.02705 1835} \quad 1.15$$

$$A(4.5,6) = \left(1-\frac{d^{(12)}}{12}\right)^{-12\times 1.5}$$

$$= \left(1-\frac{0.06}{12}\right)^{-18}$$

$$= 1.094421325$$

$$\therefore AV6 = 7049 \times A(0,2)\times A(2,4.5)\times A(4.5\times 6)$$

$$= 7049\times 1.127159776 \times \cancel{1.027051835} \times 1.094421325$$

$$1.15$$

$$= ₹\ 9999.89$$ //

∴ Accumulated amount after 6 years will be ₹ 9999.89.

Q.13] Calculate the time in years for an investment to double at :-

a) An effective rate of interest of 10% p.a

$\therefore i = 10\%$ p.a.

Let the amount invested be ₹100

∴ It will double to ₹200 i.e 100×2.

$\therefore AV = C \times A(t)$

$\therefore 200 = 100 \times (1+i)^n$

$\therefore \frac{200}{100} = (1+0.1)^n$

Taking ln on both sides

$\therefore \ln\left(\frac{200}{100}\right) = n \cdot \ln(1.1)$

$\therefore \ln(2) = n \cdot \ln(1.1)$

$\therefore n = \frac{\ln(2)}{\ln(1.1)}$

$\boxed{n \approx 7.27254 \approx 7.27 \text{ years.}}$

ii) A nominal rate of discount of 10% p.a convertible monthly.

$d^{(12)} = 10\%$ p.a

Similarly

$200 = 100(1-d)^{-n} = 100\left(1 - \frac{d^{(12)}}{12}\right)^{-n \times 12}$

$\therefore \frac{200}{100} = \left(1 - \frac{0.1}{12}\right)^{-12n}$

Taking ln on both sides

$\ln\left(\frac{200}{100}\right) = -12n \cdot \ln(0.9916667)$

$\therefore \frac{\ln(2)}{\ln(0.9916667)} = -12n$

$\therefore n = \frac{\ln(2)}{\ln(0.9916667)} \times \frac{-1}{12}$

$\boxed{\therefore n = 6.9025 \approx 6.9 \text{ years.}}$