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Roll. no: 27.
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16/4/2022.

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Q.1. Ar. of circular plate = πr^2. sq. cm.

$A(r) = 12.65\,cm$, $A'(r) = 12.5\,cm$.

$$\frac{dA}{dr} = 2\pi r.$$

$$dA = 2\pi r \cdot dr$$

$dr = A^2(r) - A'(r)$

$= 12.65 - 12.5$

$= 0.15$

$dA = 2\pi \times 12.5 \times 0.15$.

$dA = 3.75\pi\,cm^2$. (Apporximate)

Change in area (Actual) $= A(12.65) - A(12.5)$

$= (\pi \times 12.65 \times 12.65) - (\pi \times 12.5 \times 12.5)$

$= (160.0225\pi - 156.25\pi)\,cm^2$.

$= 3.7725\pi\,cm^2$.

i. Absolute error $= 3.7725\pi - 3.75\pi$.

$= 0.0225\pi\,cm^2$. //

ii. Relative error $= \frac{\text{Actual} - \text{Appoximate}}{\text{Actual}}$.

$$= \frac{3.7725\pi - 3.75\pi}{3.7725\pi}.$$

$$= \frac{0.0225\pi}{3.7725\pi}$$

$= 0.006\,cm^2$. //

iii. Percentage error = Relative error x 100.
= 0.006 x 100.
= 0.6 % //

Q2 a)

x	f(x)
4.0	0.60206
6.0	0.7781513.

$$\log(5) = \left[\frac{5-6}{4-6} \times 0.60206\right] + \left[\frac{5-4}{6-4} \times 0.7781513\right]$$

$$= 0.30103 + 0.38907.$$

$$\log(5) = 0.6901 //$$

b

x	fx
4.5	0.6532125
5.5	0.7403627.

$$\log(5) = \left[\frac{5-5.5}{4.5-5.5} (0.6532125)\right] + \left[\frac{5-4.5}{4.5-5.5} (0.7403627)\right]$$

$$= 0.32660625 + 0.37018135.$$

$$= 0.6967 //$$

3. $x^4 - x - 10 = 0$.

$\Rightarrow f(x) = x^4 - x - 10 = 0$.

x	y
1.5	-6.4375
1.6	-5.0464
1.7	-3.3479
1.8	-1.3024
1.9	1.1321
2.0	4

$x_1 = 1.8$ $\quad y_1 = -1.3024$

$x_2 = 1.9$ $\quad y_2 = 1.1321$

$$x_3 = \frac{1.8 + 1.9}{2} = 1.85$$

$x_3 = 1.85$ //

$y_3 = (1.85)^4 - 1.85 - 10$

$y_3 = -0.13649375$

$x_3 = 1.85$ $\quad y_3 = -0.13649375$

$x_4 = 1.90$ $\quad y_2 = 1.1321$

$x_4 = 1.875$

$y_4 = 0.484619140625$ //

$x_4 = 1.875$ $\quad y_4 = 0.4846191406$

$x_3 = 1.85$ $\quad y_3 = -0.13649375$

$x_5 = 1.8625$

$y_5 = 0.1708105713$ //

~~$x_4 = 1.875$~~ $\quad y_4 = 0.4846191406$

$x_5 = 1.8625$ $\quad y_5 = 0.1708105713$.

$x_3 = 1.85$ $\quad y_3 = -0.13649375$

$x_5 = 1.8625$ $\quad y_5 = 0.1708105713$.

$x_6 = 1.85625$

$y_6 = 0.016350831$6 //

$x_3 = 1.85$ $\quad y_3 = -0.13649375$

$x_6 = 1.85625$ $\quad y_6 = 0.0163508316$

$x_7 = 1.853125$

$y_7 = 0.00099387$1 //

4. $f(x) = x^3 - x - 1$

$f'(x) = 3x^2 - 1$

x	$f(x)$
0	-1
1	-1
2	5

$x_0 = 2$

$f(x_0) = 5$

$f'(x_0) = 11$

$x_1 = 2 - \frac{5}{11}$

$x_1 = 1.5454$

$f(x_1) = 1.145755069$

$f'(x_1) = 6.165289252.$

$x_2 = 1.359614916.$

$f(x_2) = 0.153704939$

$f'(x_2) = 4.545658159$

$x_3 = 1.32580135$

$f(x_3) = 0.00462\ 4917$

$f'(x_3) = 4.273247619$

$x_4 = 1.324719049$

$f(x_4) = 0.00000\ 465594$ //

5. $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^2 = A.$

~~$7A = 6$~~ $7A - (1+A)^3$

$= 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}^3$

$= \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} - \left\{ \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \right\}^3$

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$$= \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} //$$

6. $$\begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$$

$$\Rightarrow |A| = [(1)(0-6)] - [(-1)(-4-0)] + [(2)(4-0)]$$

$$= -6 - 4 + 8$$

$$|A| = -2$$

~~$|A| = 2$~~

$$adj.A = \begin{bmatrix} -6 & 4 & 4 \\ 1 & -3 & -2 \\ -6 & 2 & 4 \end{bmatrix}$$

$$A^T = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -3 & 2 \\ 4 & -2 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} . adj.A .$$

$$= \frac{1}{-2} \begin{bmatrix} -6 & 1 & -6 \\ 4 & -3 & 2 \\ 4 & -2 & 4 \end{bmatrix} //$$

7. $A = 8\hat{i} - 9\hat{j}$
$B = 7\hat{i} - 9\hat{j}$.

$$\theta = \cos^{-1}\left[\frac{\vec{A}\cdot\vec{B}}{|\vec{A}|\cdot|\vec{B}|}\right]$$

$$= \cos^{-1}\left[\frac{(8\hat{i}-9\hat{j})\cdot(7\hat{i}-9\hat{j})}{|(8\hat{i}-9\hat{j})|\cdot|(7\hat{i}-9\hat{j})|}\right]$$

$$= \cos^{-1}\left[\frac{(8)(7)+(-9)(-9)}{\sqrt{(8)^2+(-9)^2}\ \sqrt{(7)^2+(-9)^2}}\right]$$

$$= \cos^{-1}\left[\frac{56+81}{\sqrt{145}\ \sqrt{130}}\right]$$

$$\boxed{\theta = 3.7585^\circ}$$ //

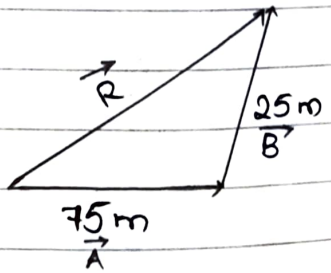
8. R is resultant vector

$$R = \sqrt{A^2 + \vec{B} + 2AB\cos\theta}$$

$$= \sqrt{(75)^2 + (25)^2 + 2\times(75)(25)\cos 30^\circ}$$

$$= \sqrt{9497.595}$$

$$\boxed{R = 104.1287\ m}$$ //



9. The lowest 3 digit number is 101 & highest 3 digit no. is 998

AP is 101, 104, 107 ..... 998.

$a = 101$

$d = 3$

$a_n = 998$

$a_n = a + (n-1)d$

$998 = 101 + (n-1)3$

$998/3 = 101 + (n-1)$

$\frac{998}{3} - 101 = (n-1)$

$n-1 = \frac{897}{3}$

$n-1 = 299$

$\boxed{n = 300.}$

$S = \frac{n}{2}(a_1 + a_n)$

$= \frac{300}{2}(101 + 998)$

$\boxed{S = 1,64,850}$

∴ Sum of all 3 digit no. that leaves a remainder 2 when divided by 3 is 1,64,850.

10. Let 'a' be first term.
Let common ratio of GP be 'r'.

$a_n = ar^{n-1}$

$a_6 = 32$

$ar^5 = 32$. —— (1)

$a_8 = 128$

$ar^7 = 128$ —— (2)

÷ (2) by (1)

$\frac{ar^7}{ar^5} = \frac{128}{32}$

$r^2 = 4$

$\boxed{r = 2}$ //

11. $(4+3x)^5$

$= 4^5 + 5(4)^4(3x) + 10(4)^3(3x)^2 + 10(4)^2(3x)^3 + 5(4)(3x)^4 + 3x^5$

$= 1024 + 3840x + 1920x^2 + 480x^3 + 60x^4 + 3x^5$

$= 3x^5 + 60x^4 + 480x^3 + 1920x^2 + 3840x + 1024$ //

12. $$A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$A - \lambda I$$

$$= \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$$

$$= 3-\lambda \begin{bmatrix} 2-\lambda & -3 \\ 2 & -5-\lambda \end{bmatrix}$$

$$= (3-\lambda)\left[(2-\lambda)(-5-\lambda)+6\right]$$

$$= (3-\lambda)(-10+5\lambda-2\lambda+\lambda^2+6)$$

$$= (3-\lambda)(\lambda^2+3\lambda-4)$$

$$= -(\lambda-3)(\lambda+4)(\lambda-1)$$ //

13. $f(x) = x^3 - 7x^2 + 8x - 3$

$\Rightarrow f'(x) = 3x^2 - 14x + 8$

$x_0 = 5$

$f(x_0) = -13$

$f'(x_0) = 55$

$x_1 = 5 - \left(\frac{-13}{55}\right)$

$x_1 = 5.236$ //

$f(x_1) = -9.473$

$f'(x_1) = 16.943088$

$x_2 = 5.236 - \left(\frac{-9.473}{16.943088}\right)$

$x_2 = 5.7951$ //

14. $(1-x+x^2)^4$

Let $1-x=y$.

$(y+x^2)^4$

$= y^4 + 4y^3x^2 + 6y^2x^4 + 4yx^6 + x^8$

$= (1-x)^4 + 4x^2(1-x)^3 + 6x^4(1-x)^2 + 4x^6(1-x) + x^8$

$= 1-4x+10x^2-16x^3+19x^4-16x^5+10x^6-4x^7+x^8$ //

15. $e^{-x} = 3\log(x)$

$\Rightarrow e^{-x} - 3\log(x) = 0$

x	$f(x)$
1	0.36788
2	~~1.9441~~ -0.76775

$x_1 = 1 \qquad f(x_1) = 0.36788$

$x_2 = 2 \qquad f(x_2) = -0.76775$

$x_3 = \frac{1+2}{2}$

$x_3 = 1.5$

$f(x_3) = -0.30514$ //

$x_1 = 1$ $\quad f(x_1) = 0.36788$

$x_3 = 1.5$ $\quad f(x_3) = -0.30514$

$$x_4 = \frac{1+1.5}{2}$$

$x_4 = 1.25$ //

$f(x_4) = -0.00423$ //