Probability And Statistics - 1

Q1)

(1) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

$$\text{Mean} = \frac{4+7+9+9+11+15+18+18+18+25}{10}$$

$$= \underline{\underline{13.4}}$$

$$\text{Mode} = \underline{\underline{18}}$$

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{ term}$$

$$= \left(\frac{10+1}{2}\right) = 5.5^{th} \text{ term}$$

$$= \frac{11+15}{2} = \underline{\underline{13}}$$

$$\underline{\text{IQR}} \Rightarrow q_1 = \left(\frac{n}{4}\right)^{th} \text{ term}$$

$$= \left(\frac{10}{4}\right)^{th} \text{ term} = 2.5^{th} \text{ term}$$

$$= \frac{7+9}{2} = \underline{\underline{8}}$$

$$q_3 = \left(\frac{3n}{4}\right)^{th} \text{ term} = \left(\frac{30}{4}\right)^{th} \text{ term} = 7.5^{th} \text{ term}$$

$$= \frac{18+18}{2} = 18 //$$

$\therefore \text{IQR} = q_3 - q_1$
$= 18 - 8$
$= 10$

(2)

N.O.H	0	1	2	3	4
N.O.P	5	11	26	15	3

f	x	f.x
0	5	0
1	11	11
2	26	52
3	15	45
4	3	12

$$\bar{x} = \frac{\sum f.x}{\sum f}$$

$$= \frac{120}{60} = 2$$

Median $\Rightarrow n = 60$

$\therefore$ Median $= \cancel{\left(\frac{n}{2}\right)^{th} = \left(\frac{60}{2}\right)^{th} \text{term}}$

$= \left(\frac{n+1}{2}\right)^{th}$ term $= \left(\frac{61}{2}\right) = 30.5^{th}$ term

$\therefore$ Median $= 2$

Mode $= 2$

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$\underline{IQR} \Rightarrow q_1 = \left(\frac{n}{4}\right)^{th}$ term

$= 15^{th}$ term

$= 1$

$q_3 = \left(\frac{3n}{4}\right)^{th}$ term

$= 45^{th}$ term

$= 3$

$\underline{IQR} = 3 - 1$

$= \underline{\underline{2}}$

Standard deviation

$$S = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{n}} \Rightarrow \sqrt{\frac{58}{60}} = 0.98319208$$

Q2) $X \sim P(0.5)$

① $P(x=0) = \frac{e^{-\lambda}\lambda^x}{x!}$

$= \frac{e^{-0.5}\lambda^0}{0!} = e^{-0.5} = 0.6065$

② Y = no of years with a claim,

then $Y \sim B(3, 0.3935)$

$\therefore P(Y=1) = {}^3C_1 \times (0.3935)^1 \times (0.6065)^2$
$= 0.434$

③ $X \sim \exp(0.5)$

$$P(x>2) = \int_2^{\infty} \lambda e^{-\lambda x}\, dx$$

$$= 0.5 \int_2^{\infty} e^{-0.5x}\, dx = \left[\frac{e^{-0.5x}}{-0.5} \times 0.5\right]_2^{\infty}$$

$$= -\left[e^{-0.5x}\right]_2^{\infty} = -\left[e^{-\infty} - e^{-1}\right]$$

$$= -\left[0 - 1/e\right] = 1/e = 0.368$$

Q3) $X \sim \text{Gamma}(\alpha, \lambda)$
$\alpha = 2$, $\lambda = 1/2$; $0 < x < \infty$

① $E(x) = \frac{\alpha}{\lambda} = \frac{2}{1/2} = 4$

$V(x) = \frac{\alpha}{\lambda^2} = \sigma^2 \Rightarrow \sigma = \sqrt{\frac{\alpha}{\lambda^2}} = \sqrt{\frac{2}{0.25}} = \sqrt{8} = 2.828$

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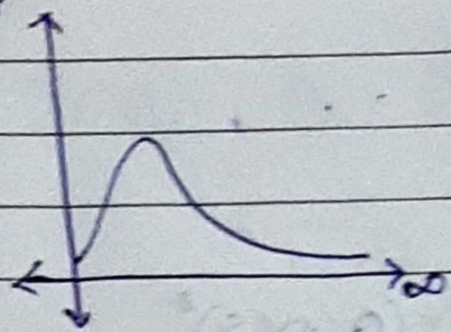
② Pdf



$$\text{Pdf} = \frac{\lambda^{\alpha} e^{-\lambda x} x^{\alpha-1}}{\Gamma\alpha}$$

increase in the start and then decreases when $\alpha = 2, \lambda = \frac{1}{2}$

③ CDF of gamma distribution,

$$F_x(x) = \int_0^{\infty} \frac{\lambda^{\alpha} e^{-\lambda x} x^{\alpha-1}}{\Gamma\alpha}$$

Q4) $P(M) = \frac{10}{18}, P(F) = \frac{8}{18}, P(MQ) = \frac{6}{18}, P(FQ) = \frac{7}{18}$

10M
8F

$\frac{10}{18}$ M $\xrightarrow{9/17}$ MM $\rightarrow P(MM) = \frac{10}{18} \times \frac{9}{17} = \frac{90}{306}$

M $\xrightarrow{8/17}$ MF $\rightarrow P(MF) = \frac{10}{18} \times \frac{8}{17} = \frac{80}{306}$

$\frac{8}{18}$ F $\xrightarrow{10/17}$ FM $\rightarrow P(FM) = \frac{8}{18} \times \frac{10}{17} = \frac{80}{306}$

F $\xrightarrow{7/17}$ FF $\rightarrow P(FF) = \frac{8}{18} \times \frac{7}{17} = \frac{56}{306}$

$$P(x) = P(FF) + P(FM) + P(MF)$$

$$= \frac{80}{306} + \frac{56}{306} + \frac{80}{306} = \frac{216}{306}$$

$P(Q) = \frac{216}{806} \times \frac{7}{18} = 0.2745$

Q6) $X \sim U(1,2)$

$a = 1,\ b = 2$

$f_x(x) = \frac{1}{2-1} = \frac{1}{1} = 1$

$$f_y(y) = f_x(w^{-1}(y)) \left| \frac{d}{dy}(w^{-1}(y)) \right|$$

$$= f_x\left(6 - \frac{3}{y}\right) \cdot \frac{3}{y^2}$$

$$= 1 \cdot \frac{3}{y^2} = \frac{3}{y^2}$$

$Y = \frac{3}{6-x}$

$6 - x = \frac{3}{y}$ $\qquad\qquad \frac{dx}{dy} = \frac{3}{y^2}$

$x = 6 - \frac{3}{y} = w^{-1}(y)$

Q7)

① $P(3) = P(x \leq 3)$
$= 0.05 + 0.15 + 0.35$
$= 0.55$

② $E(x) = 1(0.05) + 2(0.15) + 3(0.35) + 4(0.4) + 5(0.05)$
$= 3.25$

③ $E(x^2) = 1(0.05) + 4(0.15) + 9(0.35) + 16(0.4) + 25(0.05)$
$= 11.45$

$V(x) = E(x^2) - [E(x)]^2 = 11.45 - (3.25)^2$
$= 0.8875$

④ Coefficient of skewness $= E(x - \bar{x})^3$

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Q8) $X \sim B(15, 0.2)$

$P(x < 3) = P(x=0) + P(x=1) + P(x=2)$

$= {}^{15}C_0 (0.2)^0 (0.8)^{15} + {}^{15}C_1 (0.2)^1 (0.8)^{14} + {}^{15}C_2 (0.2)^2 (0.8)^{13}$

$= 0.39802$

Q9)

Q10) $X \sim P(0.4)$

$P(x > 2) = 1 - P(x \leq 2)$

$= 1 - 0.99207$

$= 0.00793$

Q12) $X \sim Exp(10)$

$\lambda = 10$

$x = 0.1$ days

$F_x(0.1) = \lambda e^{-\lambda x} = 10 e^{-10(0.1)} = 10e^{-1} = 3.67879$

Q13) $X \sim U(75, 120)$ $a = 75,\ b = 120$

$$F_x(x) = \frac{1}{b-a} = \frac{1}{120-75} = \frac{1}{45}$$

$$P(80 < x < 100) = F_x(100) - F_x(80)$$
$$= \frac{100}{45} - \frac{80}{45}$$
$$= \frac{20}{45} = \frac{9}{5} = 0.4444$$

Q14) $X \sim \text{Gamma}(5, 0.4)$ $\lambda = 0.4,\ \alpha = 5$

$P(x < 22.89) = ?$

$$2\lambda x \sim \chi^2_{10} = 0.8x \sim \chi^2_{10}$$

$$\therefore P(x < 22.89) = P(0.8x < 18.312)$$
$$= P(\chi^2_{10} < 18.312)$$
$$= 0.9499296$$