Numarical Method of algebra.

Que 1) $r = 12.5$ cm

$r_1 = 12.65$ cm

of circular plate.

Area (Acutal length) $= \pi r^2$

$= (12.5)^2 \pi$

$= 156.25 \pi$ cm^2

Area of Circular plate (measured length) $= \pi r'^2$

$= \pi (12.65)^2$

$= 160.0225 \pi$ cm^2

ii) Absolute error = measured value − Actual value.

$= 160.0225 \pi - 156.25 \pi$

$= 3.5225 \pi$

Absolute error = 11.066

iii) Relative Error $= \frac{\text{Absolute Error}}{\text{Actual value}} = \frac{3.5225 \pi}{156.25 \pi}$

Relative Error = 0.0225

iii) Percentage error = Relative error $\times 100$

$= 0.0225 \times 100$

Percentage error = 2.25 %.

Que 2)

x	$f(x) = \log x$
4.0	0.60206
4.5	0.6532125
5.5	0.7403627
6.0	0.7781513

at $x=5$

i) $y = y_1 + (x-x_1)\dfrac{(y_2-y_1)}{(x_2-x_1)}$

$$y = 0.60206 + (x-4)\left(\frac{0.7781513 - 0.60206}{6-4}\right)$$

$$y = 0.60206 + (5-4)\left(\frac{0.1760913}{2}\right)$$

$$y = 0.60206 + 1\ (0.08804565)$$

$$y = 0.69010505$$

$$\log(5) = 0.69010565$$

2) $x_1 = 4.5 \quad y_1 = 0.6532125$

$x_2 = 5.5 \quad y_2 = 0.7403627$

$\boxed{x=5}$

$$y = y_1 + (x-x_1)\frac{(y_2-y_1)}{(x_2-x_1)}$$

$$= 0.6532125 + (5-4.5)\left(\frac{0.7403627 - 0.6532125}{5.5-5}\right)$$

$$= 0.6532125 + 0.5\left(\frac{0.0871502}{0.5}\right)$$

$$y = 0.7403627$$

Que3) $y = x^4 - x - 10$

$a = 1.5 \quad b = 2$

$$x_1 = \frac{a+b}{2} = \frac{1.5+2}{2} = 1.75$$

$$y_1 = (1.75)^4 - (1.75) - 10$$

$$y_1 = -2.371094$$

$$x_2 = \frac{x_1+b}{2} = \frac{1.75+2}{2} = 1.875$$

$y_2 = (1.875)^4 - (1.875) - 10$

$y_2 = 0.48462$

$x_3 = \frac{x_1 + x_2}{2} = \frac{1.75 + 1.375}{2}$

$x_3 = 1.8125$

$y_3 = (1.8125)4 - (1.8125) - 10$

$y_3 = -1.0202484$

$x_4 = \frac{x_1 + x_3}{2} = \frac{1.75 - 1.8125}{2} = 1.78125$

$y_4 = (1.78125)^4 - (1.78125) - 10$

$= -1.7142629$

$x_5 = \frac{x_1 + x_4}{2} = \frac{1.75 + 1.78125}{2} = 1.765625$

$y_5 = (1.765625)4 - (1.765625) - 10$

$= -2.0472449$

Que4) $f(x) = x^3 - x - 1$

$y = x^3 - x - 1 = 0$

$x_1 = x - \frac{f(x)}{f'(x)}$

$x_1 = x - \frac{(x^3 - x - 1)}{3x^2 - 1}$

Let $x = 1$

$x_1 = 1 - \frac{(1 - (-1)}{3(1) - 1}$

$x_1 = 1 - \frac{1}{2}$

$x_1 = 0.5$

$y_1 = 0.5^3 - 0.5 - 1$

$y_1 = -1.375$

$y_1' = 3(0.5)^2 - 0.5$

$= 3(0.25) - 0.5$

$= 0.75 - 0.5$

$= 0.25$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.5 - \frac{(-1.375)}{0.25}$$

$= 0.5 + 5.5$

$x_2 = 6$

$f(x_2) = 6^3 - 6 - 1 = 209$

$f'(x_2) = 3(6)^2 - 1 = 107$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 6 - \frac{209}{107}$$

$x_3 = 4.04673$

$y_3 = 61.22261$

$y_3 = (4.04673)^3 - (4.04673) - 1$

$= 66.2693463 - (4.04673) - 1$

$y_3 = 61.22261$

Que5) $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$|A| = 1 \begin{vmatrix} 0 & 6 \\ 1 & -1 \end{vmatrix} - 4 \begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix}$

$= 1(-6) - 4(1-2)$

$= -6 - 4(-1)$

$= -6 + 4 = -2$

AS $|A| \neq 0$

$$\text{adj } A = \begin{bmatrix} -6 & +4 & 4 \\ +1 & -1 & -1 \\ -6 & +2 & 4 \end{bmatrix}^T$$

$$\text{ddj } A = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$= \frac{-1}{2} \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & 1/2 & -2 \end{bmatrix}$$

Que7) $\vec{a} = 8i - 9j$

$\vec{b} = 7i - 9j$

$\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$

$56 = 17 \times 32 \times \cos\theta$

$\frac{56}{17 \times 32} = \cos\theta$

$\cos\theta = 0.102941$

$\theta = \cos^{-1}(0.102941)$

$\theta = 84.09143$ ..... Required angle

Ques) $A = 7.5$ $B = 25$ $\theta = 30$

$R \rightarrow$ Resultant vector

$R = A^2 + B^2 + 2AB\cos\theta$

$= (7.5)^2 + (2.5)^2 + 2(7.5)(2.5)\cos 30$

$R^2 = 5625 + 625 + 3750 \times \frac{\sqrt{3}}{2}$

$R^2 = 6250 + 3427.5953$

$R^2 = 9497.5953$

$R = 97.46$ m

Que 9) 101, 104, 107, ... 998

$a = 101$, $d = 3$, $n = ?$ $a_n = 998$

$a_n = a + (n-1)d$

$\frac{998 - 101}{3} = n - 1$

$299 + 1 = n$

$n = 300$

$S_n = \frac{n}{2}[a + a_n] = 150\,(101 + 998)$

$S_n = 164850$

Que 10) $a_6 = 32$, $a_8 = 128$] G.progression.

$ar^5 = 32$, $ar^7 = 128$

$\frac{ar^7}{ar^5} = \frac{128}{32}$ $r^2 = 4$

$r = 2$

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Que 11) $(4+3x)^5$ | $(1+x)^p = 1 + px + \frac{p(p-1)}{2!}x^2 + \frac{p(p-1)(p-2)}{3!}$

$$(4+3x)^5 = 4 + 5(3x) + \frac{5(5-1)}{2!}(3x)^2 + \frac{5(5-1)(5-2)(3x)^3}{3!} + \frac{5(5-1)(5-2)(5-3)(3x)^4}{4!} + \frac{5(5-1)(5-2)(3x)^5}{5!}$$

$$= 4 + 5x + 90x^2 + 270x^3 + 405x^4 + 243x^5$$

$$= 4 + x(5 + 90x + 270x^2 + 405x^3 + 243x^4)$$

Que 12) $$A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$(2-\lambda)\begin{vmatrix} -5-\lambda & 0 \\ 0 & 3-\lambda \end{vmatrix} + 3\begin{vmatrix} 2 & 0 \\ 0 & 3-\lambda \end{vmatrix} = 0$$

$$-(2-\lambda)(5+\lambda)(3-\lambda) + 3(6-2\lambda) = 0$$

$$(\lambda-2)(5-2\lambda-\lambda^2) + 18 - 6\lambda = 0$$

$$-\lambda^3 - 13\lambda - 12 = 0$$

$$15\lambda - 2\lambda^2 - \lambda^3 - 30 + 4\lambda + 2\lambda^2 + 13 - 6\lambda = 0$$

$$-\lambda^3 + 13\lambda - 12 = 0$$

$$\lambda^3 - 13\lambda + 12 = 0$$

$$\lambda = 1$$

$$\lambda - 1 \;\overline{)\; \lambda^3 - 13\lambda + 12}$$

$\lambda^2 + \lambda - 12$

$-\lambda^2$

$\lambda^2 - 13\lambda + 12$

$-\lambda^2 + \lambda$

$-12\lambda + 12$

$12\lambda + 12$

0

$(\lambda-1)(\lambda^2+\lambda-12)$

$(\lambda-1)(\lambda^2+4\lambda-3\lambda-12)$

$(\lambda-1)[\lambda(\lambda+4)+3(\lambda+4)]$

$(\lambda-1)(\lambda-3)(\lambda+4)=0$

$\lambda = 1, 3, -4$

↳ eigen value.

Eigen value

$$\begin{bmatrix} 2-1 & -3 & 0 \\ 2 & -5-1 & 0 \\ 0 & 0 & 3-1 \end{bmatrix} \begin{bmatrix} x \\ y \\ -z \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & -3 & 0 \\ 2 & -6 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} x-3y \\ 2x-6y \\ 2z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x - 3y = 0$ $\quad$ $2x - 6y = 0$ $\quad$ $2z = 0$

$x = 3y$ $\quad$ $y = \frac{x}{3}$ $\quad$ $z = 0$

At eigen value

$$\begin{bmatrix} -2-3 & -3 & 0 \\ 2 & -5-3 & 0 \\ 0 & 0 & 3-3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -5 & -3 & 0 \\ 2 & -8 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -5x-3y \\ 2x-8y \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-5x-3y=0$ $\qquad$ $2x-8y=0$

$5x=-3y$ $\qquad$ $2x=8y$

$x=\frac{-3}{5}y$ $\qquad$ $y=\frac{x}{4}$ $\qquad$ $z=0$

At eigen value, -4

$$\begin{bmatrix} 2+4 & -3 & 0 \\ 2 & -1+4 & 0 \\ 0 & 0 & 3+4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -3 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$6x-3y=0$ $\qquad$ $2x-y=0$ $\qquad$ $7x=0$

$6x=3y$ $\qquad$ $y=2x$ $\qquad$ $x=0$

$x=y$

Q13) $f(x) = x^3 - 7x^2 + 8x - 3$

$f'(5) = 5^3 - 7\times5^2 + 8\times5 - 3 = 13$

$f'(x) = 3x^2 - 14x + 8$

$f'(5) = 3\times5^2 - 14(5) + 8 = 13$

$x_0 = 5$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 5 - \left(\frac{13}{13}\right) = \boxed{5 + 6 = 6}$

$f(x_1) = 6^3 - 7\times6^2 + 8\times6 - 3 = 9$

$f'(x_1) = 3\times6^2 - 14\times6 + 8 = 32$

$x_2 = 6 - \frac{9}{32} = \frac{192 - 9}{32} = \frac{183}{32}$

$x_2 = 5.7135$

Q14) $(1 + x + x^2)^4$

$${}^4C_0(1-x)^4 + {}^4C_1(1-x)^3x^2 + {}^4C_2(1-x)^2(x^2)^2 + {}^4C_3(1-x)(x^2)^3 + {}^4C_4(x^3)^4$$

$$(1-x)^4 + 4(1-x)^3x^2 + 6(1-x)^2x^4 + 4(1-x)x^6 + x^{12}$$

$$(1-x)^4 + 4x^2(1-x)^3 + 6x^4(1-x) + 4x^6(1-x) + x^{12}$$

Q15) $e^{-x} = 3\log x$

x	
0.5	
1.5	

$x_1 = \frac{0.5 + 1.5}{2} = 1$

$x_2 = \frac{x_1 + 5}{2} = 1.25$

$x_2 = 1.25 \quad y_2 = -0.00115$

$x_3 = \frac{x_1 + x_2}{2} = \frac{1.25 + 1}{2}$ $\quad x_3 = 1.125$

$y_3 = 0.063$

$x_4 = \frac{x_3 + x_2}{2} = \frac{1.125 + 1.25}{2}$ $\quad x_4 = 1.187$

$y_4 = 0.014$

$x_5 = \frac{x_4 + x_3}{2} = \frac{1.187 + 1.125}{2} = 1.156$

$y_5 = 0.125869$