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Numerical Methods & Algebra

Assignment 1

No. of pages :-

1) let $y = \sqrt{7+\sqrt{7+\sqrt{7+\sqrt{7+\sqrt{7}}}}}$

$\therefore y = \sqrt{7+y} \Rightarrow y^2 = 7+y$

$y^2 - y - 7 = 0 \qquad y = \frac{1 \pm \sqrt{1-(4)(1)(-7)}}{2}$

$y = \frac{1 \pm \sqrt{29}}{2}$ as can't be negative

ans $\Rightarrow$ $\boxed{3.19258}$

2) $7x^2 - 6y^2 = -11x^2 - 4y^2$ (dividing by y^2)

$\frac{7x^2}{y^2} - \frac{6y^2}{y^2} = -\frac{11x^2}{y^2} - \frac{4y^2}{y^2}$

$\Rightarrow 7\left(\frac{x}{y}\right)^2 - 6 = -11\left(\frac{x}{y}\right)^2 - 4$

$\Rightarrow 19\left(\frac{x}{y}\right)^2 = 2$

$\left(\frac{x}{y}\right)^2 = \frac{2}{19}$ $\qquad \boxed{\frac{x}{y} = \pm\sqrt{\frac{2}{19}}}$

3) $x + 3y = 4$ $\qquad \boxed{x = 4 - 3y}$

$x^2 + y^2 - 4y = 12$

$(4-3y)^2 + y^2 - 4y = 12$

$(16 - 24y + 9y^2) + y^2 - 4y - 12 = 0$

$10y^2 - 28y + 4 = 0$

$y = \frac{28 \pm \sqrt{784-(160)}}{20} \Rightarrow \frac{28 \pm \sqrt{624}}{20}$

$x = -\left(\frac{1 \pm 3\sqrt{39}}{5}\right)$ & $\left(\frac{-1 \pm 3\sqrt{39}}{5}\right)$

$y = \frac{28 + \sqrt{624}}{20}$ & $\frac{28 - \sqrt{624}}{20}$

4) a. $d = 2(3.5)^2 - 16(3.5) + 34$

$= 2.5$ millions of particles per cubic foot

b. $\frac{d}{dr}(d) = 4r - 16$ $\quad r = 4$ when $\frac{d}{dr} = 0$.

$\frac{d^2}{dr^2}(d) = 4$ $\quad \therefore$ minimum $\boxed{r = 4}$

output $= 2(4)^2 - 16(4) + 34$

$= 2$ millions particles per cubic foot

c. $100 = 2r^2 - 16r + 34$

$0 = 2r^2 - 16r - 66$

$r = \frac{16 \pm \sqrt{256 + 528}}{2} \Rightarrow \frac{16 + 28}{2}$ as can't be -ve

$\boxed{r = 2200 \text{ revolutions per minute}}$

5) a. $C = 1500 + 25n$

b. $C = 2100 + 22n$

c. $2100 + 22n < 1500 + 25n$

d. $2100 - 1500 < 25n - 22n$

$600 < 3n$

$\boxed{200 < n}$.

6) $\varepsilon = 0.01$ $\quad f(x) = x^3 - 7x^2 + 14x - 6 = 0$

a. diff	a	b	t	f(a)	f(b)	f(t)
1	0	1	0.5	−6	2	−0.625
0.5	0.5	1	0.75	−0.625	2	0.984375
0.25	0.5	0.75	0.625	−0.625	0.984375	$\frac{133}{512}$
0.125	0.5	0.625	0.5625	−0.625	$\frac{133}{512}$	$-\frac{663}{4096}$
0.0625	0.5625	0.625	0.59375	$-\frac{663}{4096}$	$\frac{133}{512}$	0.05404663
0.03125	0.5625	0.59375	0.578125	$-\frac{663}{4096}$	0.0540466	−0.0526237
0.015625	0.578125	0.59375	$\frac{75}{128}$	−0.0521	0.0540466	1.03×10^{-3}
7.8×10^{-3}	0.578125	$\frac{75}{128}$		−0.0521	1.03×10^{-3}	

$\boxed{\text{Ans} \Rightarrow 0.5859375}$

b. diff

a	b	t	f(a)	f(b)	f(t)
1	3.2	2.1	2	-0.112	1.791
2.1	3.2	2.65	1.791	-0.112	0.552125
2.65	3.2	2.925	0.55125	-0.112	0.0858281
2.925	3.2	3.0625	0.0858..	-0.112	$-\frac{223}{4096}$
2.925	3.0625	$\frac{479}{160}$	0.0858..	$-\frac{223}{4096}$	6.327×10^{-3}
0.0625	~~2.925~~	~~3.0625~~ 969/320	6.327×10^{-3}	$-\frac{223}{4096}$	-0.0265
479/160	969/320	1927/640	6.327×10^{-3}	-0.0265	-0.0106
479/160	1927/640	3843/1280	6.327×10^{-3}	-0.0106	-2.33×10^{-3}
✓ 479/160	3843/1280				

Ans ⇒ 3.00234375

c. diff

a	b	t	f(a)	f(b)	f(t)
0.8	3.2	4	3.6	~~1.791~~ -0.112	2	0.336
0.4	3.2	3.6	3.4	-0.112	0.336	-0.016
0.2	3.4	3.6	3.5	-0.016	0.336	0.125
0.1	3.4	3.5	3.45	-0.016	0.125	369/8000
0.05	3.4	3.45	3.425	-0.016	369/8000	833/164000
0.025	3.4	3.425	3.4125	-0.016	833/164000	-1.99×10^{-3}
0.0125	3.4125	3.425	3.41875	-1.99×10^{-3}	833/164000	5.35×10^{-3}
✓	3.4125	3.41875				

Ans ⇒ 3.4125

f) $f(x) = 230x^4 + 18x^3 + 7x^2 - 221x - 9$

$f'(x) = 920x^3 + 54x^2 + 18x - 221$

[0, 1]

$x_0 = 0$; $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \Rightarrow 0 - \left(\frac{-9}{-221}\right) = -\frac{9}{221}$

$x_2 = -\frac{9}{221} - \left(\frac{0.014342892269}{-221.705611}\right) = -0.040659258849$

$x_3 = -0.040659258832$ ✓ precision attained.

8) a. $\frac{70+86+81+83+x}{5} \Rightarrow \frac{320+x}{5}$ = mean.

b. $90 > \frac{320+x}{5} \geq 80$ for B

~~450~~ →

c. $450 > 320 + x \geq 400$

$130 > x \geq 80$ ~~cant be >100~~

~~$\therefore 100 > x \geq 80$~~

9) $x_1 = 4\%$, $y_1 = 8.54$ at IRR, $y = 0$.

$x_2 = 20\%$, $y_2 = -11.81$

$\frac{x-4}{20-4} = \frac{0-8.54}{-11.81-8.54}$ → $\boxed{x = 10.77\%}$

10) for profitable, $R(x) - C(x) > 0$

$(32x - 0.21x^2) - (195 + 12x) > 0$

$-0.21x^2 + 20x - 195 > 0$

$0.21x^2 - 20x + 195 < 0$

$x = 20 \pm \frac{\sqrt{400 - (195 \times 4 \times 0.21)}}{0.21}$

$\boxed{x > 168.4}$

$\boxed{x < 22.05}$

+ − +

$20 - 15.31$ $20 + 15.31$