Prob. & Stat. Assignment 2

PAGE No.
DATE / /

Q1) $E(X_1) = 800$ $\quad E(X_2) = 1200$

$V(X_1) = 100^2$ $\quad V(X_2) = 300^2$

ATQ ; $Y \sim N(800, 100^2)$

& $Y \sim N(1200, 300^2)$

$\therefore (Y_1 + Y_2 + Y_3) - (X_1 + X_2 + X_3 + X_4) \sim N(400, 31000$

$$P\left(Z > \frac{800-400}{\sqrt{310000}}\right)$$

$$P[Z > 0.71842]$$

$$= 1 - 0.76424$$

$$= \underline{0.2357}$$

Q2) $N \sim Bin(K, P)$

$\therefore P(N=n) = {}^{n+2}C_n \, (0.9)^3 (0.1)^n$

$= {}^{(n+2)-1}C_{(n)-1} \, (0.9)^3 (0.1)^n$

$K = 3$; $P = 0.9$

$M_Y(t) = M_N(\log M_X t)$

$X \sim Gamma(6, 2)$

$Y \sim$ Total amt.

(i) MGF $= M_Y(t) = M_N[\log M_X t]$

$$M_X(t) = \left[1 - \frac{t}{\cancel{\ }}\right]^{\alpha}$$

$$M_Y(t) = M_N\left[\log\left(1 - \frac{t}{2}\right)^{-6}\right]$$

$$\therefore M_N(t) = \left[\frac{pe^t}{1-qe^t}\right]^k = \left[\frac{p(1-t/2)^{-6}}{1-q(1-t/2)^{-6}}\right]^3$$

$$= \left[\frac{0.9(1-t/2)^{-6}}{\cancel{1} - 0.1(1-t/2)^{-6}}\right]^3$$

PAGE No.

DATE / /

$\therefore V(Y) = E(N) \cdot V(X) + [E(X)]^2 V(N)$

$= \left(\frac{3}{0.9} \times \frac{\cancel{6}}{4}\right) + \left(\frac{\cancel{6}}{\cancel{2}}\right)^2 \times \frac{3(0.1)}{0.9^2}$

$= 8.3347$

$\therefore \underline{\underline{S.D = 2.887}}$

Q3) $f(x) = \lambda \cdot e^{-\lambda(x)} = \lambda e^{\lambda(x-\lambda)}$

(i) $MGF = \int_{\alpha}^{\infty} e^{t\lambda} \cdot \lambda e^{\lambda(x-\lambda)}$

$Mx(t) = \lambda e^{-\lambda t} \int_{0}^{\infty} e^{-(\lambda-t)x}$

$x = \frac{e^{\alpha\lambda} \lambda}{(t-\lambda)} [e^{-(\lambda-t)x}]_{\alpha}^{\infty}$

$= e^{\lambda \alpha}\left[\frac{\lambda}{\lambda - t}\right]$

(ii) $M'_\lambda(t) = \lambda[e^{t\alpha}(\lambda-t)^{-2}(+1) + (\lambda-t)^{-1} e^{t\alpha}(\alpha)]$

$= \lambda e^{t\alpha}(\lambda-t)^{-2}(\alpha(\lambda-t)+1)$

at $t=0$

$M'_x(t) = \lambda(\lambda)^{-2}[\alpha(\lambda)+1)$

$= \frac{\lambda\alpha+1}{\lambda}$

$V(X) = E(X^2) - [E(X)]^2$

$= \frac{2\alpha}{\lambda} + \alpha^2 + \frac{2}{\lambda^2} - \left[\alpha + \frac{1}{\lambda}\right]^2$

PAGE No. DATE / /

Q4] $G_Y(t) = \left(\frac{p}{1-qt}\right)^k \cdot \Sigma t^v \times P(V=v)$

$P(V=4) = \frac{\sqrt{k+4}}{\sqrt{5}\sqrt{k}} p^k q^k$

$= \frac{(k+3)!\, p^k (1-p)^k}{(k-1)!\, 4!}$

Q5] $f(x,y) = ax(x+y)$ $\quad 0<x<5$

$10<y<20$

(i) $P\left(Y > \frac{1}{3}x + 10\right]$

$= a\int_{10}^{20}\int_{0}(x^2 + xy)\, dx\, dx$

$= a\int_{10}^{20}\left[\frac{x^3}{3} + \frac{x^2y}{2}\right]_0^5 dx$

$= a\int_{10}^{20}\left(\frac{125}{3} + \frac{25y}{2}\right) dy$

$\Rightarrow \left[\frac{125y}{3} + \frac{25y^2}{4}\right]_{10}^{20} = \frac{1}{a}$

$\therefore a = 0.0004363$

(ii) $P(Y > \frac{1}{3}x + 10)$

$f(y) = \frac{3}{6815}\int_0^5 (x^2 + xy)\, dx$

$= \frac{3}{6815}\left[\frac{125}{3} + \frac{25y}{2}\right]$

$\Rightarrow \frac{3}{6875}\left[\frac{125y}{3} + \frac{25y^2}{4}\right]_{10+1/3x}^{20}$

$= \frac{3}{6875}\left[\frac{1250}{3} + 18\right]$

PAGE No.
DATE / /

(iii) $E(Y/X) = ?$

$$f(Y/X) = \frac{f(x,y)}{f(x)}$$

$$\therefore f(x) = \frac{3}{6875}\int_{10}^{20}(x^2+xy)\,dy$$

$$= \frac{3}{6875}\left[x^2y + \frac{xy^2}{2}\right]_{10}^{20}$$

$$= \frac{3}{6875}\left[20x^2 + 1200x - 10x^2 - 30x\right]$$

$$= \frac{30x}{6875}\left[x+15\right]$$

$$E(Y/X) = \int_{10}^{20} y\left(\frac{x+y}{10(x+15)}\right)dy$$

$$= \frac{1}{10(x+5)}\int_{10}^{20}(xy+y^2)\,dy$$

$$= \frac{1}{10(x+15)}\left[15x + \frac{7000}{3}\right]$$

Q6] $X \sim Poi(\lambda)$; $Y \sim Poi(\mu)$

$M_X(t) = e^{\lambda[e^t-1]}$ $\quad M_Y t = e^{\mu(e^t-1)}$

$$z = X - Y$$

$$M_X(t) = e[e^{t-1}]$$

$$= E[e^{tX-Y}]$$

$$= E(e^{tX})$$

$$= e^{(X-u)(e^t-1)}$$

$Z = X+Y$

$Z \sim Poi(\lambda+\mu)$

$Z \sim Poi(\lambda, u)$

PAGE No.
DATE / /

Q7] (i) $V(X/Y=2) = E(X^2/Y=2) - [E(X/Y=2)]^2$

$$E(X^2/Y=2) = \sum_x x^2 \frac{P[X=x, Y=2]}{P(Y=2)}$$

$$= 8.8333$$

$$E(X/Y=2) = \sum_x \frac{x P(X=x, Y=2)}{P(Y=2)} = 0.807$$

$$V(X/Y=2) = 8.8333 - 0.807$$

$$= 8.182$$

(ii) $f(u,v) = \frac{48}{67}(2uv - v^2)$

$$E(U/V=u) = \frac{f(u,u)}{f(v)}$$

$$f(v) = \frac{48}{67}\int_0^1 (2uv - v^2)\,du$$

$$= 2u^2v^2 - v^3$$

$$E(U/V=u) = \frac{3}{d4-1}\int_0^1 y(2uv^2 - v^2)\,du$$

$$= \frac{8u^2 - 3}{4(3v-1)}$$

Q8] $\lambda \sim Gamma(3,2)$

$E(\lambda) = 3/2$ $\quad$ $V(X) = 3/4$

$E(X = X = x) = 3X + 1$ $\quad$ $V(Y/X=x) = 2x^2+5$

PAGE No.
DATE / /

$E(Y) = E[E(X/x=x)] = 3x+1$
$= 3E(X)+1$
$= 3\left(\frac{3}{2}\right)+1$

$= \frac{9}{2}+1 = \frac{11}{2}$

$V(Y) = E(Y(X/Y)] + V[E(X/Y)]$
$= 2E(X^2) + 5 + 9V(X)$
$V(X) = [E(X^2) - E(X)]^2$

$\therefore E(X^2) = 3$

$V(Y) = 2(3) + 5 + 9\left(\frac{3}{4}\right)$

$V(Y) = \frac{71}{4}$

Q9] $E(X) = 3$; $V(X) = 4$; $E(Y) = 4$; $V(Y) = 1$
$Z = X+Y$
$\cancel{cov}(X,Y) = -0.6$
$corric(X,Y) = -0.354$
(i) $cov(X, X+Y) = cov(X,X) + cov(X,Y)$
$= V(X) + cov(X,Y)$
$= 4 + (-0.6)$
$= 3.4$

(ii) $Var(Z) = cov(X+Y, X+Y)$
$= cov(X,X) + 2cov(X,Y) + cov(Y,Y)$
$= V(X) + 2cov(X,Y) + V(Y)$
$= 3.8$

PAGE No.

DATE / /

Q10] $f(x, y) = k \cdot x^{-\lambda} e^{-y/\beta}$

$$\Rightarrow k \int_1 \int_1^{\infty} x^{-\lambda} e^{-y/\beta} \, dx \, dy = 1$$

$$\therefore \frac{k}{-\alpha+1} \int e^{-y/\beta} \, dy = 1$$

$$\therefore k = \frac{\alpha - 1}{\beta (e^{-1/\beta})}$$

—x—0—x—