

1. $y = f(x)$

x $y = f(x)$ Δy $\Delta^2 y$ $\Delta^3 y$

7 3 - 2

8 1 2

0

6 $\Delta^3 f(x_n)$

9 1 8 $\Delta^2 f(x_n)$

8 $\Delta f(x_n)$

10 9 $\rightarrow f(x_n)$

in ∇ , $u = \frac{x - x_n}{h} = \frac{9.5 - 10}{1} = -0.5$

$h = 1$

$\therefore 9.5$ is nearer to the end, Δ will be used.

$f(x) = f(x_n) + u \Delta f(x_n) + \frac{u(u-1)}{2!} \Delta^2 f(x_n) + \frac{u(u-1)(u-2)}{3!} \Delta^3 f(x_n)$

$= 9 + (-0.5)(8) + \frac{(-0.5)(-1.5)}{2 \times 1}(8) + \frac{(-0.5)(-1.5)(-2.5)}{3 \times 2}(6)$

$= 9 - 4 + 3 + (-1.875)$

$= 6.125$

6.125

$$2. \quad y = f(x)$$

$$\begin{array}{ccccccc} x & y & 168 & \leftarrow 48 \rightarrow & 120 & \leftarrow 48 \rightarrow & 72 & \leftarrow 9 \rightarrow & 63 \\ y & x & 3 & & 7 & & 9 & & 10 \end{array}$$

$$\begin{aligned} f(6) = & \frac{(6-120)(6-72)(6-63)}{(168-120)(168-72)(168-63)} \cdot 3 \\ & + \frac{(6-168)(6-72)(6-63)}{(120-168)(120-72)(120-63)} \cdot 7 \\ & + \frac{(6-120)(6-168)(6-63)}{(72-120)(72-168)(72-63)} \cdot 9 \\ & + \frac{(6-120)(6-168)(6-72)}{(63-120)(63-168)(63-72)} \cdot 10 \end{aligned}$$

$$= f(6) = 147$$

3.

$$y = f(x)$$

x	1	2	7	8
$f(x)$	4	5	5	4

$$f(6) = \frac{(6-2)(6-7)(6-8)}{(1-2)(1-7)(1-8)} \cdot 4$$

$$+ \frac{(6-1)(6-7)(6-8)}{(2-1)(2-7)(2-8)} \cdot 5$$

$$+ \frac{(6-1)(6-2)(6-8)}{(7-1)(7-2)(7-8)} \cdot 5$$

$$+ \frac{(6-1)(6-2)(6-7)}{(8-1)(8-2)(8-7)} \cdot 4$$

$$f(6) = 3.666$$

4.

$$y = f(x)$$

x	=	1	2	3	4	7
$f(x)$	=	2	4	8	16	128

$$f(5) = \frac{(5-2)(5-3)(5-4)(5-7)}{(1-2)(1-3)(1-4)(1-7)} \cdot 2$$

$$+ \frac{(5-1)(5-3)(5-4)(5-7)}{(2-1)(2-3)(2-4)(2-7)} \cdot 4$$

$$+ \frac{(5-1)(5-2)(5-4)(5-7)}{(3-1)(3-2)(3-4)(3-7)} \cdot 8$$

$$+ \frac{(5-1)(5-2)(5-3)(5-7)}{(4-1)(4-2)(4-3)(4-7)} \cdot 16$$

$$+ \frac{(5-1)(5-2)(5-3)(5-4)}{(7-1)(7-2)(7-3)(7-4)} \cdot 128$$

$$= f(5) = 32.93333$$

5.

x	10	20	14	16
$f(x)$	0.25	0.2	0.15	0.10

$$\begin{aligned}
 f(15) &= \frac{(15-20)^{-5} (15-14)^{+1} (15-16)^{-1}}{(10-20)^{-10} (10-14)^{-4} (10-16)^{-6}} \cdot (0.25) \\
 &+ \frac{(15-10)^{-5} (15-14)^{-1} (15-16)^{-1}}{(20-10)^{-10} (20-14)^{-6} (20-16)^{-4}} \cdot (0.2) \\
 &+ \frac{(15-10)^{-5} (15-20)^{-5} (15-16)^{-1}}{(14-10)^{-4} (14-20)^{-6} (14-16)^{-2}} \cdot (0.15) \\
 &+ \frac{(15-10)^{-5} (15-20)^{-5} (15-14)^{-1}}{(16-10)^{-6} (16-20)^{-4} (16-14)^{-2}} \cdot (0.1) \\
 &= f(15) = \boxed{0.1288667}
 \end{aligned}$$

Homework 2 : Regula falsi

$$1. \quad x^3 + x - 1 = 0 = f(x) \quad 0.5 < x < 1$$

 x y 0.5 -0.375 $x_0 \leftarrow 0.6$ $-0.184 \rightarrow y_0$ $x_1 \quad 0.7$ $0.043 \rightarrow y_1$

$$x_2 = \frac{(0.6)(0.043) - (0.7)(-0.184)}{0.043 - (-0.184)}$$

$$x_2 = 0.681057$$

$$y_2 = -0.00304180$$

here x_0 & y_0 are replaced by x_2 and y_2

$$x_3 = \frac{(0.681057)(0.043) - (-0.00304180)(-0.184)}{0.043 - (-0.00304180)}$$

$$= 0.68230484$$

$$y_3 = -0.0000550366$$

$\therefore$ The root is $x = 0.68230484$

2. $f(x) = x^3 - 2x + 0.5$

$0 < x < 1$

x	y
0.1	0.301
$x_0 \leftarrow 0.2$	$0.108 \rightarrow y_0$
$x_1 \leftarrow 0.3$	$-0.073 \rightarrow y_1$

$$x_2 = \frac{(0.2)(-0.073) - (0.3)(0.108)}{-(0.108) + (-0.073)}$$

$$x_2 = 0.2596685083$$

$$y_2 = -0.001828157419$$

x_1 & y_1 are replaced by x_2 & y_2 .

$$x_3 = \frac{(0.2)(-0.0018281) - (0.2596685)(0.108)}{-0.0018281574 - 0.108}$$

$$x_3 = 0.2586748734$$

$$y_3 = -0.000411152$$

$\therefore$ root is $x = 0.2586748734$

3. $x^3 + 2x - 20 = 0 = f(x)$ $2 < x < 2.5$

x	y
2.3	-3.233
$x_0 \leftarrow 2.4$	$-1.376 \rightarrow y_0$
$x_1 \leftarrow 2.5$	$0.625 \rightarrow y_1$

$$x_2 = \frac{(2.4)(0.625) - (2.5)(-1.376)}{(0.625) - (-1.376)}$$

$$x_2 = 2.4687656172 \rightarrow \text{replaces } x_1$$

$$y_2 = -0.0158770149 \rightarrow \text{replaces } y_1$$

$$x_3 = ? \quad x_0 \quad 2.4 \quad -1.376 \quad y_0$$

$$x_2 \quad 2.4687656 \quad -0.0158270 \quad y_2$$

$$x_3 = \frac{2.4(-0.0158270249) - (2.4687656)(-1.376)}{0.625 - (-1.376)}$$

$$x_3 = 2.4695356143$$

$$y_3 = -0.0001747905$$

$$\text{root is } x = 2.469535$$

4. $f(x) = 3x - \cos x - 1 = 0 \quad 0 < x < 1$

	x	y
	0.5	-0.4999619231
$x_0 \leftarrow$	0.6	-0.1999457694 $\rightarrow y_0$
$x_1 \leftarrow$	0.7	0.1000746303 $\rightarrow y_1$

$$x_2 = \frac{(0.6 \times 0.1000746) - (0.7)(-0.1999457694)}{0.100074 + 0.199945694} = 0.6666439914$$

$$y_2 = -0.00003384$$

$$\therefore \text{the root is } 0.6666439914$$

5. $f(x) = x^3 - 3x - 5 = 0 \quad 2 < x < 2.5$

	x	y
	2.1	-2.039
$x_0 \leftarrow$	2.2	-0.952 $\rightarrow y_0$
$x_1 \leftarrow$	2.3	0.267 y_1

$$x_2 = \frac{(2.2 \times 0.267) - (2.3)(-0.952)}{0.267 - (-0.952)}$$

$$y_2 = -0.0715944071$$

$$x_3 = \frac{(2.278096 \times 0.267) - (2.3)(-0.011594)}{(0.267) - (-0.011594)}$$

$$= 2.279011107$$

$$y_3 = -0.00009661$$

$\therefore$ The root is $x = 2.279011107$

6. $f(x) = x \log_{10} x - 1.2 = 0 \quad 2 < x < 3$

	x	y
	2.6	-0.1210692953
$x_0 \leftarrow$	2.7	-0.0353178 $\rightarrow y_0$
$x_1 \leftarrow$	2.8	0.05204243 $\rightarrow y_1$

$$x_2 = \frac{(2.7 \times 0.05204243) - (2.3)(-0.0353178)}{(0.0520424 + 0.0353178)}$$

$$x_2 = 2.792154119$$

$$y_2 = 0.04513149$$

y_1 & x_1 are replaced by x_2 & y_2

$$x_3 = \frac{(2.7 \times 0.04513149) - (2.792154119)(-0.0353178)}{(0.04513149 + 0.0353178)}$$

$$x_3 = 2.740456302$$

$$y_3 = -0.00016552$$

$\therefore$ root is $x = 2.740456302$

Q7.

$$x^3 - 4x + 1 = 0 = f(x).$$

 x y

0

1

1

-4

-0.2

0.208 -

-0.4

-0.536 -

0.5

-0.875

$$x_1 = \frac{(0.4 \times 0.208) - (-0.536 \times 0.2)}{0.208 - (-0.536)}$$

$$x_1 = 0.255914$$

$$y_1 = -0.0068956$$

$$0.255914$$

$$-0.0068956$$

$$0.208 -$$

$$0.208$$

$$x_2 = \frac{(0.208 \times 0.255914) - (-0.006895 \times 0.2)}{0.208 - (-0.0068956)}$$

$$x_2 = 0.254119$$

$$y_2 = -0.000069028$$

$$\therefore \text{The root } x = \underline{\underline{0.254119}}$$

8. $f(x) = x^3 + 2x^2 - 8 = 0$

	x	y
	1.4	-1.336
$x_0 \leftarrow$	1.5	-0.125 $\rightarrow y_0$
$x_1 \leftarrow$	1.6	1.216 $\rightarrow y_1$

$$x_2 = \frac{(1.5)(1.216) - (1.6)(-0.125)}{(1.216) - (-0.125)}$$

$$x_2 = 1.509321402$$

$$\therefore y_2 = -0.0055865$$

replacing x_1 & y_1 by x_2 & y_2

1.5	-0.125
1.509321402	-0.0055865

$$x_3 = \frac{(1.5)(-0.0055865) - (1.509321402)(-0.125)}{(-0.0055865) - (-0.125)}$$

$$x_3 = 1.509757487$$

$$\therefore y_3 = -0.00002774$$

$\therefore$ the root is $x = 1.509757487$

9. $f(x) = x^3 - 9x + 1 = 0$

	x	y
	0	1

$x_0 \leftarrow$	0.1	0.101 $\rightarrow y_0$
$x_1 \leftarrow$	0.2	-0.792 $\rightarrow y_1$

$$x_2 = \frac{(0.1)(-0.792) - (0.2 \times 0.101)}{-0.792 - 0.101}$$

$$x_2 = 0.113101904$$

$$y_2 = -0.000412585$$

$\therefore$ Root is $x = 0.113101904$

10. $f(x) = x^3 - x - 4 = 0$

	x	y
	1.6	-1.504
$x_0 \leftarrow$	1.7	-0.787 $\rightarrow y_0$
$x_1 \leftarrow$	1.8	0.032 $\rightarrow y_1$

$$x_2 = \frac{(1.7 \times 0.032) - (1.8)(-0.787)}{0.032 - (-0.787)}$$

$$= 1.7960927961$$

$$y_2 = -0.0019884399$$

$\therefore$ The root $x = 1.7960927961$