1) $d^{(4)} = 8\%$ p.a. $\qquad \frac{d^{(4)}}{4} = 2\%$ p.a.

$$i = (1 - 2\%)^{-4} - 1$$

$$= 8.416978473\%$$

(i) $\delta = \ln(1+i)$

$\delta = 8.081082927\%$

ii) $i = 8.4165\%$

iii) $(1+i) = \left(1 - \frac{d^{(12)}}{12}\right)^{-12}$

$d^{(12)} = (1 - (1 + 8.4165\%)^{-1/12})12$

$d^{(12)} = 8.053933945\%$

2) $\delta(t) = 0.004t + 0.0002t^2$

i) $PV = 1000\, e^{-\int_0^{10} 0.004t + 0.0002t^2 dt}$

$= 1000\, e^{-}\left[0.002t^2 + 0.0002\frac{t^3}{3}\right]_0^{10}$

$= 1000\, e^{-}[0.2 + 0.0667] = 765.92$

ii) $(1+i)^{10} = e^{0.2667}$

$(1+i) = \sqrt[10]{1.305648693}$

$(1+i) = 1.02708827$

$i = \cancel{2.7}\ 2.7028827\%$

3) i) $d = iv \qquad v = \frac{1}{A} = \frac{1}{1+i}$

$$\boxed{d = \frac{i}{1+i}}$$

ii) $\frac{d^{(4)}}{4} = 2.25\%$ $d^4 = 9\%$

$d = 1 - (1 - 2.25\%)^4$

$= 8.7\%$

$d^{(2)} = 2(1-(1-8.78\%))^{1/2}$

$= 8.89875\%$

iii) 91 day government bill $= \frac{91}{365}$

4i) $d = iv$ we know $v = \frac{1}{A} = \frac{1}{1+i}$

$d = \frac{i}{1+i}$

ii) $i = 0.08$

a) $d^{(12)} = (1.08)^{-1/12} = \left(1 - \frac{d^{(12)}}{12}\right)$

$d^{(12)} = 7.671477614\%$

b) $i^{(365)} = 365\left[(1+0.08)^{1/365} - 1\right]$

$= 7.6969155 41\%$

c) $\delta = \ln(1+i) = \ln(1.08) = 7.696104114\%$

d) $i(0.5) = 8.32\%$

$= 0.5\left[(1+0.08)^{0.5} - 1\right]$

5) 1) a) $d^4 = 6\%$

$d = 1 - \left(1 - \frac{6\%}{4}\right)^4$

$d = 5.866\%$

$i = (1-d)^{-1} - 1$

$i = 6.2319\%$

b) $d^{(12)} = (1.062319)^{-1/12} = \left(1 - \frac{d^{(12)}}{12}\right)$

$d^{(12)} = 6.0302230\%$

7) $PV = 20000 \quad AV = 26000 \qquad n = \frac{17}{12}$ yrs

$$26000 = 20000(1+i)^{17/12}$$

$$20(1+i)^{17/12} = 26$$

$$(1+i)^{17/12} = 1.3$$

$$(1+i) = 1.203457067$$

$$i = 20.3457067\%$$

$$\Rightarrow 26000 = 20000\left(1+\frac{i^4}{4}\right)^{68/12}$$

$$\left(1+\frac{i^4}{4}\right)^{68/12} = \sqrt{1.3}$$

$$i^4 = 18.9552546\%$$

iii) $26000 = 20000\left(1+\frac{17}{12}i\right)$

$$1.3 = \left(1+\frac{17}{12}i\right)$$

$$0.3 = \frac{17}{12}i$$

$$i = 21.176470\%$$

9) if $i^{(2)} = 0.0775$

$$i = \left(1+\frac{0.775}{2}\right)^2 - 1$$

$$= 7.9001562 5\%$$

$$\delta = \ln(1+7.9001)$$

$$= 7.604\%$$

$$(1+7.9\%) = \left(1-\frac{d^{(12)}}{12}\right)^{-12}$$

$$(1.079)^{-1/12} = 1 - \frac{d^{(12)}}{12}$$

$$d^{(12)} = \left(1-(1.079)^{-1/12}\right)12$$

$$= 7.579\%$$

$$i(0.5) = 0.5\left[(177.9\%)^{1/0.5} - 1\right]$$

$$= 8.212\%$$

10) $PV(t \geq 10) = e^{-\int_0^5 0.08} \left(e^{-\int_5^{10} 0.06}\right) e^{-\int_{10}^{t} 0.04}$

$= e^{-[0.8t]_0^5} \; e^{-[0.06t]_5^{10}} \; e^{-[0.04]_{10}^{t}}$

$= e^{-[0.04][0.06-0.3][0.4t-0.4]}$

$= e^{-[0.4+0.3+0.04t-0.4]}$

$= e^{-[0.04t+0.3]} \quad t \geq 10$

11) a) $n = \frac{9}{12}$ years $\quad AV = 50{,}000 \quad d = 18\%$

$PV = AV \times DF = 50000\left(1 - \frac{3}{4} \times 0.18\right)$

$= 43250$

b) $\delta = 6\%$.

$\delta = \ln(1+i)$

$e^{\delta} = 1+i$

$e^{0.06} - 1 = i$

$i = 6.1836547\%$

$d^{(12)} = \left(1 - (1.061836547)^{-1/12}\right) 12$

$= 5.985025\%$

$d^{(4)} = 6\%$

$d = 1 - \left(1 - \frac{6\%}{4}\right)^4$

$= 5.863449$

$i = (1-d)^{-1} - 1$

$= 6.2319375\%$

13) Let AV be $= 2a \quad PV = a \quad i = 10\%$

$2a = a(1+i)^n$

Taking log on both sides

$\log 2 = n \ln 1.1$

$n = \frac{\log 2}{\ln 1.1}$

$n = 7.272$

ii) $AV = 2a \qquad PV = a \qquad d^{(12)} = 10\%$

$$2\alpha = \alpha\left(1 - \frac{d^{(12)}}{12}\right)^{-12n}$$

Taking log on both sides

$$\ln 2 = -12n \ln 0.99167$$

$n = 6.905$ years