Div.: Roll No.:

Date : Subject : Topic :

Calculus Assignment 1

1. $\lim_{h \to 0} \dfrac{2(-3+h)^2 - 18}{\cancel{18}h}$

$\rightarrow \lim_{h \to 0} \dfrac{2(h^2 + 9 - 6h) - 18}{h}$

$\lim_{h \to 0} \dfrac{2h^2 + \cancel{18} - 12h - \cancel{18}}{h}$

$\lim_{h \to 0} \dfrac{2\cancel{h}(h-6)}{\cancel{h}} \Rightarrow 2(0-6) \Rightarrow \boxed{-12}$

2. $g(x) = \begin{cases} 2x & x < 6 \\ x-1 & x \geq 6 \end{cases}$

a) $x = 4$

$\rightarrow \lim_{x \to 4^+} g(x) = 2(4) \Rightarrow \boxed{8}$

$\lim_{x \to 4^-} g(x) = 2(4) \Rightarrow \boxed{8}$

$\therefore$ function is continous at $x = 4$.

b) $x = 6$

$\lim_{x \to 6^+} g(x) \Rightarrow 6 - 1 \Rightarrow \boxed{5}$

$\lim_{x \to 6^-} g(x) \Rightarrow 2(6) \Rightarrow \boxed{12}$

$\therefore$ function is discontinous at $x = 6$.

3. $\lim_{x \to 9} \frac{x-9}{3-\sqrt{x}}$

$\rightarrow \lim_{x \to 9} \frac{x-9}{3-\sqrt{x}} \times \frac{3+\sqrt{x}}{3+\sqrt{x}}$

$\lim_{x \to 9} \frac{(x-9)(3+\sqrt{x})}{9-x}$

$\lim_{x \to 9} \frac{-\cancel{(9-x)}(3+\sqrt{x})}{\cancel{(9-x)}}$

$\lim_{x \to 9} -(3+\sqrt{x}) \Rightarrow -(3+\sqrt{9}) \Rightarrow \boxed{-(6)}$

4. $f(x) = \frac{4x+5}{9-3x}$

a) $x = -1$

$\lim_{x \to -1} f(x) \Rightarrow \frac{4(-1)+5}{9-3(-1)} \Rightarrow$

$\Rightarrow \boxed{\frac{1}{12}}$

$\lim_{x \to} f(-1) \Rightarrow \frac{1}{12}$

$\therefore$ function is continous.

b) $x = 0$

$\lim_{x \to 0} f(x) \Rightarrow \frac{4(0)+5}{9-3(0)} \Rightarrow \boxed{\frac{5}{9}}$

$f(0) \Rightarrow \frac{5}{9}$

$\therefore$ function is continous.

c $x = 3$

$\lim_{x \to 3} f(x) \Rightarrow \frac{4(3)+5}{9-3(3)} \Rightarrow \boxed{\frac{17}{0}}$

c) $x = 3$

$$f(3) \Rightarrow \frac{4(3)+5}{9\cancel{} - 3(3)} \Rightarrow \boxed{\frac{17}{0}}$$

$$\lim_{x \to 3^+} f(x) = -$$

5. $$\lim_{x \to \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$$

$e^{\infty} = \infty$

$e^{-\infty} = 0$

$$\rightarrow \lim_{x \to \infty} \frac{\cancel{e^{-x}}(6e^{5x} - e^{-x})}{\cancel{e^{-x}}(8e^{5x} - e^{3x} + 3)}$$

$$\frac{6\cancel{} - e^{-6x}}{8 - e^{-2x} + 3e^{-5x}}$$

$$\frac{6}{8} \Rightarrow \boxed{\frac{3}{4}}$$

6. $$g(y) = \begin{cases} y^2 + 5 & \text{if } y < -2 \\ 1 - 3y & \text{if } y \geq -2 \end{cases}$$

a) $$\lim_{y \to 6} g(y) \Rightarrow \lim_{y \to 6} 1 - 3y \Rightarrow 1 - 3(6)$$

$$\Rightarrow \boxed{-17}$$

b) $\lim_{y \to -2^-} g(y)$

$\rightarrow \lim_{y \to -2^-} y^2 + 5 \Rightarrow 4 + 5 \Rightarrow \boxed{9}$

$\lim_{y \to -2^+} 1 - 3y \Rightarrow 1 - 3(-2) \Rightarrow \boxed{7}$

$\therefore$ It is discontinous.

7. $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

Diff w.r.t 't'

$f'(t) \Rightarrow \cancel{8t-1} \frac{d}{dt}(4t^2 - t) \cdot (t^3 - 8t^2 + 12) + \frac{d}{dt}(t^3 - 8t^2 + 12) \cdot (4t^2 - t)$

$\Rightarrow (8t - 1)(t^3 - 8t^2 + 12) + (3t^2 - 16t)(4t^2 - t)$

$8t^4 - 64t^3 + 96t - t^3 + 8t^2 - 12 + 12t^4 - 3t^3 - 64t^3 + 16t^2$

$20t^4 - 132t^3 + 24t^2 + 96t - 12$

8. $R(\omega) = \dfrac{3\omega + \omega^4}{2\omega^2 + 1}$

$\frac{u}{v} \Rightarrow \dfrac{v \cdot \frac{d}{dx}(u) - u \frac{d}{dx}(v)}{v^2}$

diff w.r.t 'w'

$$R'(\omega) \Rightarrow \frac{(2\omega^2 + 1)\frac{d}{d\omega}(3\omega + \omega^4) - (3\omega + \omega^4) \cdot \frac{d}{d\omega}(2\omega^2 + 1)}{(2\omega^2 + 1)^2}$$

$$\Rightarrow \frac{(2\omega^2 + 1)(3 + 4\omega^3) - (3\omega + \omega^4)(4\omega)}{4\omega^4 + 1 - 4\omega^2}$$

$$\Rightarrow \frac{6\omega^2 + 8\omega^5 + 3 + 4\omega^3 - (12\omega^2 + 4\omega^5)}{4\omega^4 + 1 - 4\omega^2}$$

$\cancel{6w^2 + 8w^3 + 3 + 4}$

$$\Rightarrow \boxed{\frac{-6w^2 + 4w^5 + 3 + 4w^3}{4w^4 + 1 - 4w^2}}$$

9. $f(x) = 2e^x - 8^x$

diff w.r.t 'x'

$f'(x) \Rightarrow \boxed{2e^x - 8^x \log 8}$

10. $y = z^5 - e^z \ln(z)$

diff w.r.t 'z'

$\frac{dy}{dz} \Rightarrow 5z^4 - \left[e^z \ln(z) + \frac{e^z}{z}\right]$ $\left[\because \frac{d}{dx} \log x \Rightarrow \frac{1}{x}\right]$

$\Rightarrow \boxed{5z^4 - e^z \ln(z) - \frac{e^z}{z}}$

11 a) $f(x) = (6x^2 + 7x)^4$

diff w.r.t 'x' $\boxed{f'(gx) \cdot g'(x)}$

$f'(x) \Rightarrow 4(6x^2 + 7x)^3 \cdot (12x + 7)$

b) $f(t) = 5 + e^{4t + t^7}$

diff w.r.t 't'

$f'(t) \Rightarrow 0 + e^{4t+t^7} \frac{d}{dt}(4t + t^7)$

$\Rightarrow \boxed{0 + e^{4t+t^7} \cdot (4 + 7t^6)}$

12. a)

12. a) $z = \ln(7 - x^3)$

diff w.r.t x.

$$\frac{d}{dx} z' = \frac{1}{7 - x^3} \cdot \frac{d}{dx}(7 - x^3) \Rightarrow \frac{1}{7 - x^3} \cdot (-3x^2)$$

diff w.r.t 'x'

$$z'' \Rightarrow \frac{7 - x^3 \frac{d}{dx}(-3x^2) - (-3x^2) \cdot \frac{d}{dx}(7 - x^3)}{(7 - x^3)^2}$$

$$\Rightarrow \frac{7 - x^3(-6x) - (-3x^2)(-3x^2)}{\cancel{(7 - x^3)}(7 - x^3)^2}$$

$$\Rightarrow \frac{-42x + 6x^4 - 9x^4}{(7 - x^3)^2}$$

$$\Rightarrow \frac{-42x - 3x^4}{(7 - x^3)^2} \Rightarrow \boxed{\frac{-3x(14 + x^3)}{(7 - x^3)^2}}$$

b) $Q(v) = \dfrac{2}{(6 + 2v - v^2)^4}$ u v

diff w.r.t 'v'

$$\cancel{Q'(v) \Rightarrow 2 \cdot \frac{d}{dv}(6 + 2v - v^2)^4 - (6 + 2v - v^2)^{-4} \frac{d}{dv}(2)}$$

$$\cancel{Q'(v) \Rightarrow 2 \Rightarrow 2}$$

$$Q'(v) \Rightarrow \frac{-2 \times 4\cancel{(6 + 2v - v^2)^3}(2 - 2v) - 0}{(6 + 2v - v^2)^{\cancel{4}}}$$

$$\Rightarrow \frac{-8(2 - 2v)}{6 + 2v - v^2} \Rightarrow \frac{-16(1 - v)}{6 + 2v - v^2}$$ u v

$$Q''(v) \Rightarrow \frac{\cancel{(6 + 2v - v^2)} \frac{d}{dv}(-16(1 - v)) - [-16(1 - v)] \frac{d}{dx}(6 + 2v - v^2)}{(6 + 2v - v^2)^{2}}$$

$$\theta''(v) \Rightarrow \frac{0 + 16(1-v)(2-2v)}{(6+2v-v^2)^2}$$

$$\theta''(v) \Rightarrow \frac{16(1-v)\,2(1-v)}{(6+2v-v^2)^2}$$

$$\Rightarrow \boxed{\frac{32(1-v)^2}{(6+2v-v^2)^2}}$$

c) $f(t) = \ln(1+t^2)$

diff w.r.t 't'

$$f'(t) \Rightarrow \frac{1}{1+t^2} \cdot dt(1+t^2) \Rightarrow \boxed{\frac{2t}{1+t^2}}$$

diff w.r.t 't'

$$f''(t) \Rightarrow \frac{(1+t^2)\frac{d}{dt}(2t) - (2t)\frac{d}{dt}(1+t^2)}{(1+t^2)^2}$$

$$\Rightarrow \frac{(1+t^2)(2) - (2t)(2t)}{(1+t^2)^2}$$

$$\Rightarrow \frac{2+2t^2-4t^2}{(1+t^2)^2} \Rightarrow \boxed{\frac{2-2t^2}{(1+t^2)^2}}$$

13. $f(x) \Rightarrow x^3 - 3x^2 - 9x + 12$

diff w.r.t 'x'

$f'(x) \Rightarrow 3x^2 - 6x - 9$

$\Rightarrow 3(x^2 - 2x - 3)$

~~$\Rightarrow 3[(x^2 + x)(-3x - 3)]$~~

$\Rightarrow 3[x^2 + x - 3x - 3]$

$3[x(x+1) - 3(x+1)]$

$3[(x+1)(x-3)]$ $\quad x = -1$ or $+3$

Let $f'(x)$ be 0.

$f''(x) \Rightarrow 6x - 6$

At $x = -1$

$f''(-1) = \boxed{-12}$

At $x = 3$

$f''(3) \Rightarrow \boxed{12}$

$\therefore f(x)$ is maximum at $x = -1$ and $f(x)$ is minimum at $x = 3$.

14. $f(x) = 4x^3 - 18x^2 + 24x - 7$

diff w.r.t 'x'

$f'(x) \Rightarrow 12x^2 - 36x + 24$

$\Rightarrow x^2 - 3x + 2$

$(x^2 - 1x)(-2x + 2)$

$x(x-1) - 2(x-1)$

$(x-1)(x-2)$

Let $f'(x)$ be 0 $\quad \therefore \quad x = 1$ or $x = 2$

diff w.r.t 'x'

$f''(x) \Rightarrow 24x - 36$

At $x = 1$

$f''(1) \Rightarrow \boxed{-12}$

At $x = 2$

$f''(2) \Rightarrow \boxed{12}$

$\therefore f(x)$ is maximum at $x=1$ & $f(x)$ is minimum at $x = 2$.

15. $f(x) = x^2(x+3)$

$\Rightarrow x^3 + 3x^2$

diff w.r.t 'x' $\Rightarrow f'(x) = 3x^2 + 6x \Rightarrow 3x(x+2)$

~~At~~ Let $f'(x)$ be 0.

$\therefore x = \frac{0}{3} = 0$ or $x = -2$

~~At $x=0$~~

$f''(x) \Rightarrow 6x + 6$

At $x = 0$

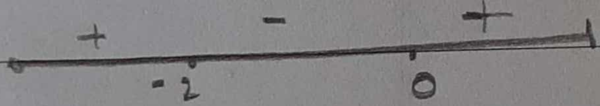
$f(0) \Rightarrow 6$

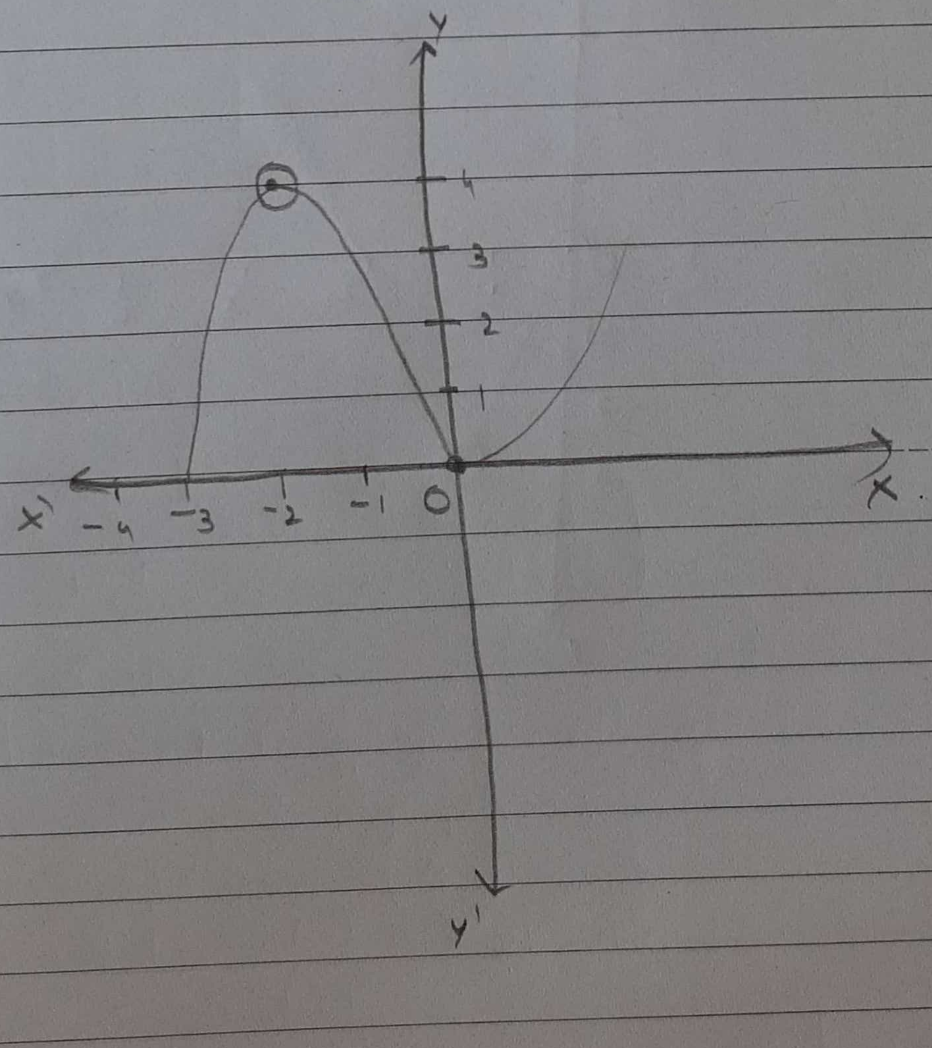
At $x = -2$

$f(-2) \Rightarrow -6$

$\therefore f(x)$ is minimum ~~maximum~~ at $x=0$ & maximum at $x=-2$.

$f(-2) \Rightarrow 4(-2+3) \Rightarrow \boxed{4}$





16. $f(x) = 3x^2 - 6x$

diff w.r.t 'x'

$f'(x) \Rightarrow 6x - 6 \Rightarrow 6(x-1)$

~~At x~~ let $f'(x)$ be 0

$\therefore$ $x = 0$ or $x = 1$

$f''(x) \Rightarrow 6$.

~~At $x = 0$~~

~~$f''(0) \Rightarrow 0$~~

At $x = 1$

$f''(1) = 6$

$f(x)$ is minimum at $x = 1$.

$f(1) \Rightarrow -3$